

ECE 310 - Homework 6

Problem 1

$$\begin{aligned} \text{a) } K'_n &= \mu_c \frac{\epsilon_{ox}}{L_{ox}} = 1000 \times 10^{-4} \times \frac{3.453 \times 10^{-11}}{7 \times 10^{-9}} \\ &= 4.933 \times 10^{-4} \text{ A/V}^2 \end{aligned}$$

$$\text{b) } V_{ov} = V_{GS} - V_{to} = 3 - 0.55 = 2.45 \text{ V. } (V_{ov} > 0)$$

$$V_{DS} = 3 \text{ V}$$

$V_{DS} > V_{ov} \rightarrow$ Mosfet in SAT.

$$I_D = \frac{1}{2} K'_n \frac{W}{L} V_{ov}^2$$

$$\frac{W}{L} = \frac{I_D}{\frac{1}{2} K'_n V_{ov}^2} = \frac{10^{-3}}{\frac{1}{2} \times 4.933 \times 10^{-4} \times 2.45^2}$$

$$= 0.675$$

$$\begin{aligned} \cancel{L} &= 0.32 \mu\text{m}; \quad \cancel{W} = 0.32 \times 0.675 \\ &= 0.216 \mu\text{m} \end{aligned}$$

$W = 0.32 \mu\text{m}$, and $L = 0.32/0.675 = 0.474 \mu\text{m}$ (L or W cannot be less than $0.32 \mu\text{m}$)

- c) • I_{D1} at $(V_{GS} = 3\text{V}, V_{DS1} = 3\text{V})$.
• $0.97 I_{D1}$ at $(V_{GS} = 3\text{V}, V_{DS2} = 2.5\text{V})$
($2.5 > 2.45 = V_{ov}$, we still in SAT).

Thus

$$\begin{cases} I_{D1} = \frac{1}{2} K' \frac{W}{L} V_{OV}^2 \left(1 + \frac{V_{DS1}}{V_A} \right) \\ 0.97 I_{D1} = \frac{1}{2} K' \frac{W}{L} V_{OV}^2 \left(1 + \frac{V_{DS2}}{V_A} \right) \end{cases}$$

$$0.97 = \frac{V_A + 2.5}{V_A + 3}$$

$$V_A = \frac{\frac{2.5}{0.97} - 3}{1 - \frac{1}{0.97}} = 13.67 \text{ V}$$

$$V'_A = \frac{V_A}{L} = \frac{13.67}{0.474} = 28.84 \text{ V}/\mu\text{m}$$

29 V/um

Problem 2

$$V_{GG} = 2.5V, R = 4.7K, V_E = 0.55V, K' \frac{W}{L} = 0.4 \text{ mA/V}^2$$

a) The mosfet is operating in the $\hat{\text{triode}}$ region.

$$\begin{cases} V_{OV} > 0 & (\text{this condition is satisfied}) \\ V_{DS} < V_{OV} \rightarrow V_{\text{output}} < V_{GG} - V_E \end{cases}$$

$$\text{Thus } V_o < 2.5 - 0.55 = 1.95V$$

Current equation:

$$I_D = \frac{1}{2} K' \frac{W}{L} (2 V_{OV} V_{DS} - V_{DS}^2)$$

$$\text{Linear term} = 2 V_{OV} V_{DS}$$

$$\text{Square term} = V_{DS}^2$$

Square term $<$ 3% of the linear term

$$V_{DS}^2 < 0.03 \times 2 \times V_{OV} \times V_{DS}$$

Since $V_{OV} > 0$, V_{DS} is necessarily positive thus:

$$V_{DS} < 0.06 V_{OV} = 0.117V$$

Thus

$$\boxed{0 < V_o < 0.117V}$$

$$\begin{aligned}
 \text{b) } V_{ov} &= V_{GS} - V_E \\
 &= V_{GG} - V_E \\
 &= 2.5 - 0.55 \\
 &= 1.95 \text{ V}
 \end{aligned}$$

$$\begin{aligned}
 g_{DS} &= K' \frac{W}{L} V_{ov} \\
 &= 0.4 \times 1.95 \\
 &= 0.78 \text{ m}\Omega^{-1}
 \end{aligned}$$

