**Name:.......................................** **Jan 6, 2001**

**VERSION A**

**(EE078) Field Theory**

**CLOSED BOOK (1 ½ HRS)**

Programmable Calculators are not allowed
Provide your answers on the computer's card only
Return the computer's card attached to the question sheet
Mark your ID-No on the computer card from left to right
Use pencil for marking your answers and your ID-No
When using eraser, be sure that you have erased well
Write with a pen your name followed by your ID-No

**!!! PENALTY 5 TO 1 !!!**

1. What is the version of your question sheet. (This question is not graded, it is used to identify your copy)

a. Version A
b. Version B
c. Version C
d. Version D

*{In all problems below take $\mu_o = 4\pi\ 10^{-7}$}*

2. Given the magnetic vector potential $A = -r^2/4\ a_z$ Wb/m, calculate the total magnetic flux crossing the surface $\varphi = \pi/2$, $1 \leq r \leq m$, $0 \leq z \leq 5$ m.

   a. 15/4 Wb
   b. 25/4 Wb
   c. 12/4 Wb
   d. 15/2 Wb
   e. None of the above

3. Identify the configuration in Fig. 1 that is not a correct representation of I and H.

   a. 1
   b. 2
   c. 3
   d. 4
   e. 5

4. For the currents and closed paths of Fig. 2, calculate the value of $\oint_L H.dl$

   a. 1
   b. 2
   c. 3
   d. 4
   e. None of the above

5. Region $0 \leq z \leq 2$ m is occupied by an infinite slab of permeable material ($\mu_r = 2.5$). If $B = 10\ y\ a_x - 5\ x\ a_y$ mWb/m$^2$ within the slab, determine M.

   a. $-4.95y\ a_x - 7.163\ a_y$ kA/m
   b. $2.38y\ a_x - 4.75\ x\ a_y$ kA/m
   c. $4.77y\ a_x - 2.38\ x\ a_y$ kA/m
   d. $2.38x\ a_x - 4.77\ y\ a_y$ kA/m
   e. None of the above

6. The xy-plane serves as the interface between two different media. Medium 1 ($z<0$) is filled with a material whose $\mu_r = 6$, and medium 2 ($z>0$) is filled with a material whose $\mu_r = 4$. If the interface carries current $(1/\mu_o)\, a_y$ mA/m, and $B_2 = 5a_x + 8a_z$ mWb/m$^2$, find $H_1$.

   a. $(1/\mu_o)\,(2.25\ a_x + 4.3\ a_z)$ mA/m
   b. $(1/\mu_o)\,(1.5\ a_x + 8\ a_z)$ mA/m
   c. $(1/\mu_o)\,(0.25\ a_x + 1.33\ a_z)$ mA/m
   d. $(1/\mu_o)\,(1.25\ a_x + 2\ a_z)$ mA/m
   e. None of the above

7. Which of these formulas is wrong ?.

   a. $B_{1n} = B_{2n}$
   b. $B_2 = \sqrt{B^2_{2n} + B^2_{2t}}$
   c. $H_1 = H_{1n} + H_{1t}$
   d. $a_{n21} \times (H_1 - H_2) = K$ where $a_{n21}$ is a unit vector normal to the interface and directed from region 2 to region 1. K- is the surface current density.
   e. None of the above

8. Within a cylindrical conductor of radius a, the current density exponentially decreases with the radius, according to $J = A\, e^{-kr}\, a_z$, where A and k are positive constants. Determine the magnetic field intensity for $r > a$.

   a. $(A/k^2 r)\,[1-(1+ka)e^{-ka}]$
   b. $(A/k^2 r)\,[1-(1+kr)e^{-kr}]$
   c. $(A/k^2 a)\,[1-(1+kr)e^{-ka}]$
   d. $(A/k^2 a)\,[1-(1+ka)e^{-kr}]$
   e. None of the above

9. Two infinitely long wires, separated by a distance 5l, carry currents I in opposite directions, as shown in Fig. 3. Find the total magnetic field intensity at the point P. shown.

   a. $(I/8\pi l)\, a_x$
   b. $(5\, I/8\pi l)\, a_y$
   c. $(5\, I/8\pi l)\, a_x$
   d. $(I/2\pi l)\, -a_x$
   e. None of the above

10. A cylindrical conductor carries a current that produces an $H = 3r\, a_\varphi$ (A/m). Determine the current density within the conductor.

a. $6\, a_\varphi$ A/m$^2$
b. $6\, a_z$ A/m$^2$
c. $6r\, a_z$ A/m$^2$
d. $3\, a_z$ A/m$^2$
e. None of the above

