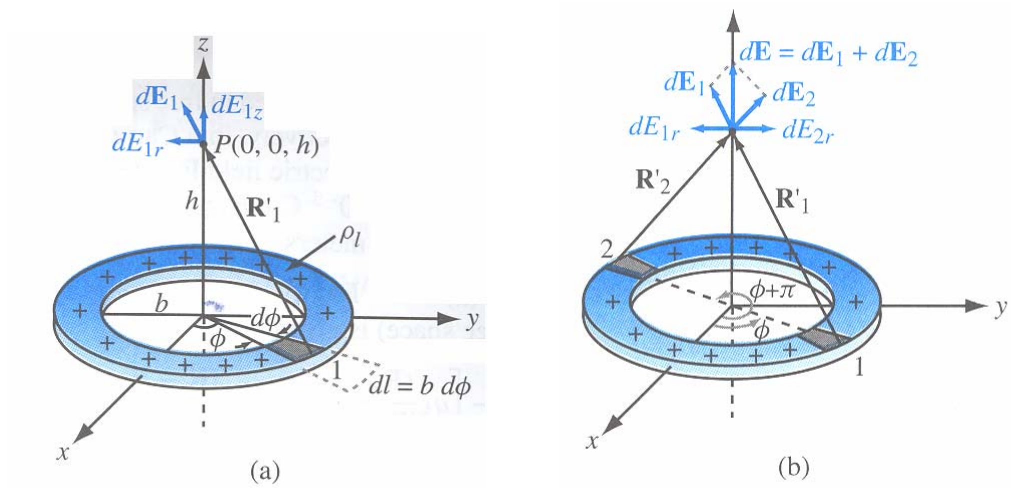
Example: Electric field of a ring of charge

A ring of charge of radius b is characterized by a uniform line charge density of positive polarity ρ_l. With the ring in free space and positioned in the x-y plane as shown in the figure below, determine the electric field intensity $\vec{\mathrm{E}}$ at the point $P(0, 0, h)$ along the axis of the ring at a distance h from its center.



Solution:
The segment has a length $dl = b\ d\phi$ and contains charge $dq = \rho_l\ dl = \rho_l b\ d\phi$.

$$\overrightarrow{\mathbf{R}'_1} = -\hat{\mathbf{r}}\,b + \hat{\mathbf{z}}\,h$$

from which we have,

$$\left|\overrightarrow{\mathbf{R}'_1}\right| = \sqrt{b^2 + h^2}, \qquad \hat{R}'_1 = \frac{\overrightarrow{\mathbf{R}'_1}}{\left|\overrightarrow{\mathbf{R}'_1}\right|} = \frac{-\hat{\mathbf{r}}b + \hat{\mathbf{z}}h}{\sqrt{b^2 + h^2}}.$$

The electric field at $P(0, 0, h)$ due to the charge of segment 1 is

$$\overrightarrow{d\mathrm{E}_1} = \frac{1}{4\pi\varepsilon_0}\hat{R}'_1\frac{\rho_l\ dl}{R'^2_1} = \frac{\rho_l}{4\pi\varepsilon_0}\frac{\left(-\hat{\mathbf{r}}\,b + \hat{\mathbf{z}}\,h\right)}{\left(b^2 + h^2\right)^{3/2}}\ d\phi.$$

From symmetry considerations, the fields $\overrightarrow{d\mathrm{E}_2}$ generated by segment 2 in the last figure which is located diametrically opposite the location of segment 1, is identical with $\overrightarrow{d\mathrm{E}_1}$ except that the $\hat{\mathbf{r}}$- component of $\overrightarrow{d\mathrm{E}_2}$ is opposite that of $\overrightarrow{d\mathrm{E}_1}$. Hence, the $\hat{\mathbf{r}}$-component of the sum cancel and the $\hat{\mathbf{z}}$-contributions add. The sum of the two contributions is

$$\overrightarrow{d\mathrm{E}} = \overrightarrow{d\mathrm{E}_1} + \overrightarrow{d\mathrm{E}_2} = \hat{\mathbf{z}}\,\frac{\rho_l\,b\,h}{2\pi\varepsilon_0}\frac{d\phi}{\left(b^2 + h^2\right)^{3/2}}.$$

so,

$$\vec{\mathrm{E}} = \hat{\mathbf{z}}\frac{\rho_l\, b\, h}{2\pi\varepsilon_0\left(b^2+h^2\right)^{3/2}}\int_0^{\pi} d\phi$$