$$r_{DS} = \frac{1}{g_{DS}} = 1.282 \text{ k}\Omega$$

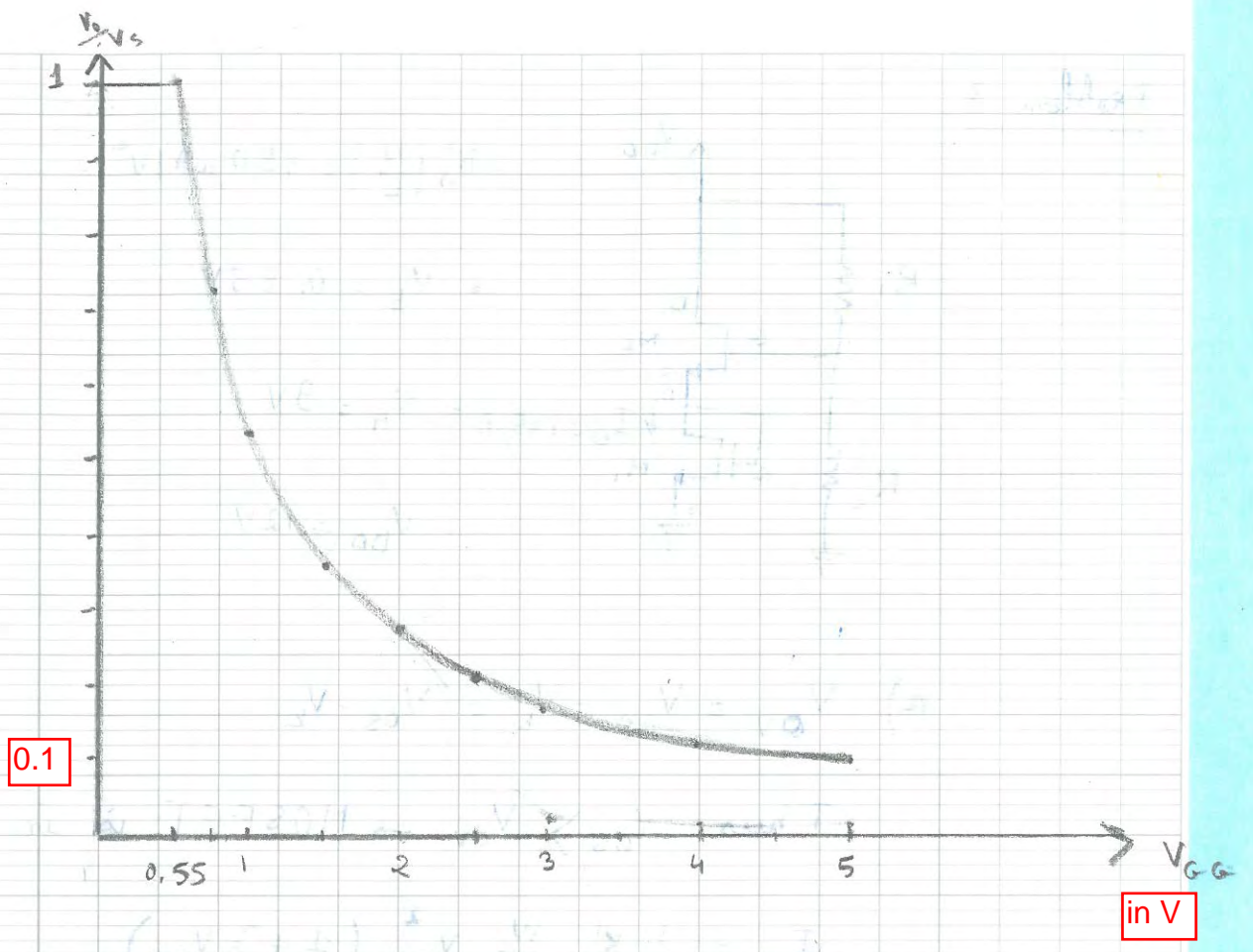
$$\begin{aligned}
 \text{c) } \frac{V_o}{V_s} &= \frac{r_{DS}}{r_{DS} + R} = \frac{1}{1 + R g_{DS}} \\
 &= \frac{1}{1 + 4.7 \times 0.4 \times (V_{GG} - 0.55)}
 \end{aligned}$$

$$\frac{V_o}{V_s} = \frac{1}{1.88 V_{GG} - 0.034} \quad \rightarrow \quad \text{W/O/S}$$

