EECE 310 _ Homework 4

Problem 1

$\begin{cases} P = 0.5\,W \\ V_Z = 9\,V \\ I_Z = 10\,mA \end{cases}$ $\quad r_Z = 25\,\Omega \quad I_{ZK} = 2\,mA \quad V_S : 10.5 - 14\,V \quad I_L : 5 - 20\,mA$

a) $$V_Z = V_{Z0} + r_Z I_Z$$

$$V_{Z0} = V_Z - r_Z I_Z = 9 - 25 \times 10 \times 10^{-3} = 8.75\,V$$

b) $$P = V_Z \cdot I_{Zmax}$$

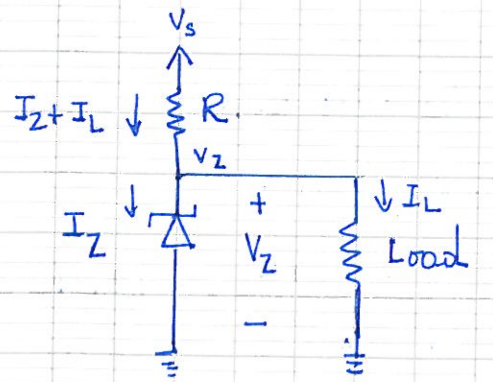
$$= (V_{Z0} + r_Z I_{Zmax})\, I_{Zmax}$$

$$= r_Z I_{Zmax}^2 + V_{Z0} I_{Zmax}$$

Thus : $0.5 = 25\, I_{Zmax}^2 + 8.75\, I_{Zmax}$

$I_{Zmax} = 0.05\,A$ (The other value obtained is $-0.4\,A$ not acceptable).

c)



$$R = \frac{V_S - V_Z}{I_Z + I_L}$$

$$R = \frac{V_S - V_{Z0} - r_Z I_Z}{I_Z + I_L}$$

The value of R must me chosen so that the zener diode remains in the breakdown region.

$$R = \frac{V_S - V_{Z0} - r_Z I_Z}{I_Z + I_L} \rightarrow \left(I_Z = \frac{V_S - V_{Z0} - R I_L}{R + r_Z} \right)$$

- When $I_Z = I_{Zmin} = I_{ZK}$, we should have: (from the above equation)

$$\begin{cases} V_{s\,min} \\ I_{L\,max} \\ R_{max} \end{cases}$$

(in fact: $\underset{min}{I_Z} = \frac{\overset{min}{V_S} - V_{Z0} - \overset{max}{R} \overset{max}{I_L}}{R + r_Z}$)

$$R_{max} = \frac{V_{s\,min} - V_{Z0} - r_Z I_{ZK}}{I_{ZK} + I_{L\,max}}$$

$$= \frac{10.5 - 8.75 - 25 \times 2 \times 10^{-3}}{2 \times 10^{-3} + 20 \times 10^{-3}} = 77.27\ \Omega$$

- When $I_Z = I_{Zmax}$, we should have:

$$\begin{cases} V_{s\,max} \\ I_{L\,min} \\ R_{min}. \end{cases}$$

$$R_{min} = \frac{V_{s\,max} - V_{Z0} - r_Z I_{Zmax}}{I_{Zmax} + I_{Lmin}}$$

$$= \frac{14 - 8.75 - 25 \times 0.05}{0.05 + 5 \times 10^{-3}} = 72.73\ \Omega$$

The range of R is: $\boxed{72.73 \leqslant R \leqslant 77.27}$

d) Resistor come in standard specific values. In this design problem, we have to choose R that is close to the nearest 5% resistor value to the upper range (i.e. 77.27Ω)

(The 5% standard resistor values ~~E24~~ / E24 come as a decade multiples (i.e. x10, x100, x1000 or x 10 000) of the values shown in the table below:)

10	11	12	13	15	16	18	20	22	24	27	30
33	36	39	43	47	51	56	62	68	75	82	91

Thus the nearest value to $77.27\,\Omega$ is 75Ω.

