AMERICAN UNIVERSITY OF BEIRUT
Mathematics Department
Math 101 - Quiz I
Fall 2005-2006

Name:......Key.......... ID:...............................

Ms. Marwa El Houri
Section 1: T 08:00 - 09:00
Section 2: T 11:00 - 12:00

Ms. Diana Audi
Section 3: Th 08:00 - 09:00
Section 4: Th 02:00 - 03:00
Section 5: Th 11:00 - 12:00
Section 6: Th 09:30 - 10:30

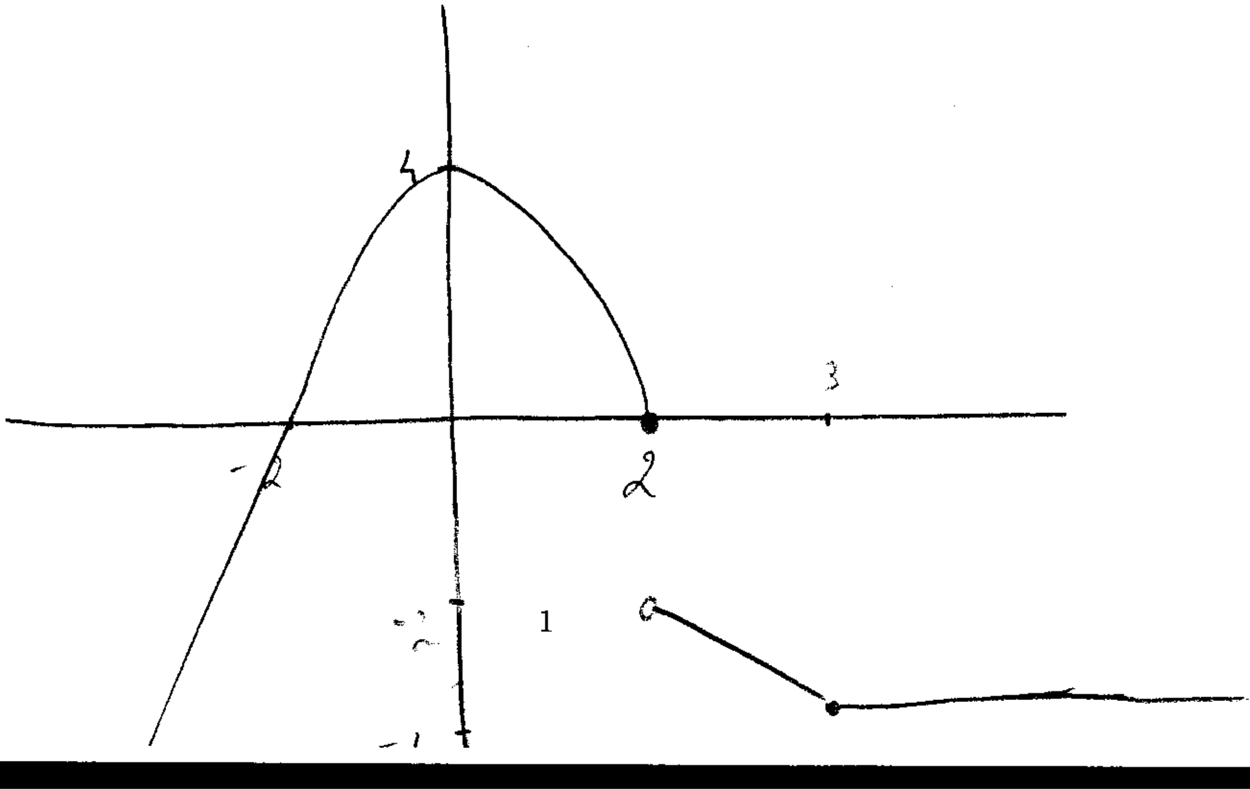
<u>Time: 60 min</u>

<u>Direction:</u> Write your name and ID number and circle your section number. Answer the questions in the allocated spaces, if more space is needed continue on the back. NO CALCULATORS ARE ALLOWED!!!

I- **(25 points)** Let

$$f(x) = \begin{cases} 4 - x^2 & x \leq 2 \\ -x - 1 & 2 < x \leq 3 \\ -4 & x > 3 \end{cases}$$

a- Draw the graph of f.



b- Find its domain and range.

