AMERICAN UNIVERSITY OF BEIRUT

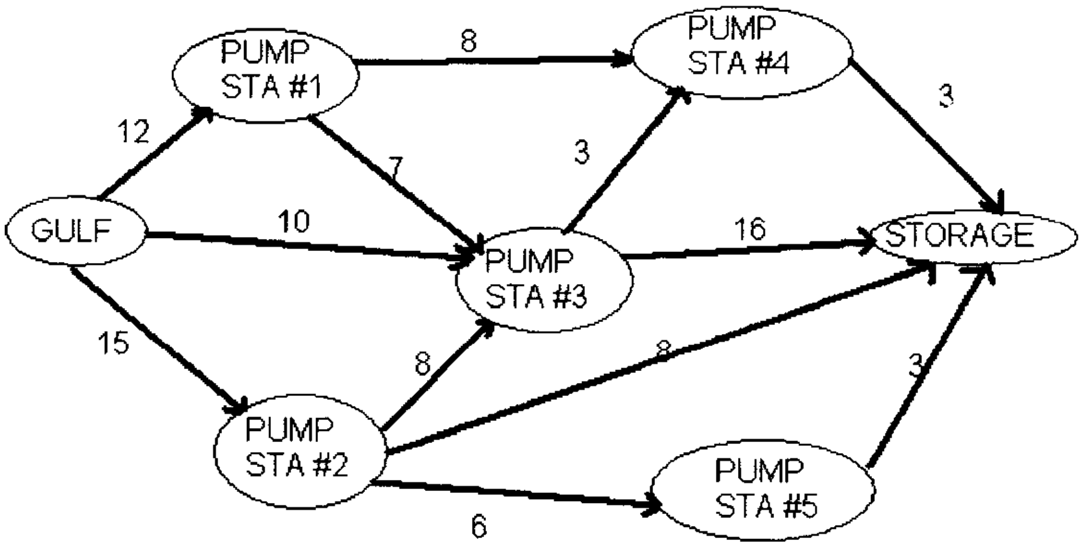
School of Business

OPIM 205: Computer Modeling for Management

Spring 2002-2003 Final exam Version A:

Date, June 7th, 2003; Time: 11:00am-1:00 pm.

(Question 1, 25 marks) Oil Company is planning to operate pipelines for transporting oil from its source in the Gulf to storage facilities in the mainland. Before reaching the storage facilities, the oil must pass through one or more pumping stations. The network below gives the maximum amount that can be transported in the pipes between sites in thousands of gallons *per hour*.



A. Assuming a 24 hour per day operation, what is the maximum volume of oil that can be pumped from the source to the storage facility per day? [10]

According to the 'Max Flow Min Cut Theorem'. The max. flow along the network is the sum of max. flows along the arcs of the min. cut. The min. cut along the network is the green cut above which has capacity 30. This can be found by identifying all cuts in the network (there are 8 or 9 of them). Alternatively, one can use the Ford-Fulkerson algorithm also as shown on the network above.

B. Where are the bottlenecks in the network?

Bottlenecks are the capacities of the four arcs leading into the storage area along the green cut of the network. Another that can be considered to be a bottleneck is the capacity of the arc from PUMP STA #1 to PUMP STA #2
[2]

C. **Assuming that the numbers on the arcs represent distance in miles, and a fire brigade station is located at the source site (GULF) with responsibility to provide emergency cover to all sites in the network. The chief officer wants to draw an emergency plan showing the list of minimum cost routes to reach each PUMP/STORAGE station from the main office located at the source site. List those routes and their associated distances.** [15]

This is a shortest path problem with source the GULF node. Dijkstra's algorithm generates the shortest paths from the GULF node to the rest of the nodes

Node	Tentative dist. d()	Tentative predecessor	Label()	Predecessor
Gulf			0	
Pump STA #1	12	GULF	12	GULF
Pump STA #2	15	GULF	15	GULF
Pump STA #3	10	GULF	10	GULF
Pump STA #4	10+3=13	Pump STA #3	13	Pump STA #3
Pump STA #5	15+6=21	Pump STA #2	21	Pump STA #2
STORAGE	10+16=26 13+3=16	Pump STA #3. Pump STA #4	16	Pump STA #4

According to the tree the shortest path from GULF to.