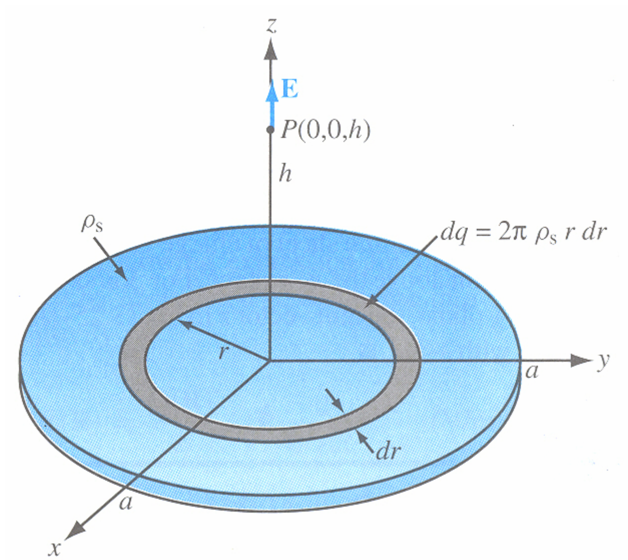
$$= \hat{\mathbf{z}}\frac{\rho_l\, b\, h}{2\varepsilon_0\left(b^2+h^2\right)^{3/2}}$$

$$= \hat{\mathbf{z}}\frac{h}{4\pi\varepsilon_0\left(b^2+h^2\right)^{3/2}}Q,$$

where $Q = 2\pi b \rho_l$.

<u>Example:</u> Electric field of a circular disk of charge

Find the electric field at a point $P(0, 0, h)$ in free space at a height h on the z-axis due to a circular disk of charge in the x-y plane with uniform charge density ρ_s, as shown in figure, and then evaluate $\vec{\mathrm{E}}$ for the infinite-sheet case by letting $a \to \infty$.



Solution:

A ring of radius r and width dr has an area $ds = 2\pi r dr$ and contains charge $dq = \rho_s ds = 2\pi\rho_s r\, dr$.

so,

$$\overrightarrow{d\mathrm{E}} = \hat{\mathbf{z}}\frac{h}{4\pi\varepsilon_0\left(r^2+h^2\right)^{3/2}}2\pi\rho_s r\ dr.$$

The total field at P is obtained by integrating the expression over the limits $r = 0$ to $r = a$:

$$\vec{\mathrm{E}} = \hat{\mathbf{z}} \frac{\rho_s h}{2\varepsilon_0} \int_0^a \frac{r\,dr}{\left(r^2 + h^2\right)^{3/2}}$$

$$= \pm\hat{\mathbf{z}} \frac{\rho_s}{2\varepsilon_0} \left[1 - \frac{|h|}{\sqrt{a^2 + h^2}}\right],$$

with the plus minus sign corresponding to when $h \succ 0$ and the minus sign to when $h \prec 0$ (below the disk).

For an infinite sheet of charge with $a = \infty$,

$$\vec{\mathrm{E}} = \hat{\mathbf{z}} \frac{\rho_s}{2\varepsilon_0} \quad \text{(Infinite sheet of charge).}$$

Gauss's Law:

$$\vec{\nabla} \cdot \vec{\mathrm{D}} = \rho_v \qquad \text{(Gauss's Law)},$$

$$\int_v \vec{\nabla} \cdot \vec{\mathrm{D}}\, dv = \int_v \rho_v \, dv = Q,$$

$$\int_v \vec{\nabla} \cdot \vec{\mathrm{D}}\, dv = \oint_s \vec{\mathrm{D}} \cdot \overrightarrow{d\mathrm{s}}.$$

$$\oint_s \vec{\mathrm{D}} \cdot \overrightarrow{d\mathrm{s}} = Q \qquad \text{(Gauss's Law)}. \qquad (*)$$

