Profs. R. Chedid
S. Abou-Chahine

FACULTY OF ENGINEERING
& ARCHITECTURE

FALL TERM 2001-02

EE 441

Name:......................................

Nov.10, 2001

TEST ID 1000

(EE078) Field Theory

CLOSED BOOK (1 ½ HRS)

Programmable Calculators are not allowed
Provide your answers on the computer's card only
Return the computer's card attached to the question sheet
Mark with a pencil your name and your ID-No
Use pencil for marking your answers
When using eraser, be sure that you have erased well

!!! PENALTY 6 TO 1 !!!

{In all problems below take $\varepsilon_o=(10^{-9}/36\pi)$}

1. A potential field is given by $V=3x^2y-yz$. Which of the following <u>is not true:</u>

   a. At point (1,0,-1), both V and E vanish
   b. $x^2y=1$ is an equipotential line on the xy plane
   c. The equipotential surface V=-8 passes through point P(2,-1,4)
   d. The electric field at P(2,-1,4) is $12a_x - 8a_y - a_z$
   e. None of the above

2. Suppose a uniform electric field exists in the room in which you are working, such that the line of force are horizontal and at right angles to one wall. As you walk toward the wall from which the lines of force emerge into the room, are you walking toward

   a. Points of lower potential ?
   b. points of higher potential ?
   c. Points of the same potential?
   d. Insufficient information to determine the direction of higher potential
   e. None of the above

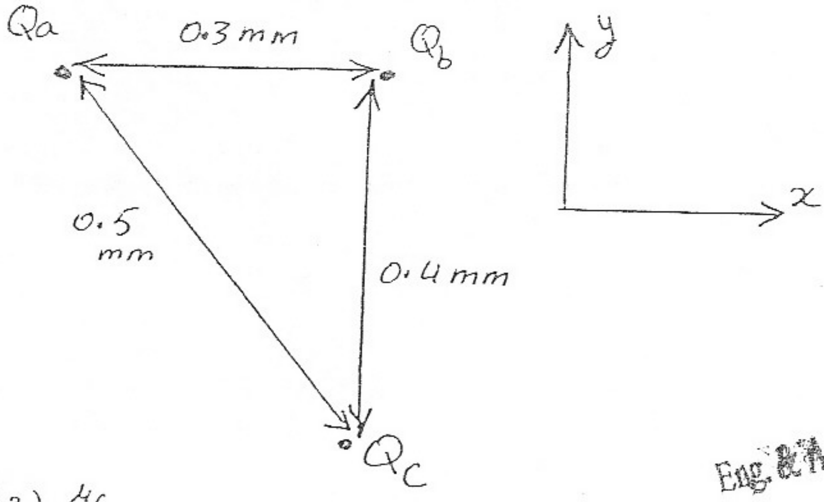
3. Plane Z=10m carries charge $20nC/m^2$. The electric field intensity at the origin is:

   a. $-10\ a_Z$ V/m
   b. $-18\pi\ a_Z$ V/m
   c. $-72\pi\ a_Z$ V/m
   d. $-360\pi\ a_Z$ V/m
   e. None of the above

4. Suppose the Z=0 plane is the interface between two regions: Z>0 has $\varepsilon_1=5\varepsilon_o$ and Z<0 has $\varepsilon_2=3\varepsilon_o$. If $E_2=10\ a_x+20\ a_z$, find D_1 and E_1.

   a. $D_1=50\ a_x+60\ a_z$, $E_1=\varepsilon_o\,(10\ a_x+12\ a_z)$
   b. $D_1=50\varepsilon_o\ a_x+60\ a_z$, $E_1=\varepsilon_o\,(10\ a_x+12\ a_z)$
   c. $D_1=\varepsilon_o(50\ a_x+60\ a_z)$, $E_1=10\ a_x+12\ a_z$
   d. $D_1=\varepsilon_o(30\ a_x+60\ a_z)$, $E_1=10\ a_x+12\ a_z$
   e. None of the above