1. Pump STA #1 is directly from GULF for a distance of 12
2. Pump STA #2 is directly from GULF for a distance of 15
3. Pump STA #3 is directly from GULF for a distance of 10.
4. Pump STA #4 is through Pump STA #3 and GULF for a distance of 13.
5. Pump STA #5 is through Pump STA #2 and GULF for a distance of 21.
6. STORAGE is though Pump STA #4 Pump STA #3 and GULF for a distance of 16

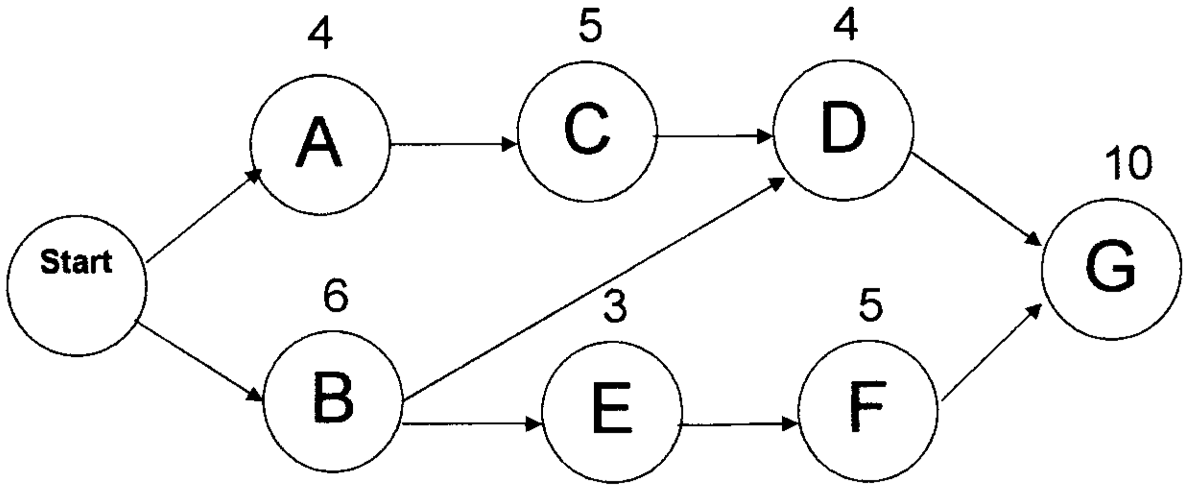
D. **What are the names of the solution methods used in solving question A,B & C? [3]**

A. Ford-Fulkerson.
B: Max-Flow/Min Cut Theorem
C: Dijkstra's algorithm

(**Question 2, 30 Marks**) Consider the following information about activities for the design of a new laptop. The times are in weeks and the costs are in '000s.

Activity	Immediate Predecessors	Normal Duration	Crashed Duration	Variance	Normal Cost	Crashed Cost
A		4	2	2	10	60
B		6	3	3	20	110
C	A	5	4	2	15	50
D	B, C	4	3	1	25	70
E	B	3	1	1	15	55
F	E	5	1	2	25	65
G	D, F	10	4	5	20	50

