American University of Beirut
Department of Electrical and Computer Engineering

EECE 311 – Electronic Circuits — Spring 2008 – 2009

Homework 2 Solution

Problem 1.

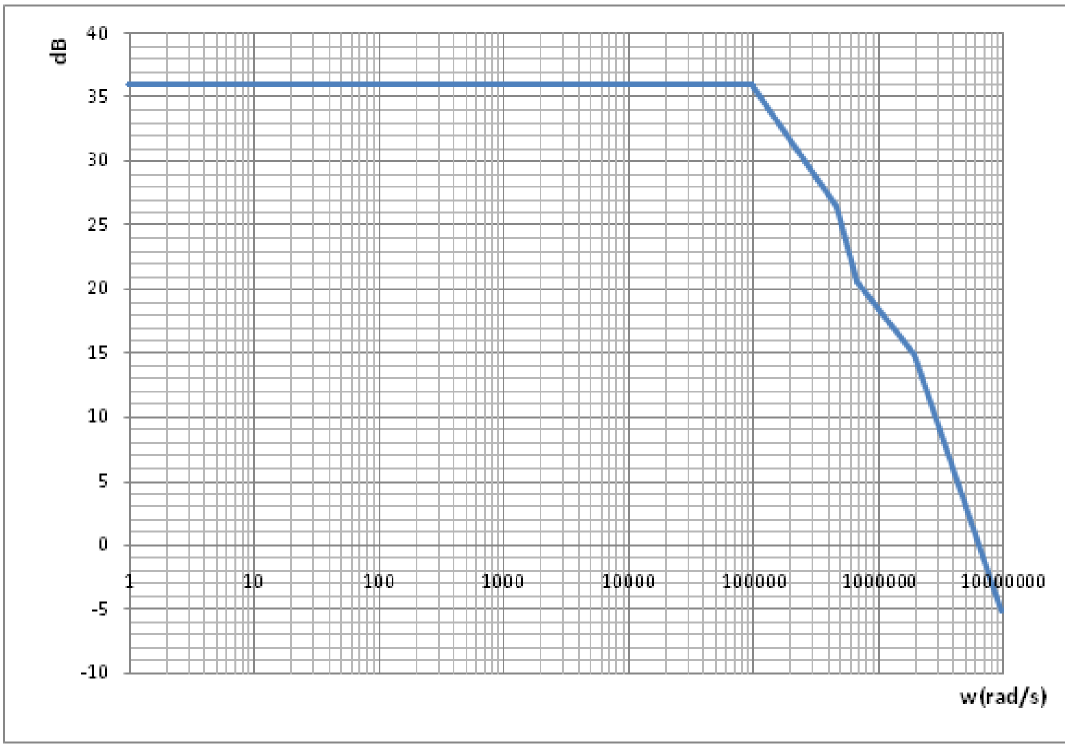
Problem 1 – Frequency Response [52 pts]

a) [2 pts] A direct-coupled amplifier has a low-frequency gain of 36 dB, poles at 15 KHz, 72 KHz, and 300 KHz, a zero at 105 KHz, and two more zeros at infinity. Express the amplifier gain function in the form:

$$A(s) = A_M \frac{\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)\cdots\left(1+\frac{s}{\omega_{zn}}\right)}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)\cdots\left(1+\frac{s}{\omega_{pn}}\right)}.$$

$$20\log A_m = 36\,dB \Rightarrow A_m = 63.1$$
$$\omega_{p1} = 2\pi \times (15K) = 94.25\ Krad/s$$
$$\omega_{p2} = 2\pi \times (72K) = 452.39\ Krad/s$$
$$\omega_{p3} = 2\pi \times (300K) = 1884.96\ Krad/s$$
$$\omega_{z1} = 2\pi \times (105K) = 659.73\ Krad/s$$
$$\Rightarrow A(s) = 63.1 \times \frac{\left(1+\frac{s}{659.73K}\right)}{\left(1+\frac{s}{94.25K}\right)\left(1+\frac{s}{452.39K}\right)\left(1+\frac{s}{1884.96K}\right)}$$

b) [8 pts] Sketch the Bode plot for the magnitude of the gain.



c) Calculate the 3-dB frequency f_H for this amplifier using:

i) [2 pts] the dominant pole approximation

Since the pole at 15 KHz is more than 2 octaves below all the other poles, the dominant pole is the one at 15 KHz ⇒ f_H = 15 KHz.