5. Consider in the xy plane the 3 point charges shown below. If $Q_A = 4.10^{-12}$ C and $Q_B = 15.10^{-12}$ C and $Q_C = 15.10^{-12}$ C, find the electric force exerted on Q_A.



a. $(7.25\ a_x + 1.72\ a_y)$ μC
b. $(-7.25\ a_x - 1.72\ a_y)$ μC
c. $(1.72 a_x - 7.25\ a_y)$ μC
d. $(-7.25\ a_x + 1.72\ a_y)$ μC
e. None of the above

6. Two charges are arranged in the xy plane as follows:
$Q_1 = 10^{-9}$ C at (0,1) and $Q_2 = -10^{-9}$ C at (0,-1)

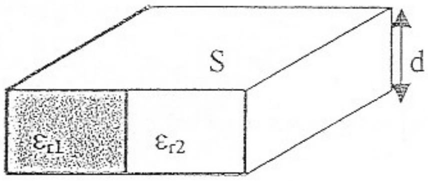
Find the electric potential V anywhere along the x-axis.

a. 0 V
b. 3.18 V
c. $(10^{-9}/4\pi\varepsilon_o) / [(1/(x-1)) - (1/(x+1))]$
d. $(10^{-9}/4\pi\varepsilon_o) / [(1/(x+1)) - (1/(x-1))]$
e. None of the above

7. In problem 6, find the electric field intensity E anywhere along the x-axis

a. $[-18/(x^2+1)^{3/2}]\ (a_x - a_y)$ V/m
b. 0 V/m
c. $[-18/(x^2+1)^{3/2}]\ (a_y)$ V/m
d. $[-18/(x^2+1)^{3/2}]\ (a_x)$ V/m
e. None of the above

8. The figure below shows a parallel plate capacitor containing two dielectrics with permittivities $\varepsilon_{r1} = 1.5$ and $\varepsilon_{r2} = 3.5$. Each of the dielectrics occupies one-half of the volume as shown. If the plate area is $S = 2m^2$ and the dielectric thickness is $d = 10^{-3}$ m, Calculate the capacitance of this capacitor.



a- C = 13.3 nF
b- C = 44.3 nF
c- C = 31.0 nF
d- C = 9.3 nF
e- Non of the above

9. Calculate the work done in moving a point charge Q = -20 μC from the origin to point (4, 2, 0) in the field $\mathbf{E} = (x/2 + 2y)\, \mathbf{a}_x + 2x\, \mathbf{a}_y$.

a- 80 μJ
b- 320 μJ
c- 400 μJ
d- 240 μJ
e- Non of the above

10. Calculate the stored energy in a system of four identical point charges Q = 4 nC located at the corners of a square, each side of which is 1m long.

a. 780 nJ
b. 390 nJ
c. 576 nJ
d. 204 nJ
e. Non of the above

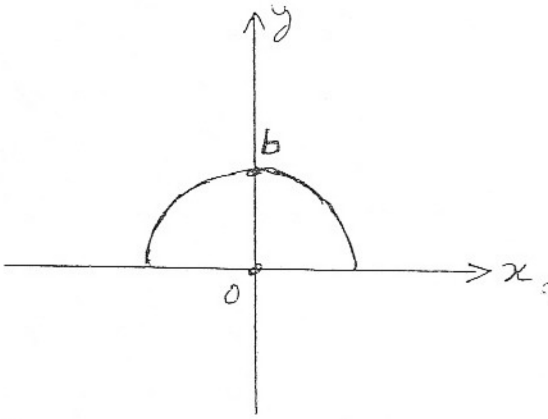
11. A uniform infinite line charge with ρ_L=5 μC/m lies along the x-axis (y=0, z=0). The electric flux density **D** at the point (3, 2, 1), m, is :

a- $0.356\,(2\mathbf{a}_y + \mathbf{a}_z)\ \mu C/m^2$
b- $0.159\,(2\mathbf{a}_y + \mathbf{a}_z)\ \mu C/m^2$
c- $0.356\,(\mathbf{a}_y + 2\mathbf{a}_z)\ \mu C/m^2$
d- $0.159\,(\mathbf{a}_y + 2\mathbf{a}_z)\ \mu C/m^2$
e- Non of the above