The integral form of Gauss's law can be applied to determine $\vec{\mathrm{D}}$ due to a single isolated charge q by constructing a closed, spherical, Gaussian surface ***S*** of an arbitrary radius ***R*** centered at q. Applying Gauss's law gives

$$\oint_s \vec{\mathrm{D}} \cdot \overrightarrow{d\mathrm{s}} = \oint_s \hat{\mathrm{R}} D_R \cdot \hat{\mathrm{R}} ds$$

$$= \oint_s D_R \cdot ds = D_R \left(4\pi R^2\right) = q.$$

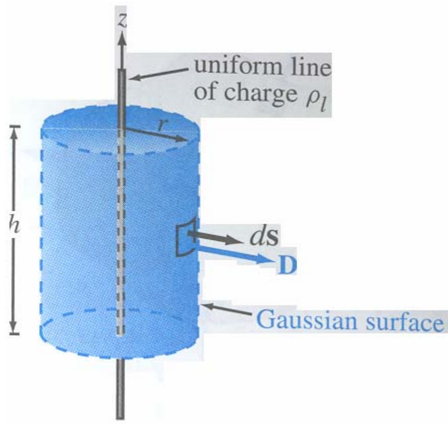
so,

$$\overrightarrow{\mathbf{E}(\mathrm{R})} = \frac{\overrightarrow{\mathbf{D}(\mathrm{R})}}{\varepsilon} = \hat{\mathbf{R}} \frac{q}{4\pi\varepsilon R^2} \qquad \text{(V/m)}.$$

Gauss's law, as given by equation (*), provide a convenient method for determining the electrostatic flux density $\vec{\mathrm{D}}$ when the charge distribution possesses symmetry properties that allow us to make valid assumptions about the variations of the magnitude and direction of $\vec{\mathrm{D}}$ as a function of spatial location. Because at every point on the surface, only the direction of $\overrightarrow{d\mathrm{s}}$ is the outward normal to the surface, only the normal component of $\vec{\mathrm{D}}$ at the surface contribute to the integral in equation (*). To successfully apply Gauss's law, the surface ***S*** should be chosen such that, from symmetry considerations, the magnitude of $\vec{\mathrm{D}}$ is constant and its direction is normal or tangential at every point of each subsurface of ***S*** (the surface of a cube, for example, has six subsurface).

<u>Example:</u> Electric field of an infinite line of charge

Use Gauss's law to obtain an expression for $\vec{\mathrm{E}}$ in free space due to an infinitely long line of charge with uniform charge density ρ_l along the z-axis.



Solution:

$$\vec{\mathbf{D}} = \hat{\mathbf{r}} D_r$$

The total charge contained within the cylinder is $Q = \rho_l\, h$, where h is the height of the cylinder. Since $\vec{\mathbf{D}}$ is along $\hat{\mathbf{r}}$, the top and bottom surfaces of the cylinder do not contribute to the surface integral.

$$\int_{z=0}^{h} \int_{\phi=0}^{2\pi} \hat{\mathbf{r}} D_r \cdot \hat{\mathbf{r}} r \, d\phi \, dz = \rho_l h$$

which gives the result,

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{D}}}{\varepsilon} = \hat{\mathbf{r}} \frac{D_r}{\varepsilon_0} = \hat{\mathbf{r}} \frac{\rho_l}{2\pi\varepsilon_0 r} \qquad \text{(Infinite line of charge).}$$