Domain $(-\infty, \infty)$

Range $(-\infty, 4]$

c- For what values of x is f continuous? Explain.

at $x = 2$: $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} 4 - x^2 = 0 = f(2)$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} -x - 1 = -2 \neq \lim_{x \to 2^-} f(x)$

f is not cont at $x = 2$.

at $x = 3$ $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} -x - 1 = -4 = \lim_{x \to 3^+} f(x) = f(3)$

f is cont at $x = 3$

* f is continuous on $(-\infty, 2) \cup (2, +\infty)$.

II- (20 points) Find the asymptotes and sketch the graph of

$$f(x) = \frac{x^2 + 7x + 10}{x^2 - 4}$$

Horizontal Asymptotes:

$\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{x^2 + 7x + 10}{x^2 - 4} = 1.$

$\boxed{Y = 1}$ is a vertical asymptote.

Vertical Asymptotes:

$x^2 - 4 = 0 \Rightarrow x = \pm 2.$

$\lim_{x \to -2} f(x) = \lim_{x \to -2} \frac{x^2 + 7x + 10}{x^2 - 4} = \lim_{x \to -2} \frac{\cancel{(x+2)}(x+5)}{\cancel{(x+2)}(x-2)}$

$= -\frac{3}{4}$ ($x = -2$ is not a vertical asymp)

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \frac{x^2 + 7x + 10}{x^2 - 4} = \lim_{x \to 2^+} \frac{x+5}{x-2} = +\infty$

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \frac{x+5}{x-2} = -\infty$ $\boxed{x = 2}$ is a Vertical asymp)

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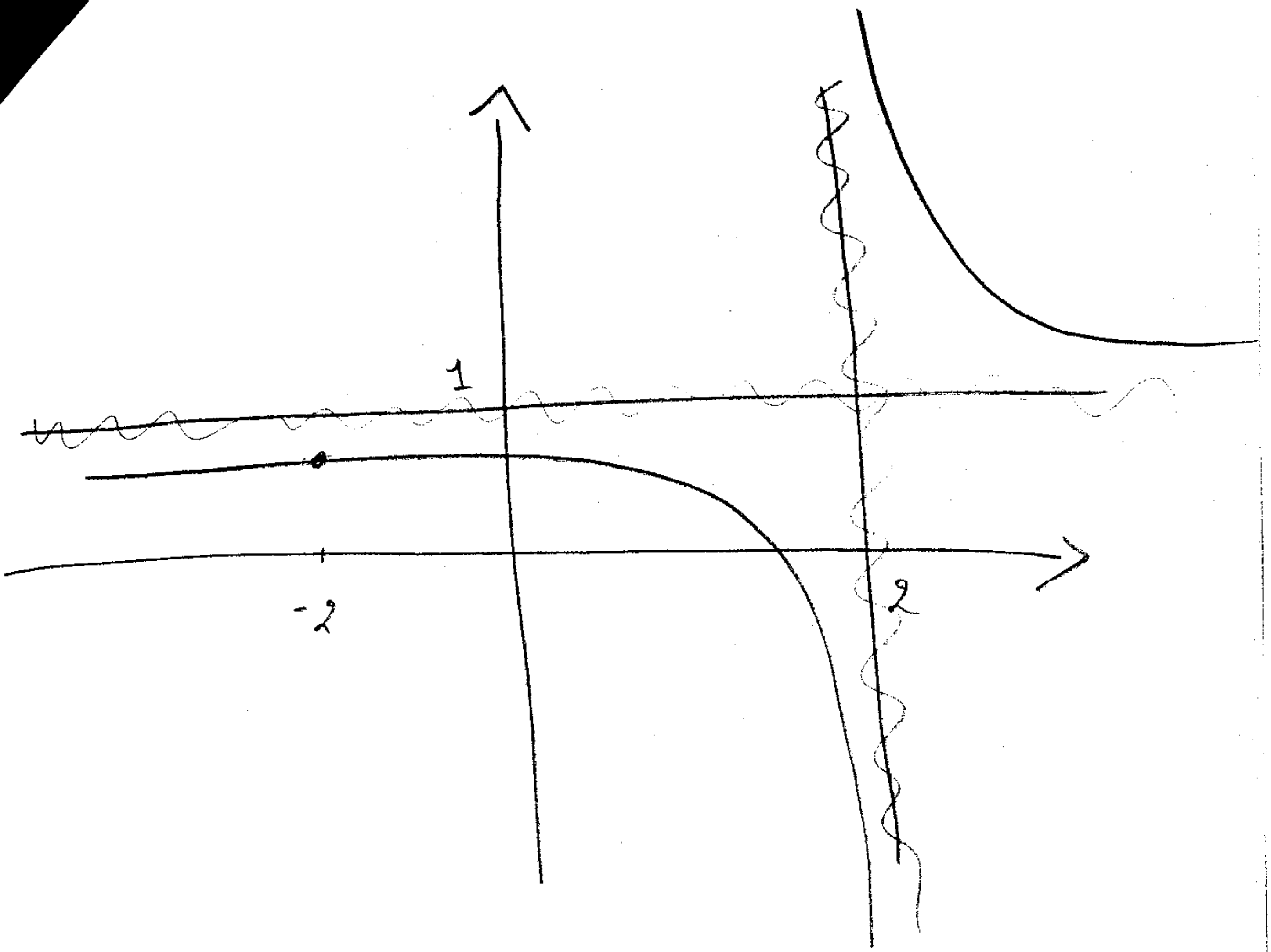

