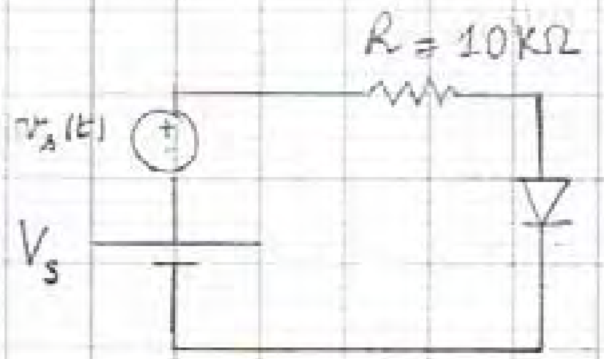
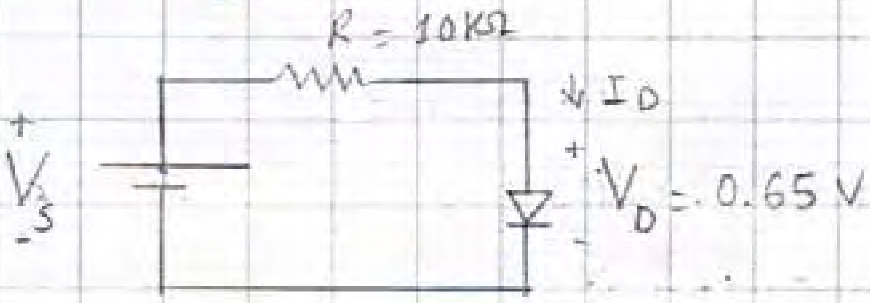
EECE 310 - Homework 3

Problem 1



DC analysis

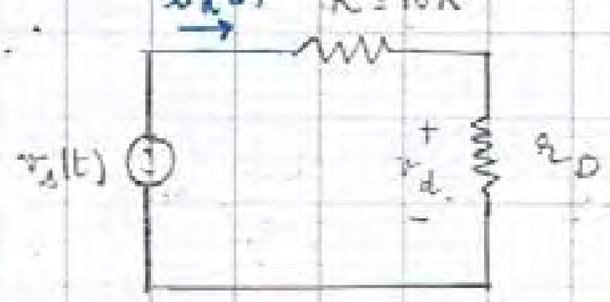


The diode is conducting:

$V_D = 0.65$ V

$$I_D = \frac{V_S - V_D}{R} = \frac{9 - 0.65}{10K} = 0.835 \text{ mA}.$$

Small signal analysis



The diode can be replaced by a resistor $r_D = \frac{nV_T}{I_D}$

$$v_d = \frac{r_d}{r_d + R} v_s(t) \quad ; \quad r_d = \frac{nV_T}{I_d} = \frac{1.7 \times 25}{0.835} = 50.898\ \Omega$$

$$v_d = \frac{50.888}{50.898 + 10\,000} v_s(t) = 5.064 \times 10^{-3} v_s(t)$$

$v_s(t)$ varies between $1.5\,V$ and $-1.5\,V$.

$$v_{d(max)} = 5.064 \times 10^{-3} \times 1.5 = 7.596\ mV$$
$$v_{d(min)} = -7.596\ mV.$$

Thus $v_d = \pm 7.596\,mV$, $v_{d_{p\text{-}p}} = 7.596 \times 2 = 15.192\ mV$

For $n = 1.7$, we have $|v_d(t)|_{max} = 7.596\ mV$; the small signal analysis is justified:

$$\cancel{i_D = I_S e^{\frac{v_D}{nV_T}} = I_S e^{\frac{V_D + v_d}{nV_T}} = I_D e^{\frac{v_d}{nV_T}}}$$

~~Taylor series:~~

$$\cancel{i_D = I_D \left(1 + \frac{v_d}{nV_T} + \frac{1}{2} \cdot \left(\frac{v_d}{nV_T}\right)^2 + \cdots \right)}$$

$$\left(\frac{v_d}{nV_T}\right)_{max} = \frac{7.596}{1.7 \times 25} = 0.178$$

< 0.2 (limit of small-signal analysis for diodes)

$$\cancel{\frac{1}{2}\left(\frac{v_d}{nV_T}\right)^2_{max} = 0.0156}$$ ~~(≈ 10 times lower than the above term)~~

$$\cancel{\text{Thus } i_D \simeq I_D \left(1 + \frac{v_d}{nV_T}\right)}$$

~~Thus the small signal analysis is justified (The higher non-linear terms are neglected because of their small values relative to the linear term).~~