Electric potential as a function of electric field

We begin by considering the simple case of a positive charge q in a uniform electric field $\vec{\mathbf{E}} = \hat{\mathbf{y}}\,E$, parallel to the (-y)-direction, as shown in figure. The presence of the field $\vec{\mathbf{E}}$ exerts a force $\vec{\mathbf{F}_e} = q\,\vec{\mathbf{E}}$ on the charge in the negative y-direction. If we attempt to move the charge along the positive y-direction (against the force $\vec{\mathbf{F}_e}$), we will need to provide an external force $\overrightarrow{\mathbf{F}_{ext}}$ to counter-act $\vec{\mathbf{F}_e}$, which requires the expenditure of energy. To move q without any acceleration (at a constant speed), it is necessary that the net force acting on the charge be zero, which means that $\overrightarrow{\mathbf{F}_{ext}} + \vec{\mathbf{F}_e} = 0$, or

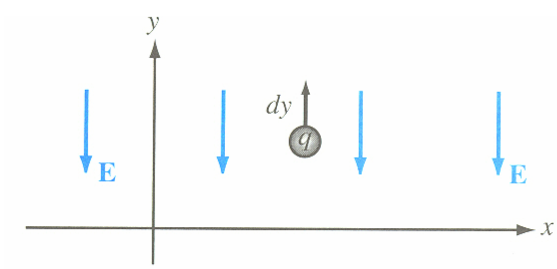
$$\overrightarrow{\mathbf{F}_{ext}} = -\vec{\mathbf{F}_e} = -q\,\vec{\mathbf{E}}.$$

The work done, or energy expended, in moving any object a vector differential distance $d\mathbf{l}$ under the influence of a force $\overrightarrow{\mathbf{F}_{ext}}$ is

$$d\,W = \overrightarrow{\mathbf{F}_{\text{ext}}} \cdot \overrightarrow{d\mathbf{l}} = -q\vec{\mathbf{E}} \,.\, \overrightarrow{d\mathbf{l}} \quad \text{(J)}$$

Work, or energy, is measured in joules (J). In the present case, if the charge is moved a distance dy along $\hat{\mathbf{y}}$, then

$$dW = -q(-\hat{\mathbf{y}}\,E) \cdot \hat{\mathbf{y}}\,dy = q\,E\,dy.$$



$$dV = \frac{dW}{q} = -\vec{\mathrm{E}} \cdot \overrightarrow{d\mathrm{l}} \quad (\text{J/C } or \text{ V}). \qquad (**)$$

The potential difference between any two points P_2 and P_1 is obtained by integrating the equation (**) along any path between them. That is,

$$\int_{P_1}^{P_2} dV = -\int_{P_1}^{P_2} \vec{\mathrm{E}} \cdot \overrightarrow{d\mathrm{l}},$$

or,

$$V_{21} = V_2 - V_1 = -\int_{P_1}^{P_2} \vec{\mathrm{E}} \cdot \vec{d\mathrm{l}} \qquad (***)$$

Usually, the reference-potential point is chosen to be at infinity. That is, in equation (***) we assume that $V_1 = 0$ when P_1 is at infinity, and therefore the electric potential V at any point P is given by

$$V = -\int_{\infty}^{P_2} \vec{\mathrm{E}} \cdot \vec{d\mathrm{l}}$$

Electric potential due to point charges

For a point charge q located at the origin of a spherical coordinate system, the electric field at a distance R is given by

$$\vec{\mathbf{E}} = \hat{\mathbf{R}} \frac{q}{4\pi\varepsilon R^2} \qquad \text{(V/m).}$$

Hence, we will conveniently choose the path to be along the radial direction , in which case $d\mathbf{l} = \hat{\mathbf{R}} dR$ and

$$V = -\int_{\infty}^{R} \left(\hat{\mathbf{R}} \frac{q}{4\pi\varepsilon R^2} \right) \cdot \hat{\mathbf{R}} dR = \frac{q}{4\pi\varepsilon R} \qquad \text{(V).}$$