$$R = 75\,\Omega.$$

i) Line regulation : $\dfrac{\Delta V_L}{\Delta V_s} = \dfrac{r_z}{R + r_z}$

$$\Delta V_L = \frac{25}{75+25} \cdot (14 - 10.5) = 0.875\,V$$

ii) Load regulation : $\dfrac{\Delta V_L}{\Delta I_L} = -(r_z \parallel R)$

$$\Delta V_L = -\frac{25 \times 75}{25+75}\,(20-5)\times 10^{-3} = -0.28125\,V.$$

iii) $I_{z\,max}$ occurs when $\begin{cases} V_s \text{ is max} \\ I_L \text{ is min} \end{cases}$ (as we have previously explained)

$$I_{Z\,max} = \frac{V_{s\,max} - V_{Z0} - R\, I_{L\,min}}{R + r_Z}$$

$$= \frac{14 - 8.75 - 75 \times 5 \times 10^{-3}}{75+25} = 0.04875\,A.$$

Under this condition,

$$P_{zener} = V_z \times I_z$$

$$= (8.75 + 25 \times 0.04875) \times 0.04875$$

$$= 0.486 \text{ W}.$$

The worst case power dissipation in R occurs when P_R is max i.e when $\begin{cases} V_s \text{ is max} \\ I_L \text{ is max.} \end{cases}$

$$P_R = R\left(I_{L_{max}} + I_Z\right)^2$$

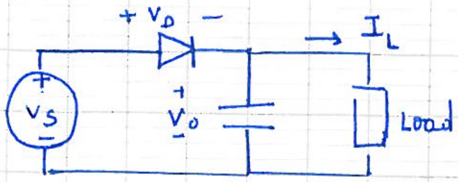
$$= R\left(I_{Lmax} + \frac{V_{smax} - V_{Z0} - R I_{Lmax}}{R + r_Z}\right)$$

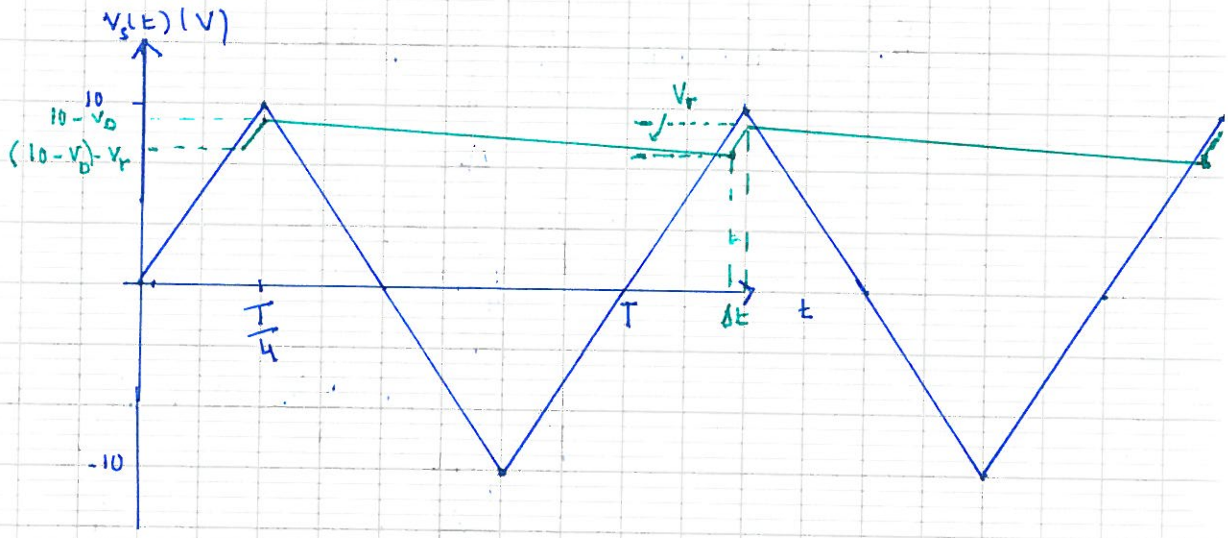
$$= 75\left(20 \times 10^{-3} + \frac{14 - 8.75 - 75 \times 20 \times 10^{-3}}{25 + 75}\right)$$

$$= 0.248 \text{ W}$$

Problem 2

$T = \frac{1}{50}$ s ; $V_D = 0.75V$; $I_L = 15$ mA (constant); $V_r = 0.5V$





a) $V_r = \frac{I_L}{f_C\, C} \rightarrow C = \frac{I_L}{f_C V_r} = \frac{15 \times 10^{-3}}{50 \times 0.5} = 6 \times 10^{-4}$ F.

b) In Δt, v_o rises linearly from $(\overbrace{V_{smax}}^{V_{omax}} - V_D - V_r)$ to $\underbrace{(V_{smax} - V_D)}_{V_{omax}}$;

The rate of this increase = rate of increase of $V_s(t)$ (same slope) sine when the diode is conducting $V_o = V_S - V_D$ (→ V_o and V_s are proportional in this case).

