AMERICAN UNIVERSITY OF BEIRUT

FINAL EXAM-MATH 101 Jan.,2008

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NAME:…… **SECTION: 4 5 6 7**

Ques.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Exam Gr.**
Pts.											

1. Solve the two independent parts: **(4+4=8points)**

a) Let $I = \int_3^5 \sqrt{1+8\cos^2 x}\, dx.$

Without attempting to integrate, show that the value of the above integral lies between 2 and 6. Hint: Use the Min-Max inequality.

b) Find the average value of the function $y = \sin 2x$, over the interval $\left[0, \frac{\pi}{4}\right]$.

2. **(4+4+4=12points)**Consider the region in the first quadrant limited by the y-axis and the curves :

   $y = x^2$ *and* $y = 4$. Answer the following three questions:

   a) Draw the region and find its area.

   b) Find the volume of the solid generated when the region is revolved about the x-axis. (Drawing is not a must)

   c) The cross sections of a solid perpendicular to the y-axis are equilateral triangles with a base running from the y-axis to the curve $y = x^2$.
   Find the volume of this solid. (Drawing is not a must).

3. **(4+4+4=12points)**Evaluate the integrals: a) $I = \int_0^1 x^4 (1+8x^5)^{-\frac{1}{2}} dx$

b) $J = \int_0^{\frac{\pi}{5}} \sin^2(5x)\, dx$

c) $K = \int_{-2}^{3} |x|\, dx$

4. **(3+3+3+3=12points)**Find $\frac{dy}{dx}$ in the following :

a) $y = \int_{x}^{1} \frac{6}{3+t^4}\,dt$

b) $y = u^2 - 4u + 3$; $u = \tan(x)$; *at* $x = \frac{\pi}{4}$.

c) $y = \sqrt{1+\sec(4x)}$

d) $xy^2 + 2x - 3y = 5$

5. (6points)

Express the limit :

$\lim_{\|P\|\to 0} \sum_{k=1}^{k=n} (c_k{}^3 - 5c_k)\Delta x_k$, *as a definite* int*egral*, *where* P *is a partition of* $[-3,4]$.

6. **(3+3+3+3=12points)** a) Solve the inequality: $\left|2-\frac{x}{3}\right|>\frac{5}{3}$.

b) Given the function: $y=x-\frac{1}{2x}$. Find the points on the curve where the tangent is parallel to the line $y=9x$.

c) For what values of a and b will $f(x)=\begin{cases} ax \quad ; \quad x>2 \\ ax^2-bx+3 \; ; \; x\geq 2 \end{cases}$ be differentiable for all values of x.

d) Find an equation of the normal line to the curve whose parametric equation is:
$x = 1 + \frac{1}{t^2}$; $y = 1 - \frac{3}{t}$ *at* $t = 2.$

7. **(6points)** *Find y as a function of x if* $\frac{dy}{dx} = -\pi \sin(\pi x)$ *and* $y(0) = 0.$

8. **(4+4+4=12points)**Given the function : $y = f(x) = x\sqrt{3-x}$

a) What is the domain? Find the value(s) of x for which the derivative is zero.

b) For what values of x is the function i) increasing?
ii)decreasing?

c) Sketch the graph of the function. Show important steps.

9. **(4+4+4=12points)**Evaluate the limits:

a) $\lim_{x\to\infty}\frac{3x}{2x+5\sqrt{x}}$

b) $\lim_{x\to 5^+}\frac{8-\lceil x\rceil}{\cos\left(\frac{\pi x}{5}\right)}$

c) $\lim_{x\to 0}\frac{\sec x-1}{x^2}$

10. Solve the two independent parts: **(4+4=8points)**

a) Assume that f is an even function and g is an odd function.

$Let \int_0^2 f(x)dx = a \quad , \quad \int_2^5 g(x)dx = b \ , \int_2^5 f(x)dx = c$

$Find \int_{-2}^{5} (3f(x) - 7g(x))dx \quad in\,terms\,of \ a, b, and\ c.$ (*Explain*).

b) (Independent part) Evaluate: $\lim_{n\to\infty} \frac{(1+2+3+.....+n)}{n^2}$