11. Refer to Fig. 4. Given $B_2 = 1.2\, a_x + 0.8\, a_y$, $\mu_1 = \mu_o$ and $\mu_2 = 15\, \mu_o$, determine $B_1$

a. $1.2\, a_x + 0.8\, a_y$
b. $1/15\, (1.2\, a_x + 0.8\, a_y)$
c. $(0.8/15)\, a_x + 1.2\, a_y$
d. $(1.2/15)\, a_x + 0.8\, a_y$
e. None of the above

12. Refer to Fig. 5. Which one of these statements is correct.

a. $\theta_4$ is independent of $\mu_2$
b. $\theta_1$ is independent of $\mu_2$
c. $\theta_3$ is independent of $\mu_1$
d. $\theta_2$ is independent of $\mu_2$
e. None of the above

13. For the regions shown in Fig. 6, we have $\mu_1 = 4\, \mu_o$, and $\mu_2 = 6\, \mu_o$. If $B_1 = 2\, a_x + a_y$ T. Find $B_2$.

a. $3\, a_x + 0.75\, a_y - 0.75\, a_z$, T
b. $3\, a_x + 1.25\, a_y$ T
c. $3\, a_x + 1.25\, a_y - 0.25\, a_z$, T
d. $5\, a_x + 1.25\, a_y - 1.25\, a_z$, T
e. None of the above

