AMERICAN UNIVERSITY OF BEIRUT
FACULTY OF ENGINEERING & ARCHITECHTURE SPRING 2007-08

COURSE NAME: EECE380, ENGINEERING ELECTROMAGNETICS

COURSE INSTRUCTOR: DR. KAMALI

QUIZ 1: CLOSED BOOK 1 ½ HOUR

DATE: 14/3/2008

STUDENT NAME: WALID KAMALI

STUDENT ID:

SECTION :

INSTRUCTIONS

12 questions 10 points each. No rise, no curve.

Programmable Calculators are not allowed

Use pencil to write on your answer sheet.

When using eraser, be sure that you have erased well.

Cellular phones are strictly forbidden in the exam room.

{In all problems below take $\varepsilon_o=10^{-9}/36\pi$}

1. A cube 2 m on a side with 2 of its sides parallel to the x-y plane of Cartesian coordinates system, and its center located at the origin. Find the total charge contained in the cube if the charge density is given by $\rho_v=x^4y^2\cos(z\pi/3)$ (C/m^3). (show your work)

$$Q=\int_v \rho_v\,dv = \int_v x^4 y^2 \cos\left(3\frac{\pi}{3}\right) dv = \int_{z=-1}^{1}\int_{y=-1}^{1}\int_{x=-1}^{1} x^4 y^2 \cos\left(3\frac{\pi}{3}\right) dx\,dy\ldots$$

$$= \int_{x=+1}^{1} x^4 dx \int_{y=-1}^{+1} y^2 dy \int_{z=-1}^{1} \cos\left(3\frac{\pi}{3}\right) dz = \left[\frac{x^5}{5}\right]_{-1}^{1} \times \left[\frac{y^3}{3}\right]_{-1}^{1} \left[\frac{3}{\pi}\sin\frac{3}{3}\ldots\right.$$

$$= \frac{2}{5}\times\frac{2}{3}\times\frac{3}{\pi}\frac{2\sqrt{3}}{2} = 0.44\,C = \frac{4\sqrt{3}}{5\pi}$$

2. Find the total charge on a circular disk defined by $r \le a$ and $z=0$, given that: $\rho_s=\rho_{so}\,e^{-r}$(C/m^2) where ρ_{so}.=4nC/m^2, and a the radius of the disk is 50cm. (show your work)

$$Q=\int_s \rho_s\,ds = \rho_{so}\int_0^{2\pi} d\phi \int_{r=0}^{a} r e^{-r} dr = 2\pi\rho_{so}\left[-re^{-r}-e^{-r}\right]_0^{a}$$

$$= 2\pi\rho_{so}\left[-(a+1)e^{-a}+1\right] = 2\times\pi\times 4n\left[1-1.5e^{-\frac{1}{2}}\right] = 2.27\,nC$$

3. Find the total charge contained in a cone defined by $R \leq 3$m and $0 \leq \theta \leq \pi/4$, given that $\rho_v = 20R^2 \cos^2\theta$ (nC/m^3).

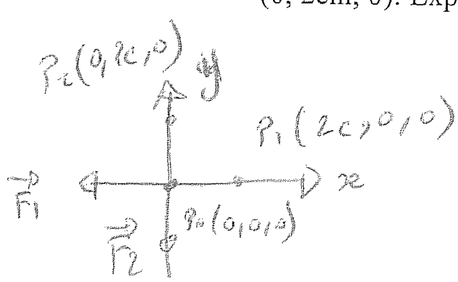
$$Q = \int_v \rho_v \, dv = \int \rho_v R^2 \sin\theta \, dR \, d\theta \, d\phi = \int_{R=0}^{3} \int_{\theta=0}^{\frac{\pi}{4}} \int_{\phi=0}^{2\pi} 20\,10^{-9} R^4 \sin\theta \cos^2\theta$$

$$= 20\,10^{-9} \int_0^{2\pi} d\phi \int_0^{\frac{\pi}{4}} \cos^2\theta \sin\theta \, d\theta \int_0^3 R^4 \, dR$$

$$= -20\,10^{-9} \times 2\pi \times \frac{3^5}{5} \left[\frac{\cos^3\theta}{3} \right]_0^{\frac{\pi}{4}} = -\frac{6107.25\,10^{-9}}{3} \left(\cos^3\frac{\pi}{4} - \cos 0 \right)$$

$$= 1316 \text{ nC}$$

4. Three point charges, each with q=9nC, are located at the corners of a triangle in the x-y plane, with one corner at the origin, another at (2cm, 0, 0), and the third at (0, 2cm, 0). Express the vector force acting on the charge located at the origin.