Therefore $\frac{V_{smax} - 0}{\frac{T}{4}} = \frac{(V_{smax} - V_D) - (V_{smax} - V_D - V_r)}{\Delta t}$

$\Delta t = \frac{\frac{T}{4} \cdot V_r}{V_{smax}} = \frac{\frac{1}{50 \times 4} \times 0.5}{10} = 2.5 \times 10^{-4}$ s $= 0.25$ ms

Fraction of the cycle $= \frac{\Delta t}{T} = \frac{0.25 \times 10^{-3}}{\frac{1}{}} = 0.0125$ (1.25 %)

c)

$$i_D = \begin{cases} i_C + I_L & \text{when it is conducting i.e. for } \Delta t \\ 0 & \text{when it is off} \end{cases}$$

$$i_{D\,average} = \frac{1}{T}\int_0^T i_D \, dt$$

$$= \frac{1}{T}\int_{\frac{T}{4}-\Delta t}^{\frac{T}{4}} i_D \, dt$$

$$= \frac{1}{T}\int_{\frac{T}{4}-\Delta t}^{\frac{T}{4}} \left(C\frac{dv_c}{dt} + I_L \right) dt$$

$$= \frac{1}{T}\left(\int_{\frac{T}{4}-\Delta t}^{\frac{T}{4}} \left(C\frac{dv_c}{dt}\right) dt + \int_{\frac{T}{4}-\Delta t}^{\frac{T}{4}} I_L \, dt \right)$$

$$= \frac{1}{T}\left(\int_{V_{sm}-V_D-V_r}^{V_{sm}-V_D} C \, dV_c + I_L \Delta t \right)$$

$$= \frac{1}{T}\left(C V_r + I_L \Delta t \right)$$

Thus $i_{D\,av} = \frac{1}{\frac{1}{50}} \left(6\times10^{-4} \times 0.5 + 15\times10^{-3} \times 2.5\times10^{-4} \right)$

$= 15.1875 \text{ mA}$

Note

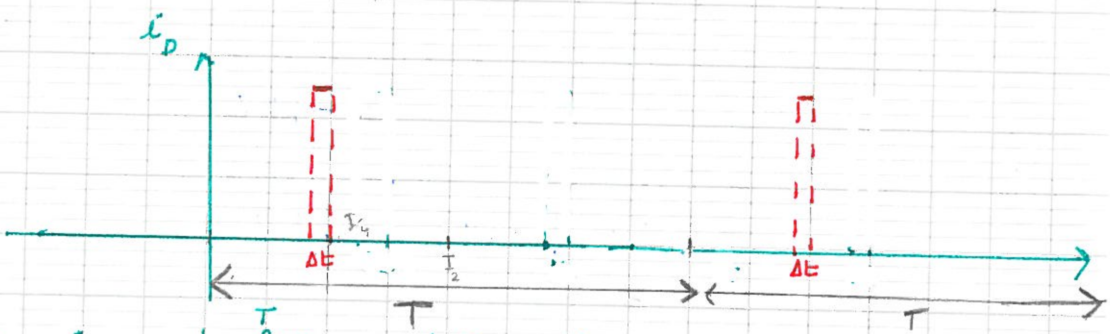
* When the diode is conducting,

$$i_D = I_C + I_L$$

$$= C\frac{dv_c}{dt} + I_L \quad ; \quad \frac{dv_c}{dt} = \text{slope of straight line} = ct$$

$$= C \cdot \frac{V_{smax}}{\frac{T}{4}} + I_L = ct$$

Thus
$$i_D = \begin{cases} C\,\dfrac{V_{smax}}{\frac{T}{4}} + I_L & \text{during } \Delta t \\ 0 & \text{else} \end{cases}$$