ii) [4 pts] the definition of the 3-dB frequency (i.e. find the exact value of f_H).

$$|F(j\omega)| = \frac{1}{\sqrt{2}} = \frac{\sqrt{\left(1+\left(\frac{\omega}{659.73K}\right)^2\right)}}{\sqrt{\left(1+\left(\frac{\omega}{94.25K}\right)^2\right)}\sqrt{\left(1+\left(\frac{\omega}{452.39K}\right)^2\right)}\sqrt{\left(1+\left(\frac{\omega}{1884.96K}\right)^2\right)}}$$

$By\ using\ MATLAB\ we\ get{:}\ \omega_H = 92.02\ Krad/s \ \Rightarrow f_H = 14.645\ KHz$

iii) [2 pts] What is the error (in %) in the value of f_H due to the dominant pole approximation?

$The\ \%\ error\ is{:} \frac{|14.645-15|}{15} \times 100\% = 2.37\%$ (over-estimated in this case by the dominant pole approximation)

d) Calculate the frequency $\boldsymbol{f_t}$ at which the gain of the amplifier becomes unity (or 0 dB) using:

i) [2 pts] the dominant pole approximation

By the dominant pole approximation we can use $f_t = A_m \times f_H = 946.5\ KHz$

ii) [5 pts] the definition of f_t.

$$|F(j\omega)| = 1 = 63.1 \times \frac{\sqrt{\left(1+\left(\frac{\omega}{659.73K}\right)^2\right)}}{\sqrt{\left(1+\left(\frac{\omega}{94.25K}\right)^2\right)}\sqrt{\left(1+\left(\frac{\omega}{452.39K}\right)^2\right)}\sqrt{\left(1+\left(\frac{\omega}{1884.96K}\right)^2\right)}}$$

$By\ using\ MATLAB\ we\ get{:}\ \omega_t = 2498.11\ Krad/s \ \Rightarrow f_t = 397.587\ KHz$

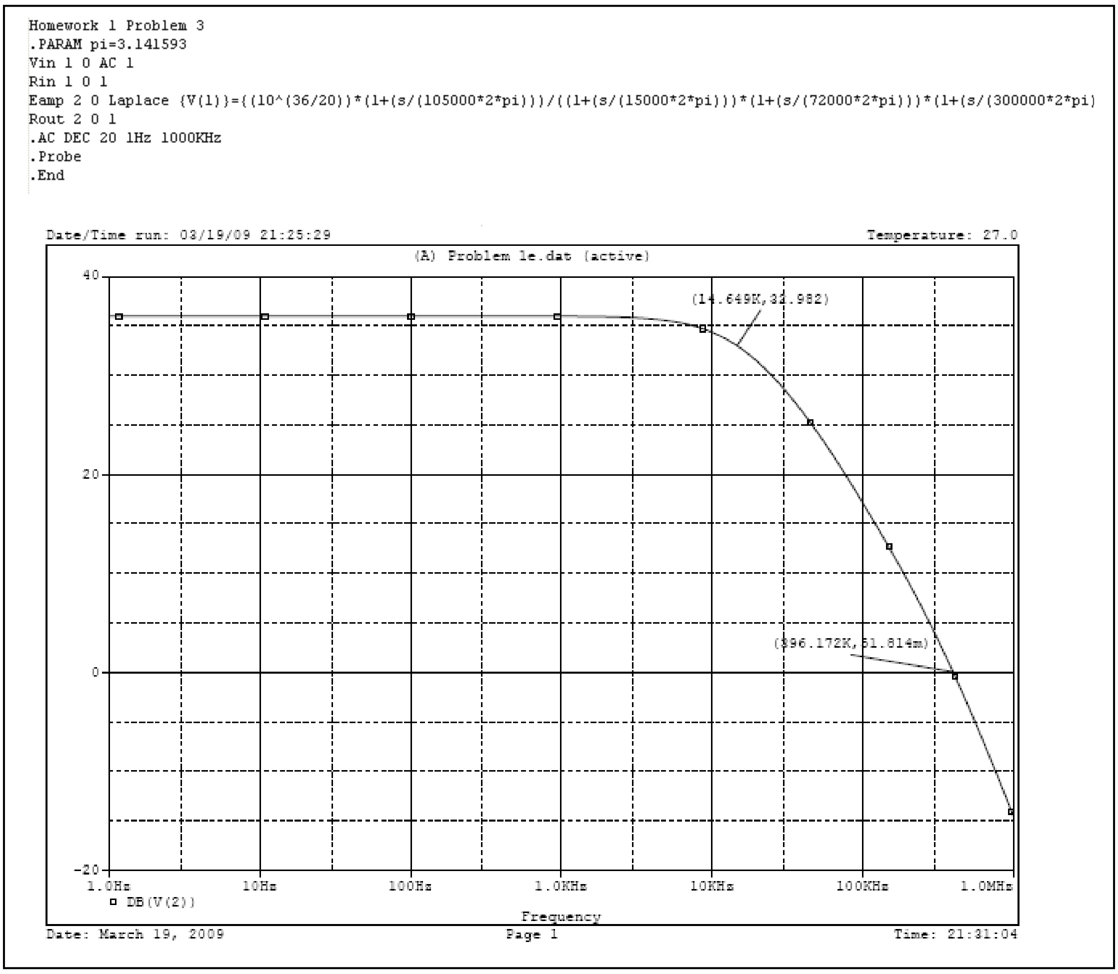
iii) [5 pts] Comment on the usefulness of the dominant pole approximation in estimating f_t.