If the charge q is at a location other than the origin, specified by a source position vector $\vec{\mathbf{R}_1}$, then V at observation position vector $\vec{\mathbf{R}}$ becomes

$$V(\vec{\mathbf{R}}) = \frac{q}{4\pi\varepsilon \left| \vec{\mathbf{R}} - \vec{\mathbf{R}_1} \right|} \qquad \text{(V),}$$

where $\left| \vec{\mathbf{R}} - \vec{\mathbf{R}_1} \right|$ is the distance between the observation point and the location of the charge q. The principle of superposition that we applied previously to the electric field $\vec{\mathbf{E}}$ also applies to the electric potential V. Hence, for N discrete point charges $q_1, q_2, \ldots, q_N$ having position vectors $\vec{\mathbf{R}_1}$, $\vec{\mathbf{R}_2}$, . . . , $\vec{\mathbf{R}_N}$, the electric potential is

$$V(\vec{\mathbf{R}}) = \frac{1}{4\pi\varepsilon}\sum_{i=1}^{N}\frac{q_i}{\left|\vec{\mathbf{R}}-\vec{\mathbf{R}}_{\mathbf{i}}\right|} \qquad \text{(V).}$$

Electric potential due to continuous distributions

Based on previous analysis:

$$V(\vec{\mathbf{R}}) = \frac{1}{4\pi\varepsilon}\int_{v'}\frac{\rho_v}{R'}\,dv' \qquad \text{(volume distribution).}$$

$$V(\vec{\mathbf{R}}) = \frac{1}{4\pi\varepsilon}\int_{S'}\frac{\rho_S}{R'}\,ds' \qquad \text{(surface distribution).}$$

$$V(\vec{\mathbf{R}}) = \frac{1}{4\pi\varepsilon}\int_{l'}\frac{\rho_l}{R'}\,dl' \qquad \text{(line distribution).}$$

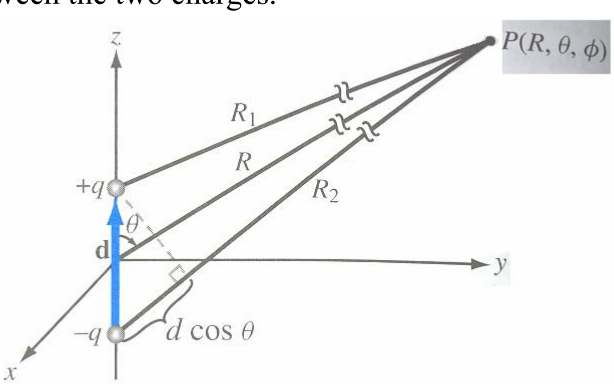
Electric field as a function of electric potential

$$\boxed{\vec{\mathbf{E}} = -\nabla\vec{V}. \qquad (\circ)}$$

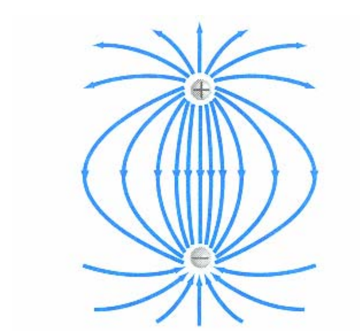
From mathematical reasoning any vector whose curl = 0 ($\vec{\nabla}\times\vec{\mathbf{E}}=0$) can be expressed as $\vec{\mathbf{E}}=\vec{\nabla}V$. The minus sign is to indicate that the work done to move q is against the field $\vec{\mathbf{E}}$.

<u>Example:</u> Electric field of an electric dipole

An electric dipole consists of two point charges of equal magnitude and opposite polarity, separated by a small distance as shown in figure. Determine V and $\vec{\mathbf{E}}$ at any point P in free space, given that P is at a distance $R \succ\succ d$, where d is the spacing between the two charges.



(a) Electric dipole


(b) Electric-field pattern

Solution:

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{R_1} + \frac{-q}{R_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{R_2 - R_1}{R_1 R_2}\right).$$

Since $R \succ\succ d$, the lines labeled R_1 and R_2 in the first figure are approximately parallel to each other, in which case the following approximations apply:

$$R_2 - R_1 \approx d\cos\theta, \quad R_2 R_1 \approx R^2$$

Hence,

$$V = \frac{qd\cos\theta}{4\pi\varepsilon_0 R^2}, \qquad (\otimes)$$

and

$$R_2 - R_1 \approx qd\cos\theta = q\mathbf{d}\cdot\hat{\mathbf{R}} = \mathbf{p}\cdot\hat{\mathbf{R}}. \qquad (\oplus)$$

where $\mathbf{p}=q\mathbf{d}$ is called the dipole moment of the electric dipole. Using the equation $(\oplus)$ in equation $(\otimes)$ then gives

$$V = \frac{\mathbf{p}\cdot\hat{\mathbf{R}}}{4\pi\varepsilon_0 R^2} \qquad \text{(electric dipole).}$$