12. A spherical capacitor consists of an inner conducting sphere of radius R_i and an outer conductor with a spherical inner wall of radius R_o. The space in between is filled with a dielectric of permittivity ε. The capacitance C is :

a- $4\pi\varepsilon(1/R_i - 1/R_o)^{-1}$
b- $4\pi\varepsilon\ (1/R_i + 1/R_o)^{-1}$
c- $4\pi\varepsilon(1/R_i - 1/R_o)$
d- $4\pi\varepsilon(1/R_i + 1/R_o)$
e- Non of the above

13. A line charge of uniform density ρ_L= 5 μC/m forms a semicircle of radius b in the upper half of the xy-plane as shown. The electric field at the center of the semicircle is :



a- $-(\rho_L/2\pi\varepsilon_o b)\ a_y$
b- $-(\rho_L/4\pi\varepsilon_o b)\ a_y$
c- $(\rho_L/2\pi\varepsilon_o b)\ (a_x + a_y)$
d- $(\rho_L/2\pi\varepsilon_o b)\ (a_x - a_y)$
e- Non of the above

EE441 Profs. R. Chedid, S. A. Chehine

EE410. Quiz #1, Fall 2001-2002

1- A potential field is given by $V = 3x^2y - yz$.

Which of the following is not true?

→ a) At point (1,0,-1), V & E vanish.

b) $x^2y = 1$ is an equipotential line in the xy-plane

c) the equipotential surface V=-8 passes through point P(2,-1,4)

d) the electric field at P(2,-1,4) is $12\vec{a}_x - 8\vec{a}_y - \vec{a}_z$

e) none of the above

2- suppose a uniform electric field exists in the room in which you are working, such that the lines of force are horizontal and at right angles to one wall. As you walk towards the wall from which the lines of force emerge into the room, are you walking toward

→ a) points of higher potential?

b) points of lower potential?

c) points of the same potential

d) More information is needed to determine ~~points~~ the direction of higher potential

e) none of the above.

3) plane Z = 10m carries charge 20nC/m². the electric field intensity at the origine is:

a) $-10\vec{a}_z$ V/m

b) $-18\pi\vec{a}_z$ V/m

c) $-72\pi\vec{a}_z$ V/m

→ d) $-360\pi\vec{a}_z$ Vm/m

e) none of the above

$\left\{ E = \frac{\rho_s}{2\varepsilon_0} = \frac{20 \times 10^{-9}}{2 \cdot \frac{10^{-9}}{36\pi}} = 360\pi \; (\vec{a}_z) \right.$

4- Suppose the $z = 0$ plane is the interface between 2 regions: $z > 0$ has $\varepsilon_1 = 5\varepsilon_0$ and $z < 0$ has $\varepsilon_2 = 3\varepsilon_0$.
If $\vec{E}_2 = 10\vec{a}_x + 20\vec{a}_z$, find $\vec{D}_1$ & $\vec{E}_1$.

a) $\vec{D}_1 = 50\vec{a}_x + 60\vec{a}_z$; $\vec{E}_1 = \varepsilon_0(10\vec{a}_x + 12\vec{a}_z)$

b) $\vec{D}_1 = 50\varepsilon_0\vec{a}_x + 60\vec{a}_z$; $\vec{E}_1 = \varepsilon_0(10\vec{a}_x + 12\vec{a}_z)$

→ c) $\vec{D}_1 = \varepsilon_0(50\vec{a}_x + 60\vec{a}_z)$; $\vec{E}_1 = 10\vec{a}_x + 12\vec{a}_z$

d) $\vec{D}_1 = \varepsilon_0(30\vec{a}_x + 60\vec{a}_z)$; $\vec{E}_1 = 10\vec{a}_x + 12\vec{a}_z$

e) none of the above

$D_2 = \varepsilon_2 E_2 = \varepsilon_0(30\vec{a}_x + 60\vec{a}_z)$

Tangential E & normal D are continuous, so

$E_{x1} = E_{t1} = E_{t2} = 10$

& $D_{z1} = D_{n1} = D_{n2} = +60\varepsilon_0$

now: $D_{x1} = \frac{\varepsilon_1}{\varepsilon_2} D_{x2} = \frac{5}{3}(30\varepsilon_0) = 50\varepsilon_0$