The dominant pole approximation is **not** useful in approximating f_t, unless there is only one

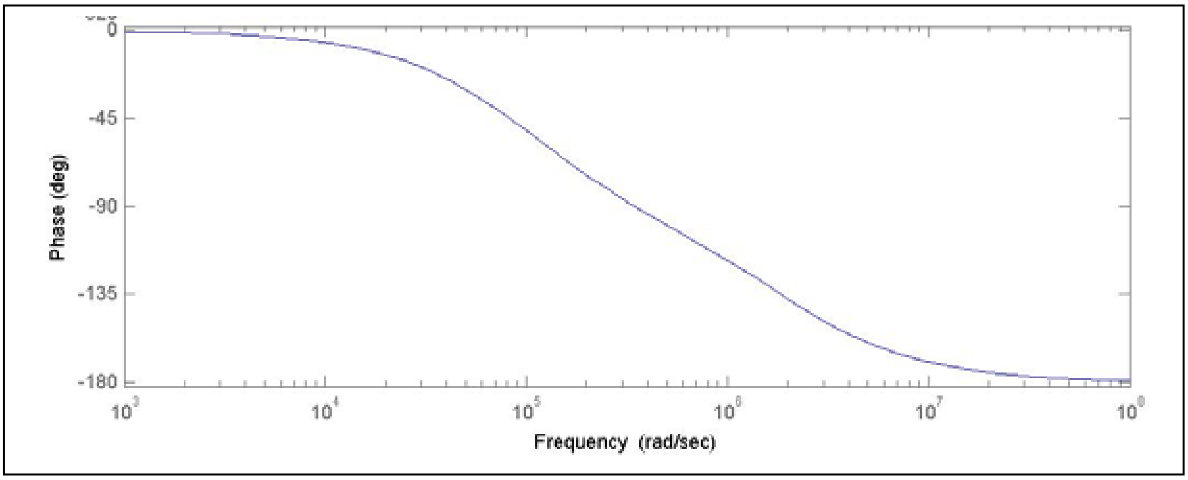
e) Verify using PSpice the values of f_H [5 pts] and f_t [5 pts] calculated above. Use

```
Homework 2 Problem 1
.PARAM pi=3.141593
Vin 1 0 AC 1
Rin 1 0 1
Eamp 2 0 Laplace {V(1)}={ your gain function here }
Rout 2 0 1
.AC DEC 20 1Hz 1000KHz
.Probe
.End
```