A. Determine the critical path and the expected completion time for this project using normal times? [8]



'Start' is a dummy activity of duration 0 and cost 0

ES(Start) = 0

ES(A) = 0, EF(A) = 0 + 4 = 4,

ES(B) = 0, EF(B) = 0 + 6 = 6,

ES(C) = EF(A) = 4, EF(C) = 4 + 5 = 9

ES(D) = max{EF(C), EF(B)} = max{9, 6} = 9, EF(D) = 9 + 4 = 13

ES(E) = EF(B) = 6, EF(E) = 6 + 3 = 9,

ES(F) = EF(E) = 9, EF(F) = 9 + 5 = 14,

ES(G) = max{EF(D), EF(F)} = max{13, 14} = 14, EF(G) = 14 + 10 = 24

The completion time of the project under normal times is then 24

LF(G) = 24, LS(G) = 14,

LF(D) = LS(G) = 14 LS(D) = 14 − 4 = 10

LF(F) = LS(G) = 14, LS(F) = 14 − 5 = 9,

LF(E) = LS(F) = 9 LS(E) = 9 − 3 = 6

LF(C) = LS(D) = 10, LS(C) = 10 − 5 = 5,

LF(B) = min{LS(D), LS(E)} = min{10, 6} = 6, LS(B) = 6 − 6 = 0.
LF(A) = LS(C) = 5, LS(A) = 5 − 4 = 1.
LF(Start) = min{LS(A), LS(B)} = min{1, 0} = 0, LS(Start) = 0 − 0 = 0.

Slack(Start) = LS(Start) − ES(Start) = 0 − 0 = 0.
Slack(A) = LS(A) − ES(A) = 1 − 0 = 1
Slack(B) = LS(B) − ES(B) = 0 − 0 = 0
Slack(C) = LS(C) − ES(C) = 5 − 4 = 1.
Slack(D) = LS(D) − ES(D) = 10 − 9 = 1.
Slack(E) = LS(E) − ES(E) = 6 − 6 = 0.
Slack(F) = LS(F) − ES(F) = 9 − 9 = 0.
Slack(G) = LS(G) − ES(G) = 14 − 14 = 0.

The critical path is Start − B − E − F − G.

B. What is the simultaneous effect of a 1-week delay to activity C and a 1-week delay to activity D on the completion time of the projects? **[2]**

The slack of activity C is 1 and that of activity D is 1 also. The combined effect of delaying both activities by one time unit is to delay the project by 1 unit

C. What would be the net expected gain or loss if the design department will get a bonus of \$30,000 if the project takes less than 22 weeks, \$0 if the project is completed within 22 and 25 weeks, and a penalty of US\$10,000 if it takes longer than 25 weeks? **[8]**

Let X be the random variable associated with the completion time of the project and r.v.(I) denote the random variable associated with the duration of activity I, I = Start, ..., G. Assume the activity durations of the project are independent and that the critical path does not change with changes in the activity durations

Expected net gain = 30,000*P(X<22) + 0*P(22 ≤ X ≤ 25) − 10,000*P(X > 25)
Var(X) = var[r.v.(Start)] + var [r.v.(B)] + var [r.v.(E)] + var [r.v.(F)] + var [r.v.(G)]
= 11
Mean(X) = 24 as computed earlier. Therefore, assuming X is Normal with mean 24 and variance 11, we get
Expected net gain
= 30,000*Normdist(22,24,sqrt(11),True) − 10,000*(1-Normdist(25,24,sqrt(11), True))
= 30,000*0.273247 − 10,000*0.38151
= \$4382.279

D. Can you suggest a minimum crash cost schedule in order to complete the projects within 16 weeks? **[4]**

In this case, we need to shave off 8 weeks from the normal completion time
Notice that activity G on the critical path can be reduced from 10 to 4 weeks for a savings of 6 weeks. Similarly, activities A and B can be reduced by 2 weeks each without affecting the slackness of any of the activities, for a total savings of 8 weeks

E. Write a linear programming model to find the optimal crash cost schedule, which determines the amount by which an activity is to be crashed to complete the project within 16 weeks. [8]

Assume that the activities durations can be reduced at a constant cost. Let M(I) be the cost of reducing activity I by 1 week, I = Start, ..., G. Then,

M(Start) = 0

M(A) = (60 - 10)/(4-2) = 25

M(B) = (110 - 20)/(6-3) = 30

M(C) = (50 - 15)/(5 - 4) = 15

M(D) = (70 - 25)/(4 - 3) = 45

M(E) = (55 - 15)/(3 - 1) = 20

M(F) = (65 - 25)/(5 - 1) = 10

M(G) = (50 - 20)/(10 - 4) = 5

The LP to find the optimal crash durations for the activities for a project completion time of 16 weeks is as follows.

