Math 102–Final Exam (Spring 00) B. Shayya

Unjustified answers will receive little or no credit.

1. (26 pts) Find the volume of the solid generated by revolving the region between the line $y = x$ and the parabola $y = x^2$ about (a) the x-axis; (b) the line $x = 2$.

2. (12 pts each) Find the following limits:

$$(a)\ \lim_{x\to 0} \frac{x+\sin x}{e^x - 1} \qquad (b)\ \lim_{x\to\infty} \frac{\sqrt{x^2+x}}{x+2} \qquad (c)\ \lim_{x\to 0^+} x(\ln x)^2$$

3. (16 pts each) Evaluate the following integrals:

$$(a)\ \int_1^e \frac{(\ln x)^2}{x}\,dx \qquad (b)\ \int 2x \arctan x\,dx \qquad (c)\ \int \frac{dx}{\sqrt{1+x^2}}$$

4. (a) (15 pts) Find the standard equation of the ellipse with foci $(0, \pm 3)$ and vertices $(0, \pm 5)$, then sketch its graph.

(b) (15 pts) Find the standard equation of the hyperbola with foci $(\pm 2, 0)$ and asymptotes $y = \frac{\pm 1}{\sqrt{3}}x$, then sketch its graph. Include the foci and asymptotes in your sketch.

5. Consider the three points

$$A(2,2,1) \qquad B(7,-3,6) \qquad C(6,6,4)$$

and let

$$\mathbf{v} = \overrightarrow{AB} \qquad \mathbf{w} = \overrightarrow{AC}\,.$$

(a) (6 pts) Express $\mathbf{v}$ and $\mathbf{w}$ in the form $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$.

(b) (14 pts) Find an equation for the plane through A, B, and C.

(c) (20 pts) Express $\mathbf{w}$ as the sum of a vector parallel to $\mathbf{v}$ and a vector orthogonal to $\mathbf{v}$.

(d) (20 pts) Let $\boldsymbol{L}$ be the line through A and B. Find parametric equations for $\boldsymbol{L}$. Also find the point on $\boldsymbol{L}$ closest to C.