MATH101 mid-term quiz 2

Saturday 3 December 2011

Name: ______________________________ ID: ____________________

Section: **Khachadourian** 5 6 7 8 **Bright** 9 10 11

Question:	1	2	3	4	5	6	7	Total
Points:	20	14	10	14	15	15	12	100
Score:								

1. (20 points) For each of the following functions, find $\frac{\mathrm{d}y}{\mathrm{d}x}$. You may use any differentiation rules you like, without justification.

   (a) $y = 4x^3 + 7x - 2$

   (b) $y = (x^2 + \sin(3x))^3$

   (c) $y = (5x^2 + 3)\tan x$

   (d) $y = \dfrac{x^2 - 2}{x^2 - 3}$

2. Consider the function $f(x) = (x+5)^2(x-4)^{1/3}$.

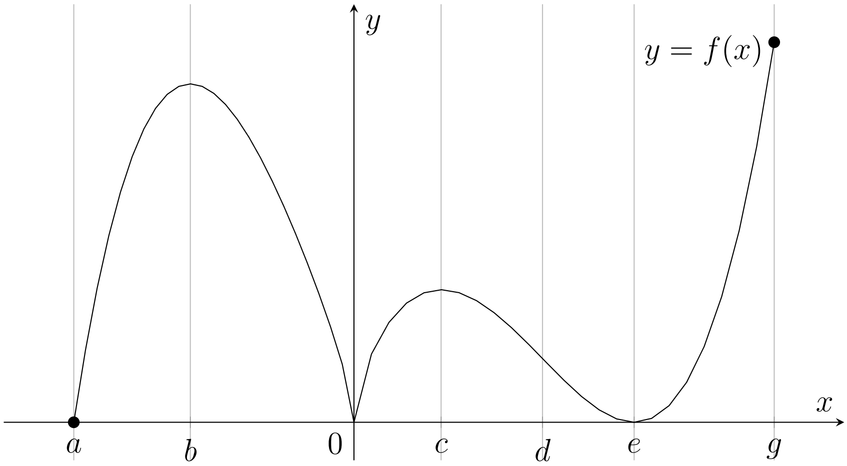
   (a) (6 points) Show that $f'(x) = \dfrac{(x+5)(7x-19)}{3(x-4)^{2/3}}$.

   (b) (2 points) Find $f'(5)$.

   (c) (6 points) What values of x are the critical points of f?

3. (10 points) Starting from the definition of the derivative as a limit, show that the derivative of the function $f(x) = \cos x$ is $f'(x) = -\sin x$. (You may assume that $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$ and that $\lim_{\theta\to 0} \frac{\cos\theta - 1}{\theta} = 0$.)

4. The picture below shows the graph of a function $f(x)$ on an interval, with various x-values labelled as a, b, c, d, e, g. This question asks you to deduce various properties of the function f by looking at its graph.



(a) (8 points) Complete the following table by putting tick (check) marks ✓ to indicate whether each x-value $a, b, 0, c, d, e, g$ corresponds to: a relative (local) minimum or maximum of f; an absolute (global) minimum or maximum of f; an inflection point of the graph. (You are allowed to mark more than one box for each x-value, if you think you should.)

x-value	a	b	0	c	d	e	g
Local minimum							
Local maximum							
Absolute minimum							
Absolute maximum							
Inflection point							

(b) (6 points) Complete the following table to indicate whether the function is increasing or decreasing, and concave-up or concave down, on each of the given intervals. (If you prefer, you may use symbols, such as ↗ and ↘ for increasing and decreasing, or ⌣ and ⌢ for concave-up and concave-down.)

Interval	(a, b)	$(b, 0)$	$(0, c)$	(c, d)	(d, e)	(e, g)
Increasing or decreasing						
Concave-up or concave-down						

5. (15 points) Find the absolute minima and maxima of the function $f(x) = x^3 - 12x$ on the interval $[-3, 5]$. (You should find both the x-value of each absolute minimum or maximum and the value of $f(x)$ there.)

6. (15 points) Let C be the curve defined by the equation $y^2 - y = x^3 + x^2$. Find the equation of the normal line to C at the point $(1, -1)$.

7. (12 points) Indicate whether each of the following statements is true or false. You do not need to show your working, and there is no penalty for wrong answers.

T **F** If $f(x)$ is a function which satisfies the conditions of Rolle's Theorem on the interval $[a, b]$, then there is at least one value $c \in [a, b]$ where the tangent to the graph of f at $x = c$ is parallel to the x-axis.

T **F** The slope of the graph of $f(x) = \cos(5x)$ is never greater than 1.

T **F** The Mean Value Theorem is a special case of Rolle's Theorem.

T **F** If a function's derivative is zero on an interval, then the function is constant on that interval.