In spherical coordinates, equation $(\circ)$ is given by

$$\begin{aligned}\vec{\mathbf{E}} &= -\nabla\vec{V} \\ &= -\left(\hat{\mathbf{R}}\frac{\partial V}{\partial R} + \hat{\boldsymbol{\theta}}\frac{1}{R}\frac{\partial V}{\partial\theta} + \hat{\boldsymbol{\varphi}}\frac{1}{R\sin\theta}\frac{\partial V}{\partial\phi}\right)\end{aligned}$$

so,

$$\vec{\mathbf{E}} = \frac{qd}{4\pi\varepsilon_0 R^3}\left(\hat{\mathbf{R}}\,2\cos\theta + \hat{\boldsymbol{\theta}}\sin\theta\right) \qquad \text{(V/m)}$$

Poisson's Equation

With $\vec{\mathbf{D}} = \varepsilon\vec{\mathbf{E}}$, the differential form of Gauss's law can be written as

$$\nabla\cdot\vec{\mathbf{E}} = \frac{\rho_v}{\varepsilon}$$

Inserting equation $(\circ)$ in the above equation gives

$$\nabla \cdot \left(\nabla \vec{V}\right) = -\frac{\rho_v}{\varepsilon}. \qquad (\times)$$

In view of the definition for the Laplacian of a scalar function *V* such as

$$\nabla^2 \vec{V} = \nabla \cdot (\nabla \vec{V}) = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Equation $(\times)$ can be cast in the abbreviated form

$$\nabla^2 \vec{V} = -\frac{\rho_v}{\varepsilon} \quad \text{(Poisson's equation).}$$

This is known as Poisson's equation. For a volume *v'* containing a volume density distribution ρ_v, the solution for *V* derived previously and expressed as

$$V = \frac{1}{4\pi\varepsilon} \int_{v'} \frac{\rho_v}{R'} dv'$$

satisfies Poisson's equation. If the medium under consideration contains no free charges, Poisson's equation reduces to

$$\nabla^2 \vec{V} = 0 \quad \text{(Laplace's equation).}$$

Conductors

The drift velocity $\overrightarrow{\mathbf{u}_{\mathrm{e}}}$ of electrons in a conducting material is related to the externally applied electric filed $\vec{\mathbf{E}}$ through

$$\overrightarrow{\mathbf{u}_{\mathrm{e}}} = -\mu_{\mathrm{e}} \vec{\mathbf{E}} \qquad \text{(m/s)},$$

where μ_{e} is a material property call the electron mobility with units of ($\mathrm{m^2/V \cdot s}$). In a semiconductor, current flow is due to the movement of both electrons and holes, and since holes are positive-charge carriers, the hole drift velocity $\overrightarrow{\mathbf{u}_{\mathrm{h}}}$ is in the same direction as $\vec{\mathbf{E}}$,

$$\overrightarrow{\mathbf{u}_{\mathrm{h}}} = -\mu_{\mathrm{h}} \vec{\mathbf{E}} \qquad \text{(m/s)},$$

where μ_{h} is the hole mobility.

