

## Math 102-Final Exam (Spring 00)



B. Shayya

Unjustified answers will receive little or no credit.

- 1. (26 pts) Find the volume of the solid generated by revolving the region between the line y = xand the parabola  $y = x^2$  about (a) the x-axis; (b) the line x = 2.
- 2. (12 pts each) Find the following limits:

(a) 
$$\lim_{x \to 0} \frac{x + \sin x}{e^x - 1}$$

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 (b)  $\lim_{x \to \infty} \frac{\sqrt{x^2 + x}}{x + 2}$  (c)  $\lim_{x \to 0^+} x(\ln x)^2$ 

$$(c) \quad \lim_{x \to 0^+} x(\ln x)^2$$

3. (16 pts each) Evaluate the following integrals:

(a) 
$$\int_{1}^{e} \frac{(\ln x)^2}{x} dx$$

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 (b)  $\int 2x \arctan x dx$  (c)  $\int \frac{dx}{\sqrt{1+x^2}}$ 

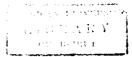
$$(c) \int \frac{dx}{\sqrt{1+x^2}}$$

- 4. (a) (15 pts) Find the standard equation of the ellipse with foci  $(0,\pm 3)$  and vertices  $(0,\pm 5)$ , then sketch its graph.
- (b) (15 pts) Find the standard equation of the hyperbola with foci ( $\pm 2, 0$ ) and asymptotes  $y = \frac{\pm 1}{\sqrt{3}}x$ , then sketch its graph. Include the foci and asymptotes in your sketch.
- 5. Consider the three points

$$A(2,2,1)$$
  $B(7,-3,6)$   $C(6,6,4)$ 

and let

$$\mathbf{v} = \overrightarrow{AB}$$
  $\mathbf{w} = \overrightarrow{AC}$ .



- (a) (6 pts) Express v and w in the form  $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ .
- (b) (14 pts) Find an equation for the plane through A, B, and C.
- (c) (20 pts) Express  $\mathbf{w}$  as the sum of a vector parallel to  $\mathbf{v}$  and a vector orthogonal to  $\mathbf{v}$ .
- (d) (20 pts) Let L be the line through A and B. Find parametric equations for L. Also find the point on  $\boldsymbol{L}$  closest to C.