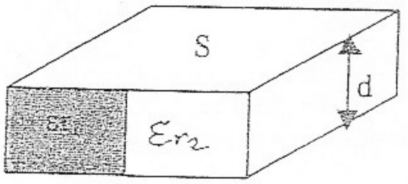
& $E_{z1} = \frac{\varepsilon_2}{\varepsilon_1} E_{z2} = \frac{3}{5}(+20) = +12$

thus, $\vec{D}_1 = \varepsilon_0(50\vec{a}_x + 60\vec{a}_z)$ ✓

& $E_1 = 10\vec{a}_x + 12\vec{a}_z$ ✓

I)

The capacitance of a parallel plate capacitor containing two dielectrics, $\varepsilon r_1 = 1.5$ and $\varepsilon r_2 = 3.5$, each comprising one-half the volume, as shown where $S = 2m^2$ and $d = 10^{-3}$ m.



a- C = 13.3 nF
b- ** C = 44.3 nF
c- C = 31.0 nF
d- C = 9.3 nF
e- Non of the above

This parallel plate capacitor is equivalent to two parallel capacitors:

$C = C_1 + C_2$.

$C_1 = \frac{\varepsilon_o \varepsilon_{r_1} S_1}{d}$, $S_1 = 1\ m^2 \Rightarrow C_1 = 13.3$ nF

Similarly $C_2 = 31.0$ nF

$\Rightarrow C = C_1 + C_2 = 44.3$ nF.

II) The work done in moving a point charge Q = -20 μC from the origin to (4, 2, 0) in the field : $E = (x/2 + 2y)\,\mathbf{a_x} + 2x\,\mathbf{a_y}$ is :

a- 80 μJ
b- 320 μJ
c- ** 400 μJ
d- 240 μJ
e- Non of the above

$W = -Q\int \vec{E}\cdot d\vec{l}$

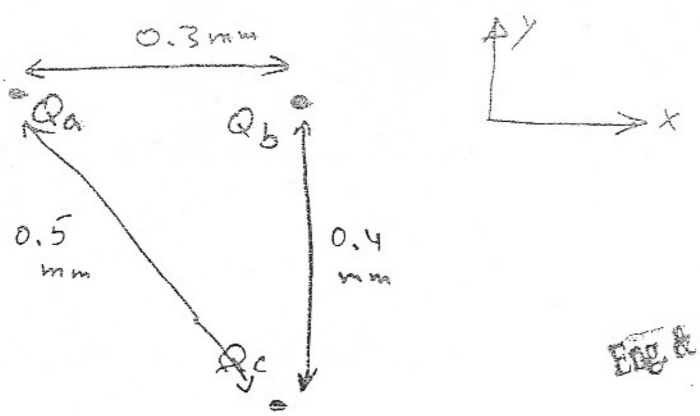
If we move the charge along the straight line connecting the points, then:

$x = 2y$ & $dx = 2dy$, $dz = 0$

$\Rightarrow W = 20\times10^{-6}\int[(x + 2y)\,\hat{a}_x + 2x\,\hat{a}_y]\cdot[dx\,\hat{a}_x + dy\,\hat{a}_y]$.

Substituting for $y = \frac{x}{2}$, $dy = \frac{dx}{2}$ & integrating from $x = 0$ to $x = 4$, we get:

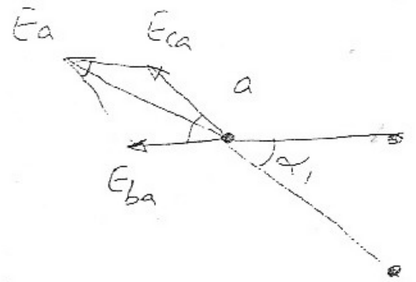
$W = 20\times10^{-6}\int_0^4 \frac{5}{2}x\,dx = 400\,\mu J$



If $Q_a = 4.10^{-12}C$; $Q_b = 15.10^{-12}C$ & Q_c $15.10^{-12}C$

find the force exerted on Qa.

a.
b.
c.
d.
e.