Thus
$$i_{Dave} = \frac{1}{T}\int_0^T i\,dt = \frac{1}{T}(i_D \cdot \Delta t)$$

$$= \frac{1}{T}\left(\frac{C\,V_{smax}}{\frac{T}{4}} + I_L\right) \cdot \Delta t$$

$$= \frac{1}{T}\left(\underbrace{C \cdot \frac{V_{smax}}{\frac{T}{4}}\Delta t}_{V_r} + I_L\,\Delta t\right)$$

$$= \frac{1}{T}(C V_r + I_L\,\Delta t) \text{ as previously obtained}$$

d) i_D remains constant during Δt,

$$i_{D\,peak} = C \cdot \frac{V_{smax}}{\frac{T}{4}} + I_L$$

$$= 6\times10^{-4} \times \frac{10}{\frac{1}{4\times50}} + 15\times10^{-3} = 1.215\text{ A}$$

or we can say that $i_{Dave} = \frac{1}{T}(i_{Dpeak} \cdot \Delta t)$

thus $i_{D\,peak} = \frac{T \cdot i_{Dave}}{\Delta t} = \frac{\frac{1}{50} \times 15.1875\times10^{-3}}{\ldots} = 1.215\text{A}$

e) PIV

When the diode is off,

- $V_{speak} = -10V = -V_{smax}$

(approximated) ← • $V_o = V_{omax} - V_r/2 = V_{smax} - V_D - \frac{V_r}{2}$; where $\begin{cases} V_{smax} = 10V \\ V_D = 0.751 \end{cases}$

$$V_D = V_s - V_o = \underbrace{V_{sp}}_{= -V_{smax}} - V_{smax} + V_D + \frac{V_r}{2}$$

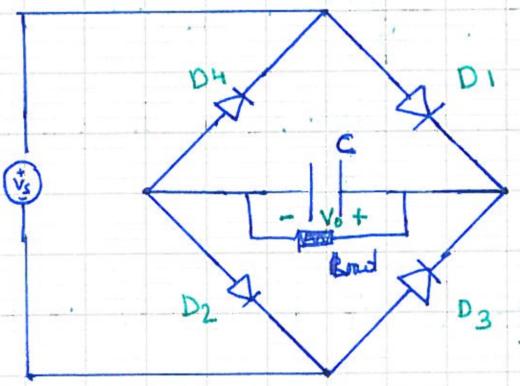
$$= -2V_{smax} + V_D + \frac{V_r}{2}$$

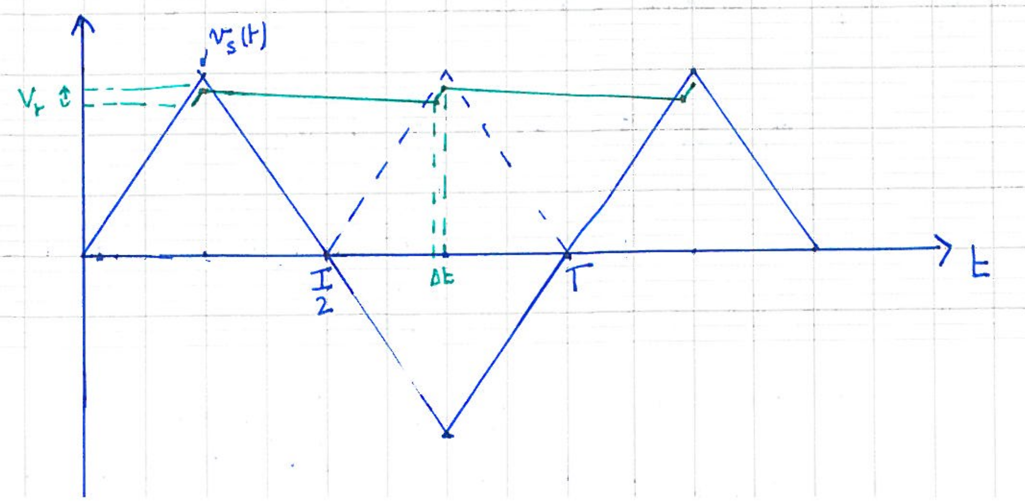
$$= -20 + 0.75 + \frac{0.5}{2}$$

$$= -19\,V$$

$$PIV = |-19| = 19\,V$$

Problem 3





a) $V_r = \frac{I_L}{(2f)\,C}$ (Now the cycle of V_o repeats every $\frac{T}{2}$ s)