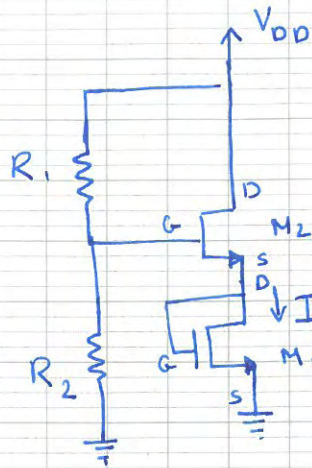
The above equation is always satisfied when $V_{ov} > 0 \rightarrow V_{GG} > V_T \rightarrow V_{GG} > 0.55\text{V}$

For $0 < V_{GG} < 0.55\text{V}$, Mosfet \rightarrow Cutoff region.

$$\frac{V_o}{V_s} = 1 \text{ in this case.}$$



Problem 3



$$\cdot K'_n \left(\frac{W}{L} \right) = 450 \mu\text{A/V}^2$$

$$\cdot V_E = 0.55\text{V}$$

$$\cdot V_A = 9\text{V}$$

$$\cdot V_{DD} = 12\text{V}$$

$$a) V_{OV} = V_{GS} - V_E = V_{DS} - V_E$$

Thus $V_{DS} > V_{OV} \rightarrow$ MOSFET is in SAT.

$$I_D = \frac{1}{2} K'_n \frac{W}{L} V_{OV}^2 (1 + \lambda V_{DS})$$

$$V_{GS} > V_E = 0.55\text{V}$$

$$I_D = \frac{1}{2} K'_n \frac{W}{L} (V_{GS} - V_E)^2 (1 + \lambda V_{GS})$$

$$(V_{GS}^2 - 2V_{GS}V_E + V_E^2) (1 + \lambda V_{GS}) = \frac{2I_D}{K'_n \frac{W}{L}}$$

$$V_{GS}^2 + \lambda V_{GS}^3 - 2V_E V_{GS} - 2V_E \lambda V_{GS}^2 + V_E^2 + \lambda V_E^2 V_{GS} = \frac{2I_D}{K'_n \frac{W}{L}}$$

$$\frac{1}{9} V_{GS}^3 + \frac{79}{90} V_{GS}^2 - 1.066 V_{GS} + 0.55^2 - \frac{2 \times 145}{450} = 0$$

$$V_{GS1} = 1.3 \text{ V}$$

b) Assume M_1 in SAT $\rightarrow \begin{cases} v_{ov1} > 0 \\ v_{ds1} > v_{ov1} \end{cases}$

$$I_D = \frac{1}{2} K'_n \frac{W}{L} (V_{GS1} - V_E)^2 \left(1 + \frac{v_{ds1}}{V_A}\right)$$

$$V_{GS1} = V_E + \sqrt{\frac{2I_D}{K'_n \frac{W}{L} \left(1 + \frac{v_{ds1}}{V_A}\right)}}$$

$$V_{G1} = \underset{V_{GS}}{V_{S1}} + V_E + \sqrt{\frac{2I_D}{K'_n \frac{W}{L} \left(1 + \frac{v_{ds1}}{V_A}\right)}}$$

$$= 1.3 + 0.55 + \sqrt{\frac{2 \times 145}{450 \left(1 + \frac{12-1.3}{9}\right)}}$$

$$= 2.39 \text{ V}$$

$$v_{ov1} = 2.39 - 0.55 = 1.84 \text{ V} < v_{ds1} = 12 - 1.3 = 10.7 \text{ V}$$

Thus M_1 : SAT.

$$c) I_{\text{gate}} = 0$$

$$\text{Thus } I_{R_1} = I_{R_2}$$

$$\frac{V_{DD} - V_{G1}}{R_1} = \frac{V_{G1}}{R_2}$$

$$R_1 = \frac{R_2 (V_{DD} - V_{G1})}{V_{G1}}$$

$$= \frac{120 (12 - 2.39)}{2.39}$$

$$= 482.51 \text{ k}\Omega$$

$$\approx 483 \text{ k}\Omega$$

$$d) P = V_{DD} (I_{R_2} + I_D)$$

$$= 12 \times \left(\frac{2.39}{120} + 0.145 \right)$$

$$= 2.78 \text{ mA}$$

$$= 1.979 \text{ mW}$$