$F_a = Q_a E_a$ where $\vec{E}_a = \vec{E}_{ca} + \vec{E}_{ba}$

$$= \frac{Q_c \, \vec{a}_{ca}}{4\pi\varepsilon_0 R_{ca}^2} + \frac{Q_b \, \vec{a}_{ba}}{4\pi\varepsilon_0 R_{ba}^2}$$

or $$\vec{E}_a = -\left(\frac{Q_c \cos\alpha_1}{4\pi\varepsilon_0 R_{ca}^2} + \frac{Q_b}{4\pi\varepsilon_0 R_{ba}^2}\right)\vec{a}_x + \frac{Q_c \sin\alpha_1}{4\pi\varepsilon_0 R_{ca}^2}$$

here: $\cos\alpha_1 = \frac{R_{ba}}{R_{ca}} = \frac{0.3}{0.5} = 0.6$

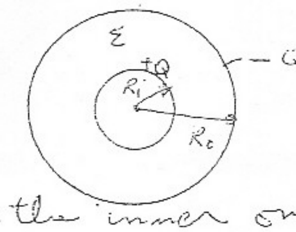
$\sin\alpha_1 = \frac{R_{cb}}{R_{ca}} = \frac{0.4}{0.5} = 0.8$

so $F_a = (-7.25\,\vec{a}_x + 1.72\,\vec{a}_y)$ ✓

V)

A spherical capacitor consists of an inner conducting sphere of radius R_i and an outer conductor with a spherical inner wall of radius R_o. The space in between is filled with a dielectric of permittivity ε. The capacitance C is :

a- ** $4\pi\varepsilon\,(1/R_i - 1/R_o)^{-1}$
b- $4\pi\varepsilon\,(1/R_i + 1/R_o)^{-1}$
c- $4\pi\varepsilon\,(1/R_i - 1/R_o)$
d- $4\pi\varepsilon\,(1/R_i + 1/R_o)$
e- Non of the above



Assume charges +Q and −Q on the inner and outer conductors, respectively.

$$\Rightarrow \vec{E} = \hat{a}_R E_R = \frac{Q}{4\pi\varepsilon_o R^2}\,\hat{a}_R$$

$$\Rightarrow V = -\int_{R_o}^{R_i} \vec{E}\cdot d\vec{l} \quad \text{But } d\vec{l} = dR\,\hat{a}_R + R\,d\theta\,\hat{a}_\theta + R\sin\theta\, d\phi\,\hat{a}_\phi$$

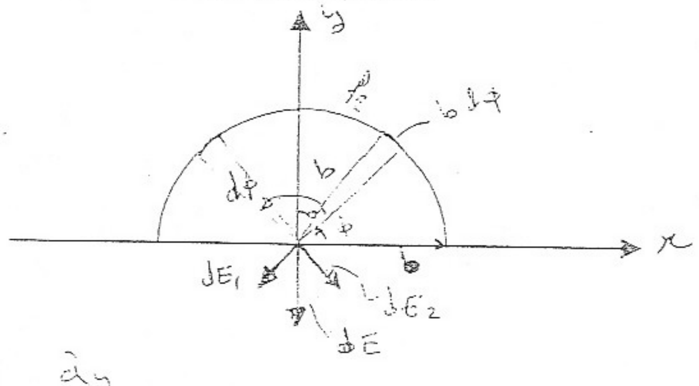
$$\Rightarrow V = -\int_{R_o}^{R_i} \frac{Q}{4\pi\varepsilon R^2} = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{R_i} - \frac{1}{R_o}\right]$$

$$\Rightarrow \frac{Q}{V} = C = \frac{4\pi\varepsilon_o\varepsilon_r}{\left[\frac{1}{R_i} - \frac{1}{R_o}\right]}$$