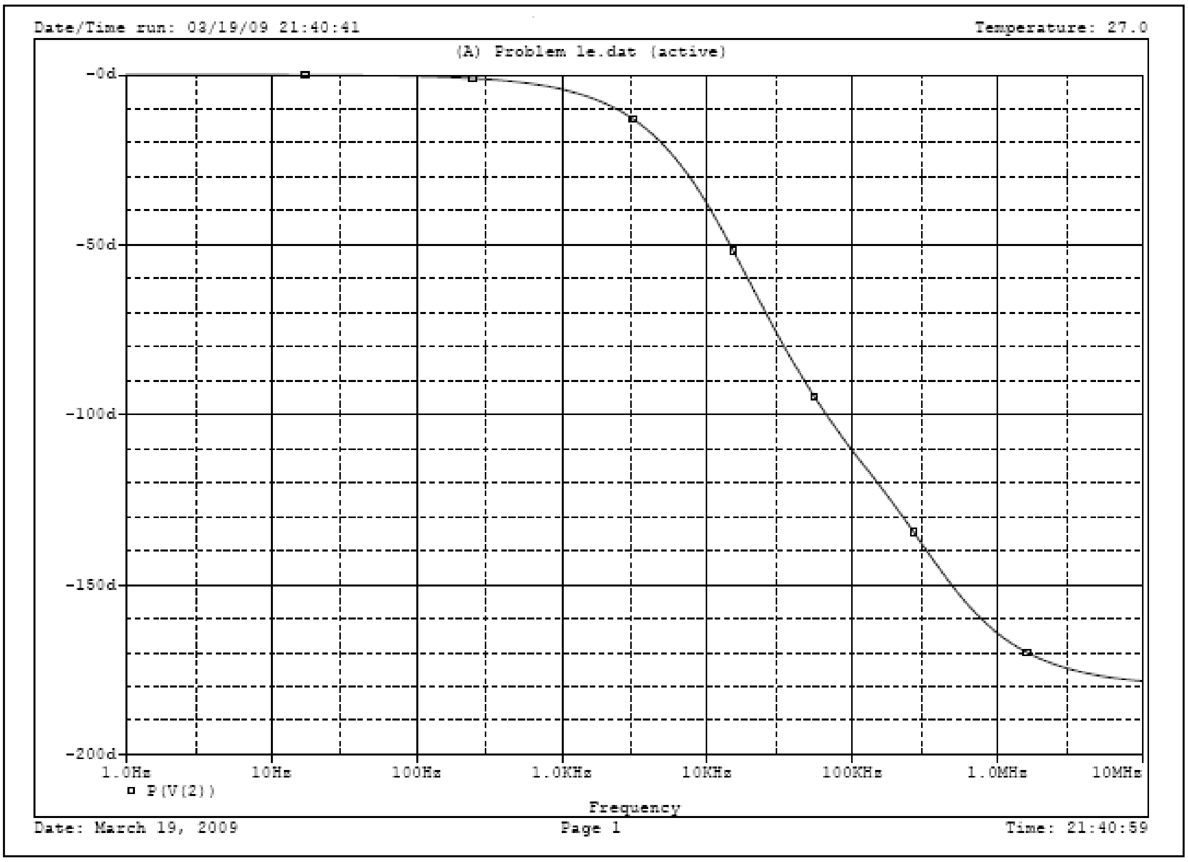
and plot `dB(v(2))`.



f) [8 pts] Sketch the Bode plot for the phase of the gain.

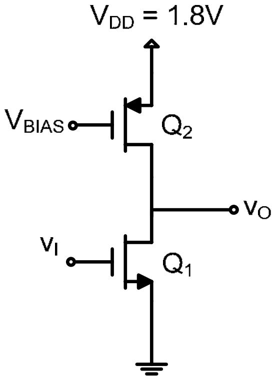


g) [4 pts] Verify using PSpice the results of part (f). Change the .AC statement to
`.AC DEC 20 1Hz 10MegHz`
The phase is obtained by plotting `p(v(2))`.



Problem 2 – Common Source Amplifier [20 pts]

The common-source stage shown below must provide a voltage gain of 10 and has a *power budget* of 500 μW.



The technology parameters are: k'_n = 330 μA/V^2, V_{tn} = 0.4 V, V_{An} = 6 V,
k'_p = 120 μA/V^2, V_{tp} = –0.45 V, and $|V_{Ap}|$ = 5 V.