Total diode current

$$i_D(t) = \underbrace{I_D}_{\substack{\text{DC} \\ \text{component}}} + i_d(t)$$

$$I_D = 0.835 \text{ mA}$$

$$i_{d}(t) = \frac{v_s(t)}{R + r_d}$$ (referring to the figure of small signal analysis)

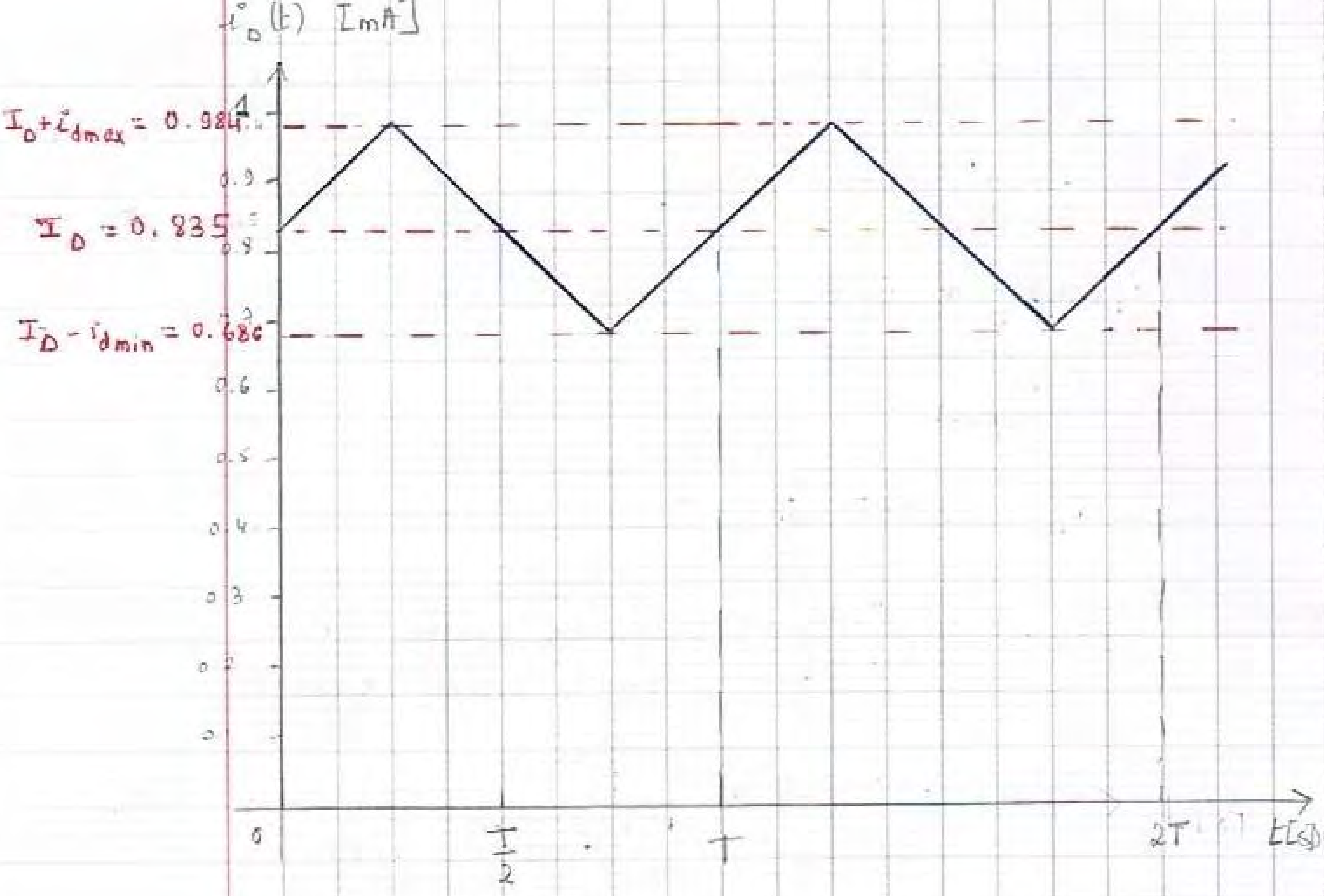
$$i_D(t) = 0.835 + \frac{1}{10 + 50.938 \times 10^{-3}} \underbrace{v_s(t)}_{[\text{in V}]} \quad [\text{in mA}]$$

$$\underbrace{i_D(t)}_{\text{triangular signal}} = 0.835 + 0.099 \underbrace{v_s(t)}_{\text{triangular signal}} \quad [\text{in mA}]$$