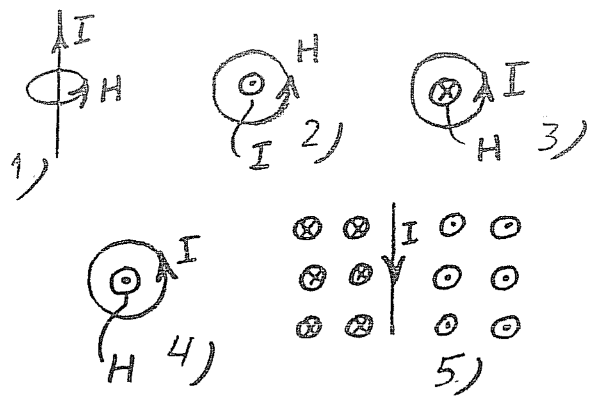



Fig.1

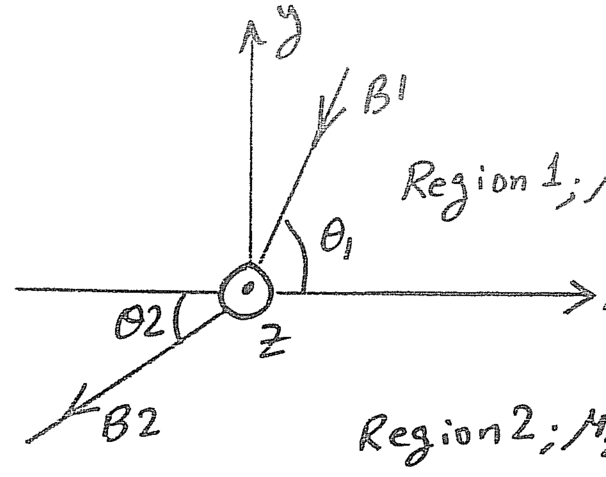



Fig.4

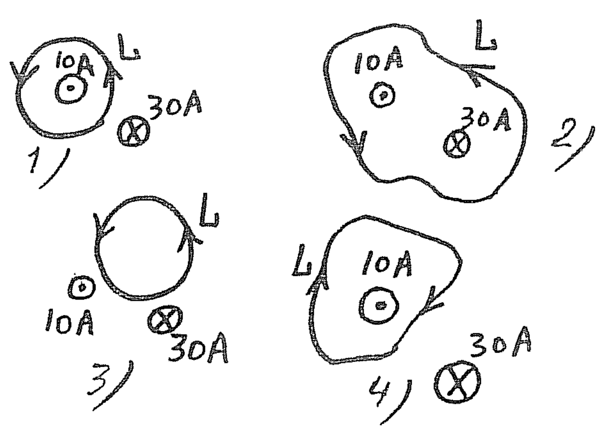



Fig. 2

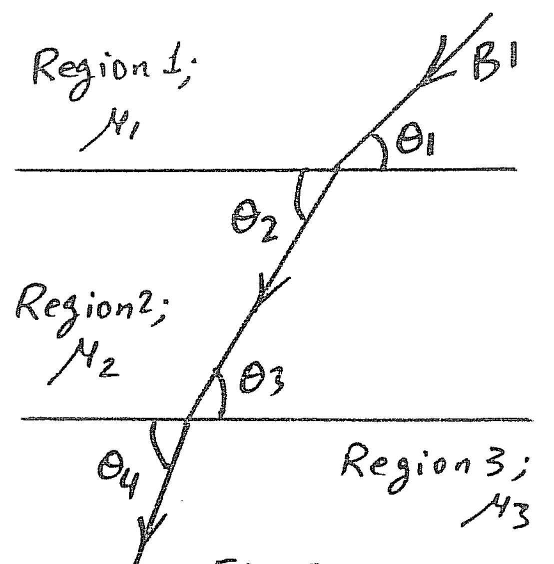



Fig.5

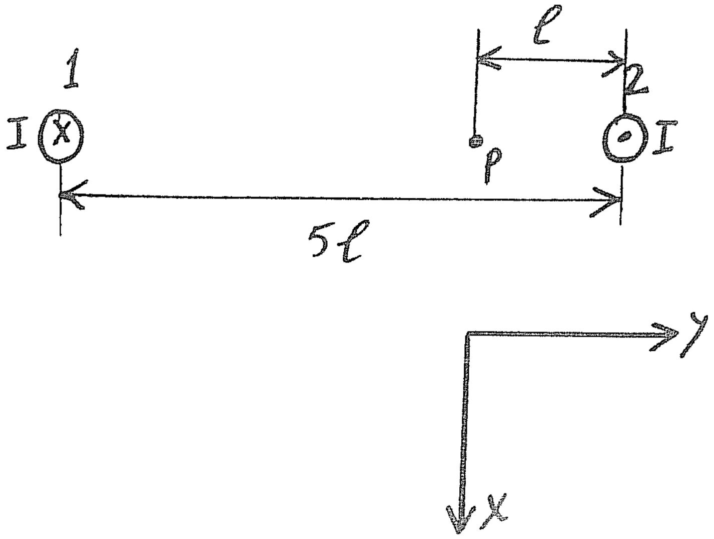



Fig.3

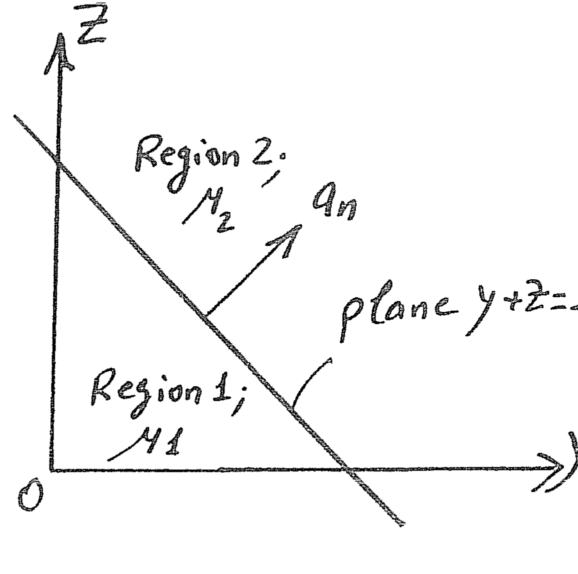



Fig. 6

Prof. R. Ghodsi EE 078 - field Theory

Quiz #8, Jan. 6, 200

#2 $B = \nabla \times A = -\frac{\partial A_z}{\partial r} a_\phi$, $ds = dr\, dz\, a_\phi$

Hence, $\psi = \int B \cdot ds = \frac{1}{2} \int_{z=0}^{5} \int_{r=1}^{2} r\, dr\, dz = \frac{1}{4} r^2 \Big|_1^2 \times 5 =$

9

$= \frac{15}{4}$

#3 $\underline{\underline{c}}$

#4 canceled

#5 $J = \nabla \times H = \nabla \times \frac{B}{\mu_0 \mu_r} = \frac{1}{4\pi \times 10^{-7} \times 2.5} \left( \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \right) \vec{a}_z$

$= \frac{10^6}{\pi} (-5-10) \times 10^{-3} \vec{a}_z = -4.775\, \vec{a}_z$ kA/m$^2$

$\vec{M} = \chi_m H = \chi_m \frac{B}{\mu_0 \mu_r} =$

9

$= \frac{1.5 (10y \vec{a}_x - 5x \vec{a}_y) \cdot 10^{-3}}{4\pi \times 10^{-7} \times 2.5} = 4.775\, y \vec{a}_x - 2.387\, x \vec{a}_y$ k

#6 $B_{1n} = B_{2n} = 8 \vec{a}_z \rightarrow B_z = 8$

but $\vec{H}_2 = \frac{\vec{B}_2}{\mu_2} = \frac{1}{4\mu_0} (5\vec{a}_x + 8\vec{a}_z)$ mA/m

