Time: 2 Hours	MATHEMATICS 101	Name -----------------------
January 31, 2002	FALL 2001-2002	ID # ----------------------
Prof. H. Abu-Khuzam	FINAL EXAMINATION	ver.

Circle your section number below:

#5 (1:00 Th). #6 (2:00 Th)

#7 (3:00 Th) #8 (4:00 Th)

	PROBLEM	GRADE
PART I	1	------- / 7
	2	------- / 7
	3	------- / 7
	4	------- / 7
	5	------- / 10
	6	------- / 6
	7	------- / 8
PART II	8-19	------- / 48
TOTAL		------- /100

PART I. Solve each of the following problems (Problems 1, 2, 3, 4, 5, 6, and 7) in the space provided for each problem.

1. Let $y = \int_2^{x^4} t^5 \sqrt{t+3}\, dt$. Find $\dfrac{dy}{dx}$

[7 points]

2. Find the area of the region bounded by the curves $x = y^2$ and $x = y+2$.

[7 points]

3. Let $f(x) = \begin{cases} 3x^3, & \text{if } x \le 2 \\ ax^2 + b, & \text{if } x > 2 \end{cases}$.

Find the values of a and b such that f is differentiable at x=2 . Explain.

[7 points]

4. Find the sides of a rectangle with perimeter 100 cm whose area is as large as possible.

[7 points]

5. Find any asymptotes, any maxima or minima , and sketch the graph for

$$f(x)=\frac{-4}{x^2+2}$$

[10 points]

6. Find $\lim_{x \to 0} 2x^2 \sin(1/x)$ (Show your work)

[6 points]

7. Find the volume of the solid generated by revolving the region bounded by the graphs of $y = x^2$ *and* $y = x$ about the y-axis. [you can use cylindrical shells or washer(Disk) method].

[8 points]

PART II. Circle the correct answer in each of the following questions (problem 8 to problem 19) [4 points for each correct answer, 0 for no answer, and –1/2 for each wrong answer] :

8. $\int_0^1 \frac{1}{(2x-1)^2}\,dx$

   a. 1
   b. 1/2
   c. - 1
   d. 0
   e. none of the above.

[4 points]

9. If $x^3 - 3xy^2 + y^3 = 3$, then y' at the point (2,1) is equal to:

   a. 1
   b. -1
   c. 0
   d. 1/2
   e. none of the above

[4 points]

10. $\int_0^1 (1+\sqrt{x})^2\,dx =$

   a. 7/2

   b. 10/3

   c. 17/6

   d. 3

   e. none of the above.

[4 points]

11. $\lim_{x \to 0^-} \frac{|x|(2x^2 + 4)}{5x}$ is equal to :

a. 2/5
b. – 4/5
c. 0
d. - 2/5
e. does not exist.

[4 points]

12. Let $f(x) = x^4 - 2x^2$, then (using the second derivative test) :

a. f(x) has a local maximum at x= -1.

b. f(x) has a local maximum at x=1.

c. f(x) has a local minimum at x= 0.

d. f(x) has a local maximum at x=0.

e. none of the above

[4 points].

13. $\int_0^{\pi/4} \frac{\sec^2 x\,dx}{\sqrt{\tan x}}$

a. $\sqrt{2}$

b. $3\sqrt{2}$

c. 3/2

d. 2

e. none of the above.

[4 points]

14. If f is continuous on an interval [a,b] and $c \in (a, b)$, then which one of the following statements is <u>TRUE</u> ?

a. If $f'(c) = 0$, then f has a local maximum or a local minimum at $x = c$.

b. If f has a local minimum at $x = c$, then $f'(c) = 0$

c. If $f'(c) = 0$, and $f''(c) < 0$, then f has a local minimum at $x = c$.

d. If $f''(c) = 0$, then f has a point of inflection at $x = c$.

e. none of the above.

[4 points].

15. If $\int_0^x f(t)\,dt = \sin 2x$, then $f(x) =$

a. 2 cos 2x
b. 2sin 2x
c. -(1/2)cos 2x
d. cos 2x
e none of the above.

[4 points]

16. The length of the curve

$$y = \frac{4\sqrt{2}}{3} x^{3/2} - 1 \quad for \quad 0 \le x \le 1$$

a. 13/6

b $\sqrt{2}$

c. 5/6

d. 4/3

e. none of the above.

[4 points].

17. If $f(x) = (5x+1)^{100}$, then $\lim_{h \to 0} \frac{f(h)-1}{h}$ is equal to :

a. 100

b. 0

c. 5

d. 500

e. does not exist.

[4 points]

18. The equation $x^3 + 7x + 1 = 0$ has

a. no solution in the interval [-2,2]

b. exactly three solutions in the interval [-2,2]

c exactly two solutions in the interval [-2,2]

d exactly one solution in the interval [-2,2]

e none of the above

[4 points].

19. The value of the constant k that will make the function

$$f(x) = \begin{cases} x^2 + 12, & if\ x \leq 2 \\ -kx^3, & if\ x > 2 \end{cases},$$

continuous, is

a. -2

b. -3

c. –5/8

d. 0

e. none of the above.

[4 points]