III- **(20 points)** Find the limit or explain why it does not exist.

a- $\lim\limits_{x \to 3^-} \dfrac{1}{9-x^2}$ $= +\infty$

$x \to 3^-$

$9 - x^2 > 0$

b- $\lim\limits_{x \to 2} \dfrac{x^2-2x}{x^2-x-2}$ $= \lim\limits_{x \to 2} \dfrac{x(\cancel{x-2})}{(\cancel{x-2})(x+1)} = \lim\limits_{x \to 2} \dfrac{x}{x+1} = \dfrac{2}{3}$

c- $\lim_{x\to-\infty} \frac{x^2+5}{|x|-3}$ $= \lim_{x\to-\infty} \frac{x^2+5}{-x-3} = +\infty$

d- $\lim_{x\to3} \frac{\sin(x-3)}{x^2-2x-3}$ $= \lim_{x\to3} \frac{\sin(x-3)}{(x-3)(x+1)}$

$$= \lim_{x\to3} \frac{\sin(x-3)}{x-3} \cdot \frac{1}{x+1}$$

$$= 1 \cdot \frac{1}{4} = \frac{1}{4}.$$

IV- **(15 points)** Let $f(x) = \dfrac{1}{4-\sqrt{x}}$ and $g(x) = 25 - x^2$

a- Find $f \circ g(x)$.

$$f(g(x)) = \frac{1}{4-\sqrt{25-x^2}}$$

b- Find the domain for $f \circ g$.

1) $4 - \sqrt{25-x^2} \neq 0$

$\Rightarrow 4 \neq \sqrt{25-x^2}$

or $16 \neq 25 - x^2$

$\Rightarrow x^2 \neq 9$ or $x \neq \pm 3$

2) $25 - x^2 \geq 0$

$\Rightarrow (5-x)(5+x) \geq 0$

		-5 $\qquad$ 5	
$5-x$	$+$	$-$	$\cancel{+}$
$5+x$	$-$	$-$	$+$
$25-x^2$	$-$	$+$	$-$

$-5 \leq x \leq 5$

Domain.

$[-5,-3) \cup (-3,3) \cup (3,5]$.

V- **(20 points)** Let $f(x) = \sqrt{x}$

a- Find the slope of the tangent line to the curve $y = f(x)$ at $x = a$.

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{\sqrt{a+h} - \sqrt{a}}{h} \cdot \frac{\sqrt{a+h} + \sqrt{a}}{\sqrt{a+h} + \sqrt{a}}$$

$$= \lim_{h \to 0} \frac{a + h - a}{h(\sqrt{a+h} + \sqrt{a})} = \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{a+h} + \sqrt{a})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{a+h} + \sqrt{x}} = \boxed{\frac{1}{2\sqrt{a}}}$$

$$\boxed{m = \frac{1}{2\sqrt{a}}}$$

b- For which value of a is the tangent line to the curve parallel to the line $4y - 2x = 31$?

The tangent line to the curve $y = f(x) = \sqrt{x}$ is // to the line $4y - 2x = 31$ when the slopes are equal.

slope of the curve $y = \sqrt{x}$ is $m_1 = \frac{1}{2\sqrt{a}}$.

slope of line : $y = \frac{31 + 2x}{4} \Rightarrow m_2 = \frac{1}{2}$.

$\Rightarrow \frac{1}{2\sqrt{a}} = \frac{1}{2} \Rightarrow \sqrt{a} = 1 \Rightarrow \boxed{a = 1}$

c- Write the equation of the tangent line at this point.

At the point $a = 1$

slope $m = \frac{1}{2}$.

$y = \sqrt{a} = \sqrt{1} = 1$

equation : $(y - 1) = \frac{1}{2}(x - 1)$

$\Rightarrow \boxed{y = \frac{1}{2}x + \frac{1}{2}}$