@ P_0;

$$\vec{E}_0 = \frac{q}{4\pi\varepsilon_0} \left[\frac{-\hat{x}}{(2\,10^{-2})^2} - \frac{\hat{y}}{(2\,10^{-2})^2} \right]$$

$$\vec{F}_0 = \frac{q^2}{4\pi\varepsilon_0 (2\,10^{-2})^2} (-\hat{x} - \hat{y})$$

$$\vec{F}_0 = \frac{81\,10^{-18} \times 36\pi}{4\pi \times 10^{-9} \times 4 \times 10^{-4}} (-\hat{x} - \hat{y})$$

$$= 182.25\,10^{-5} (-\hat{x} - \hat{y}) \text{ N}$$

$$= 1.82\,10^{-3} (-\hat{x} - \hat{y}) \text{ N}$$

5. Express Maxwell's equations for the static electric field both in their differential forms and their integral forms.

$\vec{\nabla} \cdot \vec{D} = \rho_v$

$\oint_S \vec{D} \cdot \vec{ds} = Q$

$\vec{\nabla} \times \vec{E} = 0$

$\oint_l \vec{E} \cdot \vec{dl} = 0$

6. A thin circular ring with a radius = r_1 and a total charge Q_1 uniformly distributed is placed on the -x, -y plane with it center at the origin. Another thin circular ring with a radius = r_2 and a total charge Q_2 uniformly distributed is placed on the -x, -y plane with its center also at the origin. Assume that $r_2 > r_1$, find the vector $\boldsymbol{E}$ in the point (0,0,d).

$\vec{E}_1 = \hat{z} \frac{d}{4\pi\varepsilon_0 (r_1^2+d^2)^{\frac{3}{2}}} Q_1$

$\vec{E}_2 = \hat{z} \frac{d}{4\pi\varepsilon_0 (r_2^2+d^2)^{\frac{3}{2}}} Q_2$

$\vec{E} = \vec{E}_1 + \vec{E}_2$

7. Determine the electric potential in the point (0,0,d) of the previous question.

$V = -\int_{\infty}^{d} \vec{E} \cdot dz = -\int_{\infty}^{d} \frac{Q_1}{4\pi\varepsilon_0} \frac{z\,dz}{(r_1^2+z^2)^{\frac{3}{2}}} = \frac{Q_1}{2\pi\varepsilon_0} \times \frac{2}{1} \left[\frac{1}{(r_1^2+z^2)^{\frac{1}{2}}}\right]_{\infty}^{d}$

$V_{total} = \frac{1}{2\pi\varepsilon_0}\left[\frac{Q_1}{\sqrt{r_1^2+d^2}} + \frac{Q_2}{\sqrt{r_2^2+d^2}}\right]$

$u = (r_1^2+z^2)$

$du = 2z\, \cancel{(r_1^2+z^2)}\, dz$ $\cancel{r_1^2+d^2}$

$\frac{1}{2}\int u^{-\frac{3}{2}} du = -\frac{1}{2} \times \frac{2}{1} \left[u^{-\frac{1}{2}}\right]_{\infty}$

Problem

8. Calculate the total capacitance in pF of a thin square metal plate of 1m side, placed at h=10cm above an infinite conductive plane coincident to x-y plane. (neglect the fringing effects)

$$C = \frac{\varepsilon_0 A}{d} = \frac{10^{-9}}{36\pi} \times \frac{1 \times 1}{0.1} = 88.5\,pF$$

9. Calculate the values of the voltage on the top plate and the energy stored in the capacitor when a total charge of 1nC is stored on the top plate. Express the voltage in the region between the plates.

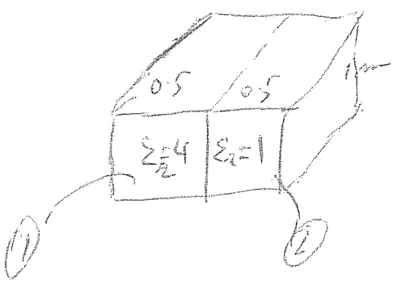
$$V = \frac{Q}{C} = \frac{10^{-9}}{88.5\, 10^{-12}} = 11.31\,V \quad ; \quad W = \frac{1}{2} C V^2 = 5.65\,nJ$$

$$V(z) = 113.1\, z$$

10. Calculate the magnitude and direction of the electric field in the region between the plates

$$\bar{E} = -\operatorname{grad} V \quad ; \quad \vec{E} = -113.1\, \hat{z}$$

11. If half the area of the top plate rests on a material with a relative dielectric constant $\varepsilon_r = 4$ and this region is called region 1, while the other half of the top plate is suspended over the air, and this region is called region 2. Sketch the new capacitor and determine the ratio of the electric flux densities in the two regions.



$$E_1 = E_2 \Rightarrow \frac{D_1}{\varepsilon_1} = \frac{D_2}{\varepsilon_2}$$

$$\frac{D_1}{D_2} = \frac{\varepsilon_1}{\varepsilon_2} = 4$$

12. Calculate the total energy stored in the new capacitor if 1nC is stored on the top plate.

$$C_{new} = \frac{88.4}{2} + 4\,\frac{88.4}{2} = \frac{5 \times 88.4}{2} = 221\,pF$$

$$W = \frac{1}{2} CV^2 = \frac{1}{2} QV = \frac{1}{2}\frac{Q^2}{C} = \frac{10^{-18}}{2 \times 221 \times 10^{-12}} = \underline{2.26\,nJ}$$

Table 2-1: Summary of vector relations.

