Name:..................................... Dec. 13, 2004

(EECE441) ELECTROMAGNETICS

CLOSED BOOK (1 ½ HRS)

Programmable Calculators are not allowed
Provide your answers on the computer's card only
Return the computer's card attached to the question sheet
Mark with a pencil your name and your ID-No
Use pencil for marking your answers
When using eraser, be sure that you have erased well

1. The B-field over the xy plane is given by $B = \sin(\pi x/2)\sin(\pi y/2)\, a_z$. (T)
Calculate the flux passing through the square $0<(x,y)<2$ m.

   a. $8/\pi^2$ Wb
   b. $16/\pi^2$ Wb
   c. $12/\pi^2$ Wb
   d. $10/\pi^2$ Wb
   e. None of the above

2. Given that $H_1 = -2a_x + 6a_y + 4a_z$ A/m in region $y - x - 2 \leq 0$ where $\mu_1 = 5\,\mu_o$, calculate H_{1n}

   a. $2a_x - 2a_y + 4\, a_z$
   b. $4a_x - 4a_y$
   c. $2a_x + 2a_y$
   d. $-4a_x + 4a_y$
   e. None of the above

3. In problem 2, calculate H_{2t} in the region $y - x - 2 \geq 0$ where $\mu_2 = 2\,\mu_o$

   a. $2a_x + 2a_y + 4\, a_z$
   b. $2a_x - 2a_y + 4\, a_z$
   c. $-4a_x + 4a_y$
   d. $4a_x - 4a_y$
   e. None of the above

4. Consider 2 infinitely long parallel wires, each carrying a current I in the z-direction, one passing through the point (x=0, y=-a) and the other through (x=0, y=a). Find B at a point P(x=b, y=0).

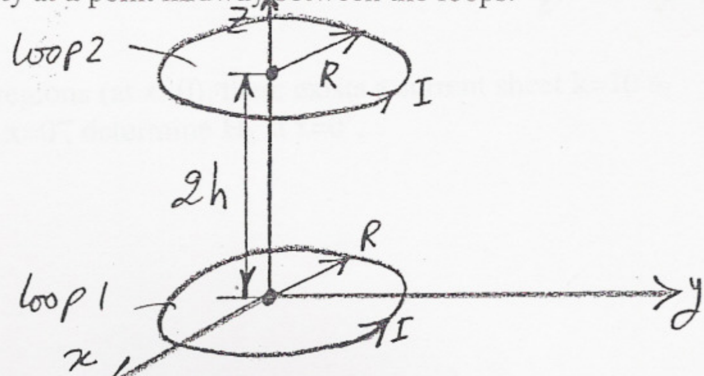
   a. 0
   b. $a_y \{\mu_0 I b/(\pi[a^2-b^2])\}$
   c. $a_y \{\mu_0 I b/(\pi[a^2+b^2])\}$
   d. $-a_x \{\mu_0 I b/(\pi[b^2-a^2])\}$
   e. None of the above

5. For the current element Idx (a_x+a_y) Ampere situated at the point (1,-2,2), find the magnetic flux density at the point (2,-1,3).

a. $\{\mu_0 I\, dx/(12\pi\sqrt{3})\}(a_x - a_y - a_z)$
b. $\{\mu_0 I\, dx/(12\pi\sqrt{6})\}(a_x - a_y)$
c. $\{\mu_0 I\, dx/(12\pi\sqrt{3})\}(a_x - a_y + a_z)$
d. $\{\mu_0 I\, dx/(12\pi\sqrt{3})\}(a_x - a_y)$
e. None of the above

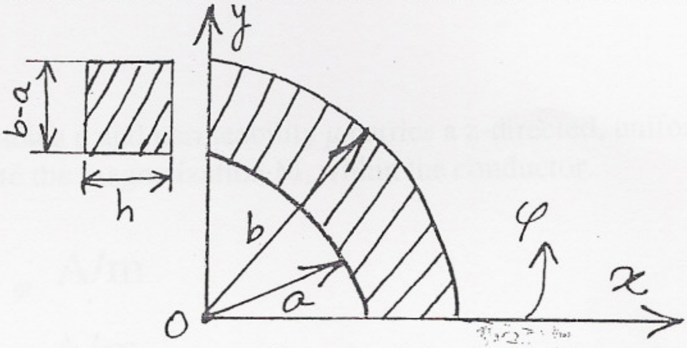
6. For the loops shown, R=0.1m and I= 100A, and the separation between the loops is 0.2m. Calculate the flux density at a point midway between the loops.