3

& $\vec{H}_1 = \frac{\vec{B}_1}{\mu_1} = \frac{1}{6\mu_0} (B_x \vec{a}_x + B_y \vec{a}_y + B_z \vec{a}_z)$ mA/m

but $(\vec{H}_1 - \vec{H}_2) \times \vec{a}_{n12} = \vec{K}$ (K or J = $\frac{1}{\mu_0} \vec{a}_y$)

so $\frac{1}{6\mu_0} (B_x \vec{a}_x + B_y \vec{a}_y + B_z \vec{a}_z) \times \vec{a}_z = \frac{1}{4\mu_0} (5\vec{a}_x + 8\vec{a}_z) \times$

$+ \frac{1}{\mu_0} \vec{a}_y$

equating components yields: $B_y = 0$, $\frac{-B_x}{6} = -\frac{5}{4} + 1$ or $B_x = 1.5$

therefore $B_1 = 1.5\vec{a}_x + 8\vec{a}_y$ mWb/m$^2$

& $H_1 = \dfrac{B_1}{\mu_1} = \dfrac{1}{\mu_0}(0.25\,\vec{a}_x + 1.33\,\vec{a}_y)$ mA/m

#7 $\underline{\underline{c}}$ because $\vec{H}_1 = \vec{H}_{1n} + \vec{H}_{1t}$ or $H_1 = \sqrt{H_{1n}^2 + H_{1t}^2}$

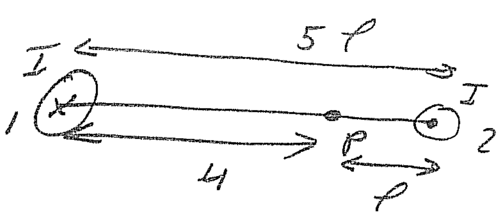
#8. $H_\phi\, 2\pi r = \displaystyle\int_{r=0}^{a}\int_{\phi=0}^{2\pi} A e^{-kr}\, r\, dr\, d\varphi$ {Ampere's law

or $H_\phi = \dfrac{A}{k^2 r}\left[1 - (1+ka)\,e^{-ka}\right]$

#9 By Ampere's law:

$\vec{H}_1 = \dfrac{I}{2\pi(4\ell)}\,\vec{a}_x$ & $H_2 = \dfrac{I}{2\pi\ell}\,\vec{a}_x$

2 $\vec{H}_T = H_1 + H_2 = \dfrac{5I}{8\pi\ell}\,\vec{a}_x$




#10 By Ampere's law:

13 $\vec{J} = \text{curl}\,\vec{H} = +\frac{1}{r}\left[\frac{d}{dr}(3r^2)\right]\vec{a}_z = 6\vec{a}_z$ A/m$^2$

#11. By continuity: $H_{t1} = H_{t2} = \dfrac{B_{t2}}{\mu_2} = \dfrac{1.2}{15\mu_0}$

hence: $\vec{H}_1 = \dfrac{1.2}{15\mu_0}\,\vec{a}_x + H_{n1}\,\vec{a}_y$

10 Also by continuity: $B_{n1} = B_{n2} = 0.8 \Rightarrow B_1 = B_{t1}\,\vec{a}_x + 0.8\,\vec{a}_y$

but $\vec{B}_1 = \mu_1 \vec{H}_1$ so $B_{t1} = \dfrac{1.2}{15}$ & $H_{n1} = \dfrac{0.8}{\mu_0}$

11 #12. a & b are correct

#13. $\vec{a}_n = \frac{1}{\sqrt{2}}\vec{a}_y + \frac{1}{\sqrt{2}}\vec{a}_z$

$B_{n2} = B_{n1} = \vec{B}_1 \cdot \vec{a}_n = \frac{1}{\sqrt{2}}$ T

$\vec{B}_{n2} = \vec{B}_{n1} = \frac{1}{\sqrt{2}}\vec{a}_n = 0.5\vec{a}_y + 0.5\vec{a}_z$ T.

then $\vec{B}_{t_1} = \vec{B}_1 - \vec{B}_{n1} = 2\vec{a}_x + 0.5\vec{a}_y - 0.5\vec{a}_z$ T

$\vec{H}_{t_2} = \vec{H}_{t_1} = \frac{1}{\mu_0}(0.5\vec{a}_x + 0.125\vec{a}_y - 0.125\vec{a}_z)$ A/

$B_{t_2} = \mu_2 H_{t_2} = 3\vec{a}_x + 0.75\vec{a}_y - 0.75\vec{a}_z$ T

12

& finally: $\vec{B}_2 = \vec{B}_{n2} + \vec{B}_{t_2}$

$= 3\vec{a}_x + 1.25\vec{a}_y - 0.25\vec{a}_z$ T.