Assume in the following that for the DC quantities: $I_D \simeq \frac{1}{2}k'\left(\frac{W}{L}\right)V_{OV}^2$.

a) [10 pts] Find the value of $(W/L)_1$

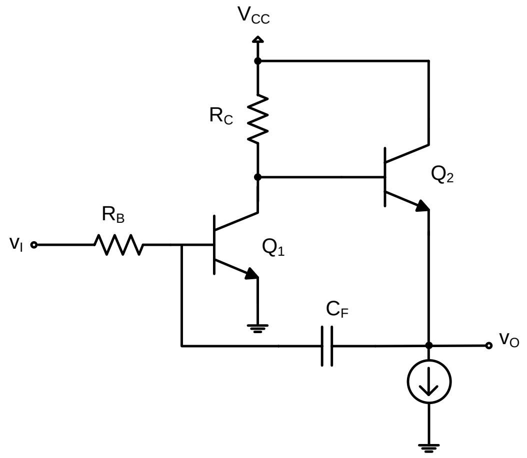
$$I_{DD} = \frac{P}{V_{DD}} = 0.277\ mA$$
$$r_{o1} = \frac{V_{AN}}{I} = 21.6\ K\Omega,\ \ r_{o2} = \frac{V_{AP}}{I} = 18.1\ K\Omega$$
$$|A_{V1}| = g_{m1}(r_{o1}//r_{o2}) \Rightarrow g_{m1} = 1.015\ mA/V$$
$$But, g_{m1} = \frac{2I}{V_{OV1}} \Rightarrow V_{OV1} = 0.546\ V$$
$$I = \frac{1}{2}k'_n\left(\frac{W}{L}\right)_n V_{OV1}^2 \Rightarrow \left(\frac{W}{L}\right)_n = 5.63$$

b) [10 pts] Find the required value of V_{BIAS} if $(W/L)_2$ is (18/0.18).

$$I = \frac{1}{2}k'_p\left(\frac{W}{L}\right)_2 V_{OV2}^2, in\ Q2$$
$$V_{OV2} = 0.215\ V \Rightarrow V_{OV2} = V_{DD} - V_{BIAS} - |V_{tp}| \Rightarrow V_{BIAS} = 1.135\ V$$

Problem 3 – Miller's Theorem [28 pts]

Use Miller's theorem to estimate the input [12 pts] and output [16 pts] poles of the circuit shown below, in terms of C_F, R_B, R_C, and the small-signal parameters of Q_1 and Q_2 (g_{m1}, g_{m2}, $r_{\pi1}$, $r_{\pi2}$, r_{o1}, & r_{o2})
You may use $\beta = g_m r_\pi$.
C_F is much larger than the *internal* BJT capacitors, and they can be neglected.
The current source is ideal.



The two BJT's in this circuit are cascaded, so the total
gain is the product of the gain of each BJT stage independently.

The gain of the first stage is $-g_{m1}$ (R_C // r_{o1} // R_{in2}). The input resistance of stage 2 is R_{in2} = (β+1)(r_{e2}+r_{o2}), which is much larger than r_{o1} // R_C and may be neglected.
The gain of the second stage is $r_{o2}/(r_{o2}+r_{e2})$, which is very close to 1, since $r_o >> r_e$
$\Rightarrow K \cong -g_{m1}(R_c//r_{o1}) \times 1$
by Miller's theorem we can seperate
C_F in to 2 different capacitors, one on the
side of the input and one on the input.
At the input: $C_{in} = C_F(1-K)$
At the output: $C_{out} = C_F\left(1-\frac{1}{K}\right)$
The resistance seen by C_{in} is:
$R_{Cin} = (R_B//r_{\pi1})$
The resistance seen by C_{out}: Replace C_{out} by a current source I_x having voltage V_x, or use the equation on page 485 in the textbook:

$$R_{Cout} = \frac{V_x}{I_x} = \frac{(r_{\pi2}+R_C//r_{o1})r_{o2}}{(1+\beta)r_{o2}+r_{\pi2}+R_C//r_{o1}}$$

$$*\ \textit{the pole at the input is:}\ \frac{1}{R_{Cin}C_{in}} = \frac{1}{(R_B//r_{\pi1})C_F(1+g_{m1}(R_c//r_{o1}))}$$

$$*\ \textit{the pole at the output is:}\ \frac{1}{R_{Cout}C_{out}}$$

$$= \frac{(1+\beta)r_{o2}+r_{\pi2}+R_C//r_{o1}}{(r_{\pi2}+R_C//r_{o1})r_{o2}C_F\left(1+\frac{1}{g_{m1}(R_c//r_{o1})}\right)}$$