Let Y(I) be the number of weeks the project is to be reduced by and S(I) be the start time of activity I, I = Start, ..., G

Min 25 Y(A) + 30 Y(B) + 45 Y(C) + 45 Y(D) + 20 Y(E) + 10 Y(F) + 5 Y(G)

(cost function)

s.t.

Y(A) ≤ 2, Y(B) ≤ 3, Y(C) ≤ 45, Y(D) ≤ 1, Y(E) ≤ 20, Y(F) ≤ 10, Y(G) ≤ 6

(maximum reductions have upper bounds)

S(C) ≥ S(A) + 4 − Y(A)

S(D) ≥ S(C) + 5 − Y(C)

S(D) ≥ S(B) + 6 − Y(B)

S(E) ≥ S(B) + 6 − Y(B)

S(F) ≥ S(E) + 3 − Y(E)

S(G) ≥ S(D) + 4 − Y(D)

S(G) ≥ S(F) + 5 − Y(F) **(precedence relationships must be respected)**

S(G) + 10 − Y(G) ≤ 16 **(the project must be finished within 16 weeks)**

S(I), Y(I) ≥ 0 for all I

(Question 3, Marks 20)

A Lebanese food company has decided to produce fruit yogurt for which the annual demand potential is an estimated 1.2 million boxes. A box contains six pieces of 250 grams each. The company has a choice of two machines to lease for producing the fruit yogurt. The annual lease costs of machines are $40,000 and $45,000 whereas the daily production capacities are 2750 and 11,000 boxes, respectively.

The factory is open 365 days a year. The production setup cost is estimated at $4000, while the holding cost is estimated at $0.10 per box per year. The company estimates that it will earn $1.20 on each box produced.

A. **Determine which machine should be leased in order to minimize the annual total cost.** **[10]**

Let D be the annual demand. The total annual costs in this case consist of inventory cost, the setup costs and the lease costs. The given setup cost is $C_s = \$4000$ and the holding cost per box is $C_h = \$0.1$

<u>Option 1</u>

Let L_1 be the lease cost of the machine.

The yearly machine capacity, $K = 2750*365 = 1{,}003{,}750$ boxes which is less than the yearly demand. Therefore, ideally, there is no inventory. Ideally, there is no setup cost over the long run either, because once the machine is setup, it never needs to be setup again. The only cost then is the lease cost.

The minimum annual total cost is: $L_1 = \$40{,}000$

<u>Option 2</u>

Let L_2 be the lease cost of the machine.

The yearly machine capacity, $K = 11000*365 = 4{,}015{,}000$ boxes which is greater than the yearly demand.

Therefore, using the optimal production lot size (section 8.5):

$Q^* = \text{sqrt}\{2*1{,}200{,}000*4000/[(1 - 1{,}200{,}000/4{,}015{,}000)*0.1]\} = 370{,}032$ boxes

The annual minimum total cost is: $(Q^*/2)(1 - D/K)C_h + (D/Q^*)C_s + L_2 = \$70{,}943.71$

Clearly, Option 1 is less costly so machine 1 should be leased.

<u>Remark:</u> Note that in practice, one would want to maximize profits rather than minimize cost. If revenue were constant in both cases, the two objectives would have been equivalent. But, it so happens that here they are not because the yearly capacity of machine 1 is less than the yearly demand. An analysis that is more relevant in practice would have compared profits not costs. This can be done using the given earnings per box.

B. **What would be the decision (with regards to the order quantity and the associated annual cost) if the fruit yogurt boxes couldn't be kept in storage for more than three weeks after production?** **[10]**

<u>Option 1</u>

In this case, the analysis does not change because the machine capacity is less than the demand.