The current density in a medium containing a volume density ρ_v of charges moving with a velocity $\vec{\mathbf{u}}$ is $\vec{\mathbf{J}} = \rho_v \vec{\mathbf{u}}$. Thus, the total conduction current density is

$$\vec{\mathbf{J}} = \vec{\mathbf{J}}_e + \vec{\mathbf{J}}_h = \rho_{ve}\vec{\mathbf{u}}_\mathrm{e} + \rho_{vh}\vec{\mathbf{u}}_\mathrm{h} \qquad (\mathrm{A/m^2}).$$

or

$$\vec{\mathbf{J}} = (-\rho_{ve}\mu_\mathrm{e} + \rho_{vh}\mu_\mathrm{h})\vec{\mathbf{E}}.$$

where $\rho_{ve} = -N_e e$ and $\rho_{vh} = -N_h e$, with N_e and N_h being the number of free electrons and the number of free holes per unit volume, and $e = 1.6\times10^{-19}$ C is the absolute charge of a single hole or electron. The quantity inside the parentheses is defined as the conductivity of the material, σ. Thus,

$$\begin{aligned}\sigma &= -\rho_{ve}\mu_\mathrm{e} + \rho_{vh}\mu_\mathrm{h} \\ &= (N_e\mu_e + N_h\mu_h)e \qquad (\mathrm{S/m}) \quad (\text{semiconductors}),\end{aligned}$$

so,

$$\vec{\mathbf{J}} = \sigma\vec{\mathbf{E}} \quad (\mathrm{A/m^2}) \quad (\text{Ohm's law}).$$

This equation is called the point form of Ohm's law. Note that, in a perfect dielectric with $\sigma = 0, \vec{\mathbf{J}} = 0$ regardless of $\vec{\mathbf{E}}$, and in a perfect conductor with $\sigma = \infty, \vec{\mathbf{E}} = \vec{\mathbf{J}}/\sigma = 0$ regardless of $\vec{\mathbf{J}}$. That is,

$$\text{Perfect dielectric}: \quad \vec{\mathbf{J}} = 0$$

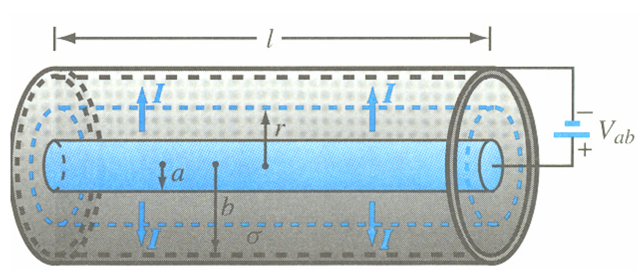
$$\text{Perfect conductor}: \ \vec{\mathbf{E}} = 0$$

Resistance

$$R = \frac{V}{I} = \frac{\int_l \vec{\mathbf{E}}\cdot d\vec{\mathbf{l}}}{\int_S \vec{\mathbf{J}}\cdot d\vec{\mathbf{s}}} = \frac{\int_l \vec{\mathbf{E}}\cdot d\vec{\mathbf{l}}}{\int_S \sigma\vec{\mathbf{E}}\cdot d\vec{\mathbf{s}}}.$$

<u>Example:</u> Conductance of coaxial cable

The radii of the inner and outer conductors of a coaxial cable of length *l* are *a* and *b* respectively. The insulation material has conductivity σ. Obtain an expression for *G'*, the conductance per unit length of the insulation layer.



Solution:

Let I be the total current flowing from the inner conductor to the outer conductor through the insulation material. At any radial distance r from the axis of the center conductor, the area through which the current flows is $A = 2\pi r l$. Hence,

$$\vec{\mathbf{J}} = \hat{\mathbf{r}}\frac{l}{A} = \hat{\mathbf{r}}\frac{I}{2\pi r l},$$

and from $\vec{\mathbf{J}} = \sigma\vec{\mathbf{E}}$,

$$\vec{\mathbf{E}} = \hat{\mathbf{r}}\frac{I}{2\pi\sigma r l}$$

In a resistor, the current flows from higher electric potential to lower potential. Hence, if $\vec{\mathbf{J}}$ is the $\hat{\mathbf{r}}$-direction, the inner conductor must be at a higher potential than the outer conductor. Accordingly, the voltage difference between the conductors is

$$V_{ab} = -\int_b^a \vec{\mathbf{E}}\cdot d\vec{\mathbf{l}} = -\int_b^a \frac{I}{2\pi\sigma l}\frac{\hat{\mathbf{r}}\cdot\hat{\mathbf{r}}\,dr}{r}$$