VI)

A line charge of uniform density ρ_L 5 μC/m forms a semicircle of radius b in the upper half xy-plane. The electric field at the center of the semicircle is :

a- ** $-(\rho_L/2\pi\varepsilon_o b)\,a_y$
b- $-(\rho_L/4\pi\varepsilon_o b)\,a_y$
c- $(\rho_L/2\pi\varepsilon_o b)\,(a_x + a_y)$
d- $(\rho_L/2\pi\varepsilon_o b)\,(a_x - a_y)$
e- Non of the above



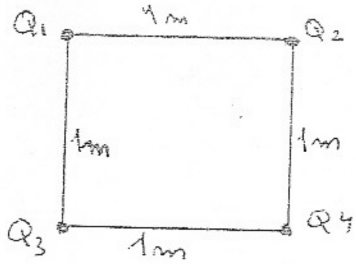
$$d\vec{E} = \frac{\rho_l\, dl}{4\pi\varepsilon_o r^2}\,\hat{a}_r$$

$$d\vec{E} = d\vec{E}_1 + d\vec{E}_2 = \frac{-\rho_l (b\,d\phi)}{4\pi\varepsilon_o b^2}\sin\phi\;\hat{a}_y$$

$$\Rightarrow E = -\frac{\rho_l}{4\pi\varepsilon_o b}\int_0^{\pi}\sin\phi\, d\phi\;\hat{a}_y = \frac{-\rho_l}{2\pi\varepsilon_o b}\,\hat{a}_y$$

iii)

The stored energy in a system of four identical point charges Q = 4 nC, at the corners of a square 1 m on a side is :



a- ** 780 nJ
b- 390 nJ
c- 576 nJ
d- 204 nJ
e- Non of the above

$$W_E = \frac{1}{2}\sum_{i=1}^{4} Q_i V_i = \frac{1}{2}\left[Q_1V_1 + Q_2V_2 + Q_3V_3 + Q_4V_4\right]$$

But $Q_1 = Q_2 = Q_3 = Q_4$.

& $V_1 = V_2 = V_3 = V_4 = \frac{4\times10^{-9}}{4\pi\varepsilon_0}\left[\frac{1}{1} + \frac{1}{1} + \frac{1}{\sqrt{2}}\right] = 97.5$

$\Rightarrow W_E = 2Q_1V_1 = 2\times 4\times10^{-9}\times 97.5 = 780$ nJ.

IV)

A uniform infinite line charge with ρ_L 5 μC/m lies along the x-axis. The electric flux density **D** at (3, 2, 1) m is :

a- 0.356 ($2\mathbf{a}_y + \mathbf{a}_z$) μC/m²
b- ** 0.159 ($2\mathbf{a}_y + \mathbf{a}_z$) μC/m²
c- 0.356 ($\mathbf{a}_y + 2\mathbf{a}_z$) μC/m²
d- 0.159 ($\mathbf{a}_y + 2\mathbf{a}_z$) μC/m²
e- Non of the above

$\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 r}\hat{a}_r$ where $\vec{r} = 2\hat{a}_y + 1\hat{a}_z$

$\Rightarrow \hat{a}_r = \frac{2\hat{a}_y + 1\hat{a}_z}{\sqrt{4+1}}$ & $r = \sqrt{4+1} = \sqrt{5}$

$\Rightarrow \vec{E} = \frac{5\times10^{-6}}{2\pi\varepsilon_0\sqrt{5}} \cdot \frac{(2\hat{a}_y + 1\hat{a}_z)}{\sqrt{5}}$ V/m

$\vec{D} = \varepsilon_0\vec{E} = \frac{10^{-6}}{2\pi}(2\hat{a}_y + 1\hat{a}_z)$.

$\Rightarrow \vec{D} = 0.159\,(2\hat{a}_y + 1\hat{a}_z)$ μC/m².