Math 101
Final Exam

January 31, 2002

1. Let $f: R \to R$ be given by $f(x) = x^3 - 2x^2 - 2x$.
   (i) Sketch the graph of f.
   (ii) Find the area of the region enclosed by the curve $y = f(x)$ and the straight line $y = x$.
2. Find the volume generated by revolving about the X-axis the region given by the curve $y = \sin x$ and the interval $0 \le x \le \pi$.
3. The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the X– axis. Find the volume generated.
4. Let $f: R \to R$ be given by $f(x) = \frac{x}{x^2 + 1}$.
   (i) Find the intervals of R on which f is increasing;
   (ii) Find the intervals of R on which f is decreasing;
   (iii) Locate the points of local minima and local maxima;
   (iv) Find the intervals of R on which the graph of f is concave up and concave down;
   (v) Locate the points of inflection of the graph of f.
5. Find the equation of the tangent line to the curve $x\sin 2y - y\cos 2x = 0$ at the point $(\pi/4, \pi/2)$
6. Evaluate the following limits :

(i) $\lim_{x \to 1} \sec(x\sec^2 x - \tan^2 x - 1)$;

(ii) $\lim_{x \to \infty} \frac{2x^2 + 7x - 25}{5x^2 - 7x + 25}$.