<u>Option 2</u>

Let T_1 be the production period for the yogurt and T2 be the period length when the yogurt is not produced (both in years). Assuming that the demand rate is constant, (K-

(K-D) T_1 = D T_2 because the production during period T_1 should be equal to the consumption during period $T_1 + T_2$. This also says that T_2 = (K/D − 1) T_1 and that $T_1 + T_2 = K T_1/D$

Now, since the yogurt should not spoil, we want $T_2 \le 3/52$ (yogurt spoils in 3 weeks)

The average inventory in the system will be (K-D) T_1/2 + DT_2/2 and the number of orders per year will be 1/($T_1 + T_2$). Hence, the variable cost is
TV(T_1) = (K-D) $T_1 C_h$/2 + C_o/($T_1 + T_2$). The problem is Min TV(T_1) s.t.
$T_1 \le 21D/(K-D)$ and $T_1 \ge 0$

Note that TV(T_1) = (K-D) $T_1 C_h$/2 + DC_o/KT_1. Without the upper bound on T_1, the optimal value of T_1 would have been 0.091 years = 33.64 days. With the upper bound, the minimum value of TV(T_1) occurs at T_1 = 8.95 days ≈ 9 days. It follows that T_2 = 21 days and TV(T_1) = $3451.27 + 48,666.67 = $52,117.94. Therefore, factoring in the lease, the annual cost of this option is $97,117.94.

Clearly, option 1 with an annual cost of $40,000 is still better.

Note that the same remark as in the first question still applies. In practice, it is more relevant to compare maximum profits rather than minimum costs.

In practice also, the machine in option 2 may be used to produce different kinds of products during the idle days of the yogurt cycle. So its lease cost could be shared among the different products on a time proportional basis. The analysis of that problem is a lot more complicated though; even if we assume the items are non perishable (in which case the problem is the so called Economic Lot Sizing Problem).

(Question 4, Marks 25) Suppose that a cell phone operator offers customers a choice of three monthly leasing plans:

Plan I: $30 per month with 20 free minutes and $.30 per minute
for additional minutes
Plan II: $40 per month with 30 free minutes and $.20 per minute
for additional minutes
Plan III: $60 per month with 100 free minutes and $.10 per minute
for additional minutes

An AUB student wanted to lease a line. He/she estimates that the total amount of talk time can be approximated by the following distribution:

Time	**Probability**
30 minutes	.30
60 minutes	.40
100 minutes	.20
150 minutes	.10

A. Construct the payoff cost matrix of leasing the phone under the three plan. [5]

	Usage=30 mins	Usage=60 mins.	Usage=100 mins	Usage=150 mins
Plan 1	30+(30-20)*0.3 = 33	30+(60-20)*0.3 = 42	30+(100-20)*0.3 = 54	30+(150-20)*0.3 = 69
Plan 2	40	40+(60-30)*0.2 = 46	40+(100-30)*0.2 = 54	40+(150-30)*0.2 = 64
Plan 3	60	60	60	60+(150-100)*0.1 = 65

B. If he/she were an optimistic student, which plan would he/she chooses? [5]

She would choose according to the minimin criterion. I.e. minimize across each row and then across the minima that were obtained. That choice would correspond to Plan 1 and the "Usage = 30 mins" state of nature.

C. If he/she bases the leasing decision on the expected value criterion, which leasing plan would be chosen?

[7]

EV(Plan 1) = 0.3 (33) + 0.4 (42) + 0.2 (54) + 0.1 (69) = 44.4
EV(Plan 2) = 0.3 (40) + 0.4 (46) + 0.2 (54) + 0.1 (64) = 47.6
EV(Plan 3) = 0.3 (60) + 0.4 (60) + 0.2 (60) + 0.1 (65) = 60.5

The least expected value is associated with Plan 1. Therefore, Plan 1 is chosen in this case.

D. If the student were able to seek an advice from an expert consultant on the amount of perfect talk time, how much the student should pay the consultant for the advice. [8]

EVPI = ERPI − EREV
We already computed EREV in part C as 44.4
ERPI = 0.3 (33) + 0.40 (42) + 0.2 (54) + 0.1 (64) = 43.9. Therefore, EVPI = 0.5 and the student should pay no more than $0.50 for the consultant