$$= \frac{I}{2\pi\sigma l}\ln\left(\frac{b}{a}\right).$$

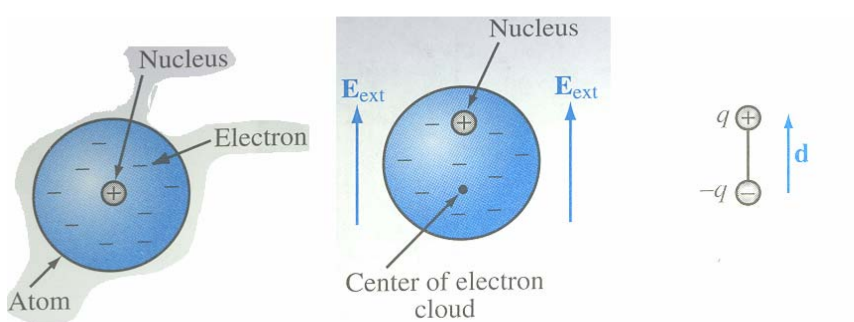
The conductance per unit length is then

$$G' = \frac{G}{l} = \frac{1}{Rl} = \frac{I}{V_{ab}l} = \frac{2\pi\sigma}{\ln(b/a)} \qquad \text{(S/m)}$$

Dielectrics

In a dielectric, an externally applied field $\overrightarrow{\mathbf{E}_{\mathbf{ext}}}$ cannot effect mass migration of charges since none are able to move freely, but it can polarize the atoms or molecules in the material by distorting the center of the cloud and the location of the nucleus.

In the absence of an external electric field $\overrightarrow{\mathbf{E}_{\mathbf{ext}}}$, the center of the electron cloud is co-located with the center of the nucleus, but when a field is



(a) $\mathbf{E}_{\text{ext}} = 0$ (b) $\mathbf{E}_{\text{ext}} \neq 0$ (c) Electric dipole applied, the two centers are separated by a distance *d*.

The induced electric field, called a polarization field, is weaker than and opposite in direction to $\overrightarrow{\mathbf{E}_{\mathbf{ext}}}$. Consequently, the net electric field present in the dielectric material is smaller than $\overrightarrow{\mathbf{E}_{\mathbf{ext}}}$.

Whereas $\vec{\mathbf{D}}$ and $\vec{\mathbf{E}}$ are related by ε_0 in free space, the presence of these microscopic dipoles in a dielectric material alters that relationship in that material to

$$\vec{\mathbf{D}} = \varepsilon_0 \vec{\mathbf{E}} + \vec{\mathbf{P}},$$

where $\vec{\mathbf{P}}$, called the electric polarization field, accounts for the polarization properties of the material. A dielectric medium is said to be linear if the magnitude of the induced polarization field is directly proportional to the magnitude of $\vec{\mathbf{E}}$, and it is said to be isotropic if the polarization field and $\vec{\mathbf{E}}$ are in the same direction.

A medium is said to be homogenous if its constitutive parameters (ε, μ and σ) are constant throughout the medium.

$$\vec{\mathbf{P}} = \varepsilon_0 \chi_e \vec{\mathbf{E}},$$

where χ_e is called the electric susceptibility of the material. Thus we have

$$\begin{aligned} \vec{\mathbf{D}} &= \varepsilon_0 \vec{\mathbf{E}} + \varepsilon_0 \chi_e \vec{\mathbf{E}} \\ &= \varepsilon_0 (1 + \chi_e) \vec{\mathbf{E}} = \varepsilon \vec{\mathbf{E}}, \end{aligned}$$

which defines the permittivity of the material as

$$\varepsilon = \varepsilon_0 (1 + \chi_e)$$

In reality, if $\vec{\mathbf{E}}$ exceeds a certain critical value, known as the dielectric strength of the material, it will free the electrons completely from the molecules and cause them to accelerate through the material in the form of a conducting current. When this happens, sparking can occur, and the dielectric material can sustain permanent damage due to electron collision with the molecular structure. This abrupt change in behavior is called dielectric breakdown.