a. 0.222 mT $(-a_z)$
b. 0.444 mT a_z
c. 1.2 mT a_z
d. 0.745 mT $(-a_z)$
e. None of the above



7. A metal bar of conductivity σ is bent to form a flat 90° sector of inner radius a, outer radius b, and thickness h as shown. Calculate the resistance of the bar between the curved sides .

a. $4h/[\sigma\pi(b^2-a^2)]$
b. $2\ln(b/a)/(\sigma\pi h)$
c. $4\sigma\pi h/[(b^2-a^2)]$
d. $\ln(b/a)/(2\sigma\pi h)$
e. None of the above



8. Determine the mutual inductance between a very long, straight wire and a conducting equilateral triangular loop as shown in the figure.

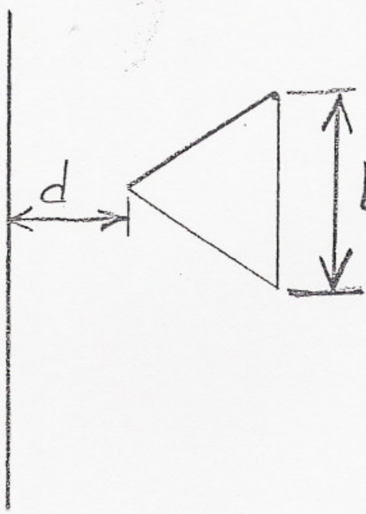
$a.\ \frac{\mu_o}{\pi}\left[\frac{b}{2} - \frac{d}{\sqrt{3}}\ln\left(1 + \frac{\sqrt{3}\,b}{2d}\right)\right]$

$b.\ \frac{\mu_o}{\pi}\left[\frac{b}{2} - \frac{d}{\sqrt{2}}\ln\left(1 + \frac{\sqrt{2}\,b}{2d}\right)\right]$

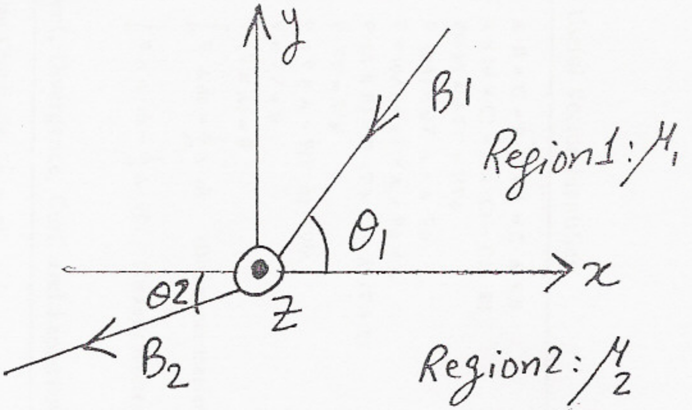
$c.\ \frac{\mu_o}{\pi}\left[\frac{b}{2} + \frac{d}{\sqrt{3}}\ln\left(\frac{\sqrt{3}\,b}{2d}\right)\right]$

$d.\ \frac{\mu_o}{\pi}\left[\frac{b}{2} + \frac{d}{\sqrt{2}}\ln\left(\frac{\sqrt{2}\,b}{2d}\right)\right]$

e. None of the above



9. At the interface shown, we have a current sheet k=-8 a_z. Given H_1 =12 a_x +9 a_y , μ_1= $3\mu_o$, and μ_2= 9 μ_o, determine H_2.



a. H_2 =(2/3) H_1
b. H_2 =(1/4) H_1
c. H_2 =(2/5) H_1
d. H_2 =(1/3) H_1
e. None of the above

10. At the interface between two regions (at x=0), there exists a current sheet k=10 a_z A/m. Given H_1 =12 a_y A/m at $x=0^-$, determine H_2 at $x=0^+$.

a. 22 a_x A/m
b. 12 a_y A/m
c. 22 a_y A/m
d. (10 a_z -12 a_y) A/m
e. None of the above

11. A cylindrical conductor of radius a and permeability μ carries a z-directed, uniformly distributed current I. Evaluate the magnetization M, within the conductor.

