AMERICAN UNIVERSITY OF BEIRUT
Mathematics Department
Math 101 - Final Exam
Fall 2005-2006

Name:.................................. **ID:**................................

Ms. Marwa El Houri
Lecture: MWF 8:00 - 9:00
Section 1: T 08:00 - 09:00
Section 2: T 11:00 - 12:00

Ms. Diana Audi
Lecture: MWF 9:00 - 10: 00
Section 3: Th 08:00 - 09:00
Section 4: Th 02:00 - 03:00
Section 5: Th 11:00 - 12:00
Section 6: Th 09:30 - 10:30

Time: 120 min

Direction: Write your name and ID number and circle your section number. Answer the questions in the allocated spaces, if more space is needed continue on the back. NO CALCULATORS ARE ALLOWED!!!

I- (10 points) Evaluate the following integrals:

a- $\displaystyle\int \frac{x^2}{(x^3-\sqrt{2})^2}\,dx$

b- $\displaystyle\int \frac{\cos^2(\sqrt{x})}{\sqrt{x}}\,dx$

c- $\displaystyle\int_{3/\pi}^{1/\pi} \frac{\sin(\frac{1}{\theta})}{\theta^2 \cos^3(\frac{1}{\theta})}\, d\theta$

d- $\displaystyle\int_{1}^{2} \frac{(t+2)^2 - 1}{t^4}\, dt$

II- (15 points) Let $f(x) = \begin{cases} 1 & x < 0 \\ x^2 + 1 & 0 \le x < 1 \\ -2x + 4 & 1 \le x < 2 \\ 0 & x \ge 2 \end{cases}$

a- Draw the graph of f.

b- Find the domain and range of f.

c- Where is f continuous? Explain briefly.

d- Where is f differentiable? Explain briefly.

e- Recall that $f(x) = \begin{cases} 1 & x < 0 \\ x^2+1 & 0 \leq x < 1 \\ -2x+4 & 1 \leq x < 2 \\ 0 & x \geq 2 \end{cases}$, and let $g(x)$ be defined by:

$$g(x) = \int_0^x f(t)dt$$

Find $g(-2)$, $g(1)$ and $g(3)$.

f- Is g differentiable? Explain.

III- (20 points) Let $f(x) = \dfrac{x^2 + 2x - 4}{x}$

a- Find the domain of f.

b- Find the asymptotes.

c- Find the critical points of f.

d- Find the intervals where f is increasing or decreasing.

e- Find the intervals where f is concave up or concave down.

f- Find the local minimum and maximum of f. Does f has any absolute extremes?

g- Does f have an inflection point?

h- Draw the graph of f and find the range of f.

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IV- (8 points) Let $x = \sin t - t + \frac{\pi}{2}$ and $y = \cos t + 3$ be the equation of a parametric curve.

a- Find the length of the parametric curve for $-\pi \leq t \leq \pi$.

b- Find $\dfrac{dy}{dx}$.

c- Find the equation of the tangent line to the curve at the point $t = \frac{\pi}{2}$.

V- (10 points) Let $f(x) = \dfrac{\tan x}{1+x^8}$ and $g(x) = x^2 + |x - x^5| + 3$ be two functions.

a- For each function, determine if the function is even or odd.

b- Deduce the value for $\displaystyle\int_{-1}^{1} \frac{\tan x}{1+x^8}\, dx$ and $\displaystyle\int_{-1}^{1} (x^2 + |x - x^5| + 3)\, dx$.

VI- (5 points) Let $f(x) = \begin{cases} c & x \leq -3 \\ \dfrac{9 - x^2}{4 - \sqrt{x^2 + 7}} & -3 < x \leq 3 \\ d & x > 3 \end{cases}$

Find the values of c and d for which the function f is continuous.

VII- (5 points) Let g be a differentiable function with $g(1) = 0$, and let

$$\sin^2(g(x)) + xg(x) = x^2 + 1$$

Find $g'(1)$.

VIII- (5 points) Find the area of the region enclosed by the curves $y = \sqrt{x}$, $y = x - 2$ and the $x-$axis.

IX- (12 points) Find the volume of revolution of the region bounded by the parabola $y = (x-1)^2$ and the line $y = 1$ around

a- $x-$axis

b- $y-$axis

c- $y = 1$

d- $x = 2$

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X- (10 points) Let f be a continuous function with $\int_0^1 f(t)dt = \frac{1}{2}$, and let

$$F(x) = \int_0^x f(t)dt - \frac{x^2}{2}$$

Use Rolle's theorem for F to show that there is a point c in (0,1) such that $f(c) = c$.

GOOD LUCK!!