

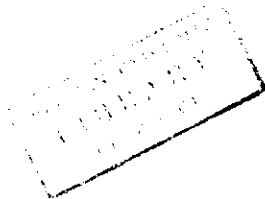
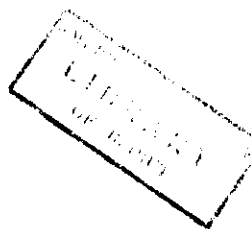
Time : 2 hours
Prof. H. Abu-Khuzam
February 3, 2001

MATHEMATICS 101
Final Examination
(First Semester 2000-01)

NAME _____
ID# _____
Section (5 or 6 or 7) _____
Ver. 2

1. Find the area of the region between the curve $y = x^3$, $-1 \leq x \leq 1$, and the x-axis.

[8 points]



2. Let $y = \int_2^x \sqrt{3t^4 + 2t^2 + 3} dt$. Find $\frac{dy}{dx}$.

[8 points]

3. Find any asymptotes, any maxima or minima, and sketch the graph for

$$f(x) = \frac{2}{x^2 + 1}.$$

{ 12 points }

4. Let $g(x) = 2x + \sin x$. Find a function $f(x)$ so that $f'(x) = g'(x)$ for all x , and $f(\pi) = 1$.
[8 points]

Circle the correct answer in each of the following questions (problem 5 to problem 20) [4 points each]:

5. $\int_0^{\sqrt{3}} \frac{4x}{\sqrt{x^2+1}} dx$

- a. 4
- b. $\sqrt{3}-1$
- c. 1
- d. $\frac{\sqrt{3}-1}{3}$
- e. none of the above.

[4 points]

6. $\lim_{x \rightarrow 0} 3x^2 \sin\left(\frac{1}{x}\right) =$

- a. 3
- b. -3
- c. 0
- d. 1
- e. does not exist .

[4 points]

7. If $x + \sin y = xy$, then y' at the point $(0,0)$ is equal to:

- a. 0
- b. $1/2$
- c. 1
- d. -1
- e. none of the above

[4 points]

8. The graph of $y = -x^3 + 1$ has

- a. a local minimum at $(0, 1)$
- b. a local maximum at $(0, 1)$
- c. an absolute minimum at $(2, -15)$
- d. a point of inflection at $(0, 1)$
- e. no extreme values.

[4 points]

9. The graph of the function $y = \frac{x^3 + 1}{x^2 + x + 1}$ has an oblique asymptote of equation :

- a. $y = x$
- b. $y = -x - 1$
- c. $y = x + 2$
- d. $y = x + 1$
- e. $y = x - 1$.

[4 points]

10. The value of the constant k that will make the function

$$f(x) = \begin{cases} 3x-1, & \text{if } x \leq 2 \\ -kx^3, & \text{if } x > 2 \end{cases},$$

continuous, is

- a. 5
- b. -3
- c. 5/8
- d. -5/8
- e. none of the above.

[4 points]

11. $\lim_{x \rightarrow 0^-} \frac{|x|(3x^2 + 4)}{|-2x^3|}$ is equal to :

- a. - 3/2
- b. 3/2
- c. 2
- d. 0
- e. does not exist.

[4 points]

12. If $f(x) = (2x+1)^{100}$, then $\lim_{h \rightarrow 0} \frac{f(h)-1}{h}$ is equal to :

- a. 200
- b. 100
- c. 0
- d. 2
- e. does not exist.

[4 points]

13. If $f(x) = \begin{cases} -|2x+2| & \text{if } x \leq -2 \\ \lfloor x \rfloor & \text{if } x > -2 \end{cases}$

then

- a. $f(x)$ is continuous at $x = -2$
- b. $\lim_{x \rightarrow (-2)^-} f(x) = -3$
- c. $f(-2)$ is not defined
- d. $\lim_{x \rightarrow -2} f(x)$ does not exist
- e. none of the above.

[4 points]

14. $\int_0^{\pi/3} \frac{\sec x \tan x dx}{\sqrt{\sec x}}$

- a. $3\sqrt{2}$
- b. $\sqrt{2}$
- c. $2\sqrt{2} - 2$
- d. 0
- e. none of the above.

[4 points]

15. If f is continuous on an interval $[a,b]$ and $c \in (a,b)$, then which one of the following statements is TRUE?

- a. If $f'(c) = 0$, then f has a local maximum or a local minimum at $x = c$.
- b. If f has a local maximum at $x = c$, then $f'(c) = 0$
- c. If $f'(c) = 0$, and $f''(c) < 0$, then f has a local minimum at $x = c$.
- d. If $f''(c) = 0$, then f has a point of inflection at $x = c$.
- e. f is integrable over $[a,b]$.

[4 points].

16. Let $f(x) = \begin{cases} 3x^2, & \text{if } x \leq 1 \\ ax+b, & \text{if } x > 1 \end{cases}$. Then, the values of a and b so that f will be differentiable at $x=1$ are :

- a. $a=6$ and $b=-3$
- b. $a=3$ and $b=0$
- c. $a=6$ and $b=6$
- d. $a=1$ and $b=1$
- e. $a=6$ and $b=1$

[4 points]

17. Let $f(x) = x^3 - 3x + 3$, then :

- a. $f(x)$ is always increasing.
- b. $f(x)$ has an inflection point at $x=0$.
- c. $f(x)$ has a local maximum at $x=1$.
- d. $f(x)$ a local minimum at $x=-1$.
- e. $f(x)$ has no extreme values.

[4 points].

18. Let $f(x)=x^3+1$. If $g(x)$ is a polynomial such that $g(0)=1$ and $g'(0)=3$, then $(f \circ g)'(0)$ is equal to:

- a. 9
- b. 2
- c. 3
- d. 6
- e. none of the above.

[4 points]

19. If x and y are two nonnegative numbers whose sum is 1, then the minimum value of the sum of their squares is :

- a. $1/4$
- b. $5/9$
- c. 0
- d. $1/2$
- e. none of the above

[4 points]

20. If $\int_0^x f(t) dt = \sqrt{x^2 + 1}$, then $f(x) =$

- a. $\frac{1}{\sqrt{x^2 + 1}}$
- b. $-\frac{1}{\sqrt{x^2 + 1}}$
- c. $2x\sqrt{x^2 + 1}$
- d. $\frac{x}{\sqrt{x^2 + 1}}$
- e. none of the above.

[4 points]