a. $\frac{I}{2\pi a^2}\left(\frac{\mu}{\mu_o} - 1\right) r\, a_\varphi$ A/m

b. $\frac{I}{2\pi a^2}\left(\frac{\mu}{\mu_o} - 1\right) r\, a_z$ A/m

c. $\frac{I}{2\pi a^2}\frac{\mu}{\mu_o} r\, a_\varphi$ A/m

d. $\frac{I}{2\pi a^2}\frac{\mu}{\mu_o} r\, a_z$ A/m

e. None of the above

Some Useful Vector Identities

$\mathbf{A}\cdot\mathbf{B}\times\mathbf{C} = \mathbf{B}\cdot\mathbf{C}\times\mathbf{A} = \mathbf{C}\cdot\mathbf{A}\times\mathbf{B}$

$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$

$\nabla(\psi V) = \psi\nabla V + V\nabla\psi$

$\nabla\cdot(\psi\mathbf{A}) = \psi\nabla\cdot\mathbf{A} + \mathbf{A}\cdot\nabla\psi$

$\nabla\times(\psi\mathbf{A}) = \psi\nabla\times\mathbf{A} + \nabla\psi\times\mathbf{A}$

$\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$

$\nabla\cdot\nabla V = \nabla^2 V$

$\nabla\times\nabla\times\mathbf{A} = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$

$\nabla\times\nabla V = 0$

$\nabla\cdot(\nabla\times\mathbf{A}) = 0$

$\int_V \nabla\cdot\mathbf{A}\,dv = \oint_S \mathbf{A}\cdot d\mathbf{s}$ (Divergence theorem)

$\int_S \nabla\times\mathbf{A}\cdot d\mathbf{s} = \oint_C \mathbf{A}\cdot d\boldsymbol{\ell}$ (Stokes's theorem)

Gradient, Divergence, Curl, and Laplacian Operations

Cartesian Coordinates (x, y, z)

$$\nabla V = \mathbf{a}_x\frac{\partial V}{\partial x} + \mathbf{a}_y\frac{\partial V}{\partial y} + \mathbf{a}_z\frac{\partial V}{\partial z}$$

$$\nabla\cdot\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \mathbf{a}_x\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \mathbf{a}_y\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \mathbf{a}_z\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

C. From cylindrical to Cartesian Coord.

$x = \rho\cos\varphi$, $Y = \rho\sin\varphi$, $Z = Z$

D. From spherical to Cartesian Coord.

$X = R\sin\theta\cos\varphi$, $Y = R\sin\theta\sin\varphi$, $Z = R\cos\theta$

Cylindrical Coordinates (r, ϕ, z)

$$\nabla V = \mathbf{a}_r\frac{\partial V}{\partial r} + \mathbf{a}_\phi\frac{\partial V}{r\,\partial\phi} + \mathbf{a}_z\frac{\partial V}{\partial z}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{\partial A_\phi}{r\,\partial\phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \frac{1}{r}\begin{vmatrix} \mathbf{a}_r & \mathbf{a}_\phi r & \mathbf{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \mathbf{a}_r\left(\frac{\partial A_z}{r\,\partial\phi} - \frac{\partial A_\phi}{\partial z}\right) + \mathbf{a}_\phi\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \mathbf{a}_z\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial}{\partial}\right.$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial\phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical Coordinates (R, θ, ϕ)

$$\nabla V = \mathbf{a}_R\frac{\partial V}{\partial R} + \mathbf{a}_\theta\frac{\partial V}{R\,\partial\theta} + \mathbf{a}_\phi\frac{1}{R\sin\theta}\frac{\partial V}{\partial\phi}$$

$$\nabla\cdot\mathbf{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\nabla\times\mathbf{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \mathbf{a}_R & \mathbf{a}_\theta R & \mathbf{a}_\phi R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix} = \mathbf{a}_R\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right]$$

$$+ \mathbf{a}_\theta\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial\phi} - \frac{\partial}{\partial R}(RA_\phi)\right]$$

$$+ \mathbf{a}_\phi\frac{1}{R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial\theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}$$

A. $d\ell = dr\,a_r + r\,d\theta\,a_\theta + r\sin\theta\,d\varphi\,a_\varphi$

$ds = r^2\sin\theta\,d\theta\,d\varphi\,a_r$

$r\sin\theta\,dr\,d\varphi\,a_\theta$

$r\,dr\,d\theta\,a_\varphi$

$dv = r^2\sin\theta\,dr\,d\theta\,d\varphi$

B. $d\ell = d\rho\,a_\rho + \rho\,d\varphi\,a_\varphi + dz\,a_z$

$ds = \rho\,d\varphi\,dz\,a_\rho$

$d\rho\,dz\,a_\varphi$

$\rho\,d\varphi\,d\rho\,a_z$

$dv : \rho\,d\rho\,d\varphi\,dz$