Time : 2 hours
Prof. H. Abu-Khuzam
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MATHEMATICS 101
Final Examination
(First Semester 2000-01)

NAME____________________
ID# ____________________
Section (5 or 6 or 7)________
Ver 2

1. Find the area of the region between the curve $y = x^3$, $-1 \le x \le 1$, and the x-axis.

[8 points]

2. Let $y = \int_2^{x^3} \sqrt{3t^4 + 2t^2 + 3}\, dt$. Find $\frac{dy}{dx}$.

[8 points]

3. Find any asymptotes, any maxima or minima , and sketch the graph for

$$f(x) = \frac{2}{x^2 + 1}.$$

[12 points]

4. Let $g(x) = 2x + \sin x$. Find a function $f(x)$ so that $f'(x) = g'(x)$ for all x, and $f(\pi) = 1$.

[8 points]

<u>Circle the correct answer in each of the following questions (problem 5 to problem 20) [4 points each]:</u>

5. $\int_0^{\sqrt{3}} \frac{4x}{\sqrt{x^2+1}} dx$

a. 4

b. $\sqrt{3}-1$

c. 1

d. $\frac{\sqrt{3}-1}{3}$

e. none of the above.

[4 points]

6. $\lim_{x \to 0} 3x^2 \sin(1/x) =$

a. 3

b. -3

c. 0

d. 1

e. does not exist .

[4 points]

7. If $x + \sin y = xy$, then y' at the point (0,0) is equal to:

a. 0
b. 1/2
c. 1
d. -1
e. none of the above

[4 points]

8. The graph of $y = -x^4 + 1$ has

a. a local minimum at (0, 1)

b. a local maximum at (0, 1)

c. an absolute minimum at (2, -15)

d. a point of inflection at (0, 1)

e. no extreme values.

[4 points]

9. The graph of the function $y = \dfrac{x^3 + 1}{x^2 + x + 1}$ has an oblique asymptote of equation :

a. y= x

b. y= -x-1

c. y= x+2

d. y= x+1

e. y= x-1.

[4 points]

10. The value of the constant k that will make the function

$$f(x) = \begin{cases} 3x-1, & if\ x \le 2 \\ -kx^3, & if\ x > 2 \end{cases} \quad ,$$

continuous, is

a. 5

b. -3

c. 5/8

d. -5/8

e. none of the above.

[4 points]

11. $\lim_{x \to 0^-} \frac{|x|(3x^2+4)}{|-2x^3|}$ is equal to :

a. - 3/2
b. 3/2
c. 2
d. 0
e. does not exist.

[4 points]

12. If $f(x) = (2x+1)^{100}$, then $\lim_{h \to 0} \frac{f(h)-1}{h}$ is equal to :

a. 200

b. 100

c. 0

d. 2

e. does not exist.

[4 points]

13. If $f(x) = \begin{cases} -|2x+2| & \text{if } x \leq -2 \\ \lfloor x \rfloor & \text{if } x > -2 \end{cases}$,

then

a. $f(x)$ is continuous at $x = -2$

b. $\lim_{x \to (-2)^-} f(x) = -3$

c. $f(-2)$ is not defined

d. $\lim_{x \to -2} f(x)$ does not exist

e. none of the above.

[4 points]

14. $\int_0^{\pi/3} \frac{\sec x \tan x \, dx}{\sqrt{\sec x}}$

a. $3\sqrt{2}$

b. $\sqrt{2}$

c. $2\sqrt{2} - 2$

d. 0

e. none of the above.

[4 points]

15. If f is continuous on an interval [a,b] and $c \in (a, b)$, then which one of the following statements is TRUE ?

a. If $f'(c) = 0$, then f has a local maximum or a local minimum at $x = c$.

b. If f has a local maximum at $x = c$, then $f'(c) = 0$

c. If $f'(c) = 0$, and $f''(c) < 0$, then f has a local minimum at $x = c$.

d. If $f''(c) = 0$, then f has a point of inflection at $x = c$.

e. f is integrable over [a,b].

[4 points].

16. Let $f(x) = \begin{cases} 3x^2, & \text{if } x \le 1 \\ ax+b, & \text{if } x > 1 \end{cases}$. Then, the values of a and b so that f will be differentiable at x=1 are :

a. a=6 and b= -3
b. a=3 and b=0
c. a=6 and b=6
d. a=1 and b= 1
e. a=6 and b=1

[4 points]

17. Let $f(x) = x^3-3x+3$, then :

a. f(x) is always increasing.
b. f(x) has an inflection point at x=0.
c. f(x) has a local maximum at x=1.
d. f(x) a local minimum at x= -1.
e. f(x) has no extreme values.

[4 points].

18. Let $f(x)=x^3+1$. If g(x) is a polynomial such that g(0)=1 and $g'(0)=3$, then $(f \circ g)'(0)$ is equal to:

a. 9
b. 2
c. 3
d. 6
e. none of the above.

[4 points]

19. If x and y are two nonnegative numbers whose sum is 1, then the minimum value of the sum of their squares is :

a. 1/4

b. 5/9

c. 0

d. 1/2

e. none of the above

[4 points]

20. If $\int_0^x f(t)\,dt = \sqrt{x^2+1}$, then $f(x) =$

a. $1/\sqrt{x^2+1}$

b. $-1/\sqrt{x^2+1}$

c. $2x\sqrt{x^2+1}$

d. $x/\sqrt{x^2+1}$

e none of the above.

[4 points]