$$C = \frac{I_L}{2f\,V_r} = 3 \times 10^{-4}\ F.$$

b) As calculated in the previous exercise
$\Delta t = 0.25$ ms

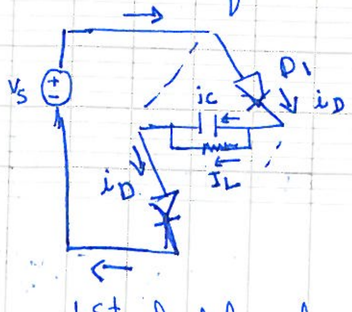
Each diode conducts ~~...~~ for Δt
(D_1 & D_2 take Δt during the first half cycle

D_3 & D_4 take Δt during the second half cycle)

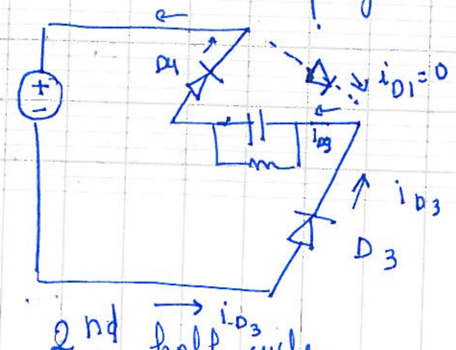
So that each diode conducts for $\frac{\Delta t}{T} = \frac{0.25 \times 10^{-3}}{\frac{1}{50}} = 0.0125$ (1.25%) fraction of the cycle; and ~~the~~ all the diodes conduct for $\frac{2\Delta t}{T} = 0.025$ (2.5%) fraction of the cycle.

c) As previously, i_D for each diode $= \begin{cases} i_c + I_L & \text{for } \Delta t \\ 0 & \text{otherwise} \end{cases}$

~~In~~ D_1 for example: during the first half cycle it conducts for Δt, during the second half cycle it is zero.



1st half cycle



2nd half cycle

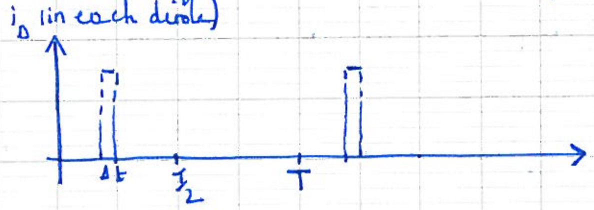
to derive this equation we proceed as we did in problem 2

Thus $i_{D\,average} = \frac{1}{T}\left(CV_r + I_L\,\Delta t\right)$

$$= \frac{1}{\frac{1}{50}}\left(3\times10^{-4}\times 0.5 + 15\times10^{-3}\times 0.25\times10^{-3}\right)$$

$$= 7.6875\ \text{mA} \quad (\text{for each diode})$$

d) During conduction, i_D in each diode remains constant for Δt (as justified in problem 2).



$$i_{D\,ave} = \frac{1}{T}\,\Delta t \,.\, i_{D\,peak}$$

$i_{D\,peak}$ for each diode $= \frac{T\,.\,i_{D\,ave}}{\Delta t} = \frac{\frac{1}{50}\times 7.6875}{0.25} = 0.615\ \text{A}$.

e) When one of the diode is not conducting (D_1 for example): (to calculate PIV for each diode)

- $V_s = -10\ \text{V}$
- $V_0 = V_{0max} = V_{smax} - V_D = 10 - 0.75 = 9.25\ \text{V}$

$$V_s = V_{D1} + V_0 + V_{D2}$$

$V_s = 2V_{D1} + V_0$; $V_{D1} = \frac{1}{2}\left(V_s - V_0\right)$

$$= \frac{1}{2}\left(-10 - 9.25\right) = -9.625$$

PIV $= 9.625\ \text{V}$ (for each diode).

Note : In problem 1 part B:

$V_z = V_{z0} + rz * I_{maax} = 8.75 + 25 * 0.05 = 10V$.