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation, $\mathbf{A} =$	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$
Magnitude of $\mathbf{A}$, $\lvert A\rvert =$	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\overrightarrow{OP_1} =$	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1$, for $P(x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1$, for $P(r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1$, for $P(R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{\mathbf{x}}\cdot\hat{\mathbf{x}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\cdot\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}}\cdot\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product, $\mathbf{A}\cdot\mathbf{B} =$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product, $\mathbf{A}\times\mathbf{B} =$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length, $d\mathbf{l} =$	$\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
Differential surface areas	$d\mathbf{s}_x = \hat{\mathbf{x}}\,dy\,dz$ $d\mathbf{s}_y = \hat{\mathbf{y}}\,dx\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}\,dx\,dy$	$d\mathbf{s}_r = \hat{\mathbf{r}}r\,d\phi\,dz$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}r\,dr\,d\phi$	$d\mathbf{s}_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$
Differential volume, $d\nu =$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

Table 2-2: Coordinate transformation relations.

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2 + y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}}\sin\theta\cos\phi + \hat{\mathbf{y}}\sin\theta\sin\phi + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}}\cos\theta\cos\phi + \hat{\mathbf{y}}\cos\theta\sin\phi - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$	$A_R = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}}\sin\theta\cos\phi + \hat{\boldsymbol{\theta}}\cos\theta\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}}\sin\theta\sin\phi + \hat{\boldsymbol{\theta}}\cos\theta\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_x = A_R\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}}\sin\theta + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}}\cos\theta - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

GRADIENT, DIVERGENCE, CURL, & LAPLACIAN OPERATORS

CARTESIAN (RECTANGULAR) COORDINATES (x, y, z)

$$\nabla V = \hat{x}\frac{\partial V}{\partial x} + \hat{y}\frac{\partial V}{\partial y} + \hat{z}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

CYLINDRICAL COORDINATES (r, ϕ, z)

$$\nabla V = \hat{r}\frac{\partial V}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial V}{\partial \phi} + \hat{z}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \frac{1}{r}\begin{vmatrix} \hat{r} & \hat{\phi}r & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \hat{r}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{z}\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

SPHERICAL COORDINATES (R, θ, ϕ)

$$\nabla V = \hat{R}\frac{\partial V}{\partial R} + \hat{\theta}\frac{1}{R}\frac{\partial V}{\partial \theta} + \hat{\phi}\frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{R} & \hat{\theta}R & \hat{\phi}R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix}$$

$$= \hat{R}\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right] + \hat{\theta}\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}(RA_\phi)\right] + \hat{\phi}\frac{1}{R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial \theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

SOME USEFUL VECTOR IDENTITIES

$\mathbf{A} \cdot \mathbf{B} = AB\cos\theta_{AB}$ Scalar (or dot) product

$\mathbf{A} \times \mathbf{B} = \hat{n}AB\sin\theta_{AB}$ Vector (or cross) product, $\hat{n}$ normal to plane containing $\mathbf{A}$ and $\mathbf{B}$

$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$

$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \times \mathbf{B})$

$\nabla(U + V) = \nabla U + \nabla V$

$\nabla(UV) = U\nabla V + V\nabla U$

$\nabla \cdot (\mathbf{A} + \mathbf{B}) = \nabla \cdot \mathbf{A} + \nabla \cdot \mathbf{B}$

$\nabla \cdot (U\mathbf{A}) = U\nabla \cdot \mathbf{A} + \mathbf{A} \cdot \nabla U$

$\nabla \times (U\mathbf{A}) = U\nabla \times \mathbf{A} + \nabla U \times \mathbf{A}$

$\nabla \times (\mathbf{A} + \mathbf{B}) = \nabla \times \mathbf{A} + \nabla \times \mathbf{B}$

$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$

$\nabla \cdot (\nabla \times \mathbf{A}) = 0$

$\nabla \times \nabla V = 0$

$\nabla \cdot \nabla V = \nabla^2 V$

$\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$

$\int_{v} (\nabla \cdot \mathbf{A})\, dv = \oint_S \mathbf{A} \cdot d\mathbf{s}$ Divergence theorem (S encloses v)

$\int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{s} = \oint_C \mathbf{A} \cdot d\mathbf{l}$ Stokes's theorem (S bounded by C)