$$i_D(t)_{max} = 0.835 + 0.099 \times 1.5 = 0.984 \text{ mA}$$

$$i_D(t)_{min} = 0.835 - 0.099 \times 1.5 = 0.686 \text{ mA}$$

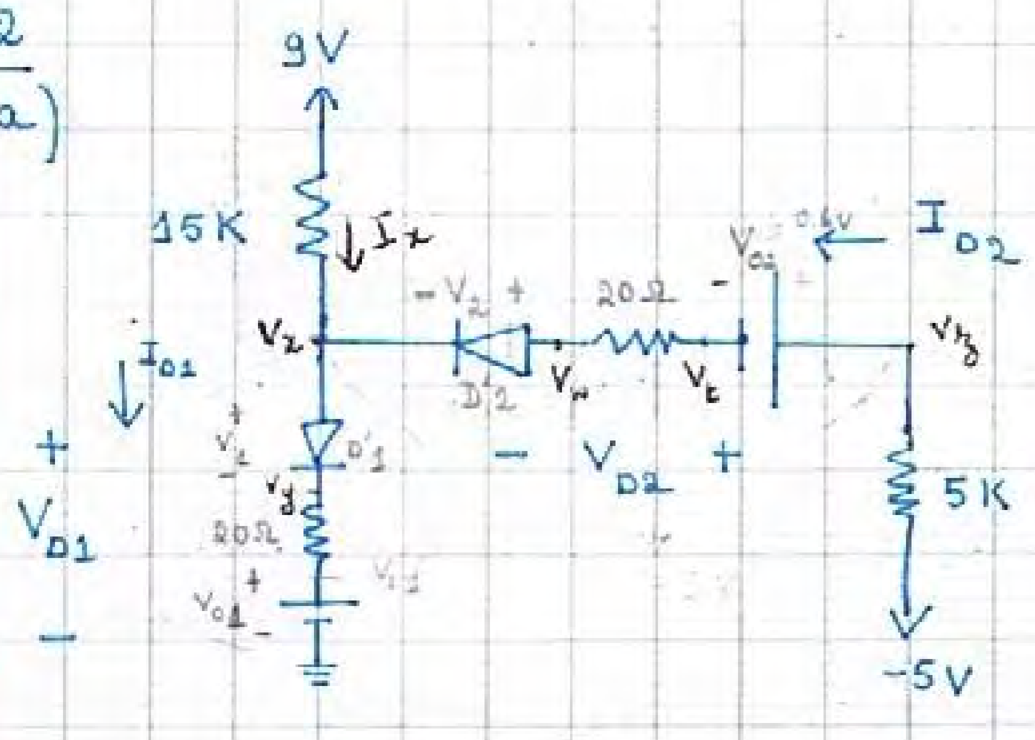
$i_D(t)$ has a triangular wave form that varies between 0.984 mA and 0.686 mA, crossing the line of 0.835 mA.



$i_D(t)$ [mA]
$I_D + i_{d\,max} = 0.984$
$I_D = 0.835$
$I_D - i_{d\,min} = 0.686$
0
$\frac{T}{2}$
T
2T
t[s]

Problem 2

a)



Assume D'_1 is ON and D'_2 is OFF

Thus $\begin{cases} V_1 = 0V \\ I_{D2} = 0A \end{cases}$ and we have to get $\begin{cases} I_{D1} > 0 \\ V_2 < 0 \end{cases}$

Since $V_1 = 0V$, $V_x = V_y$

Since $I_{D2} = 0A$, $I_x = I_{D1}$

$$I_x = \frac{9 - V_x}{15K} = \frac{V_y - V_{D1}}{20}$$

$$\frac{9 - V_x}{15\,000} = \frac{V_x - 0.6}{20}$$

$$180 - 20V_x = 15\,000V_x - 9\,000$$

$$V_x = \frac{9180}{15020} = 0.611\text{ V}$$

$$I_{D1} = \frac{V_x - 0.6}{20} = \frac{0.611 - 0.6}{20} = 0.55\text{ mA}. \ (I_{D1} > 0)$$

Since $I_{D2} = 0A$, $V_z = -5V$

$V_{D2} = V_z - V_t$; $V_t = V_z - V_{D2} = -5 - 0.6 = -5.6V$

$V_t = V_W = -5.6V$ (since $I_{D2} = 0A$).

$V_2 = V_W - V_x = -5.6 - 0.611 = -6.211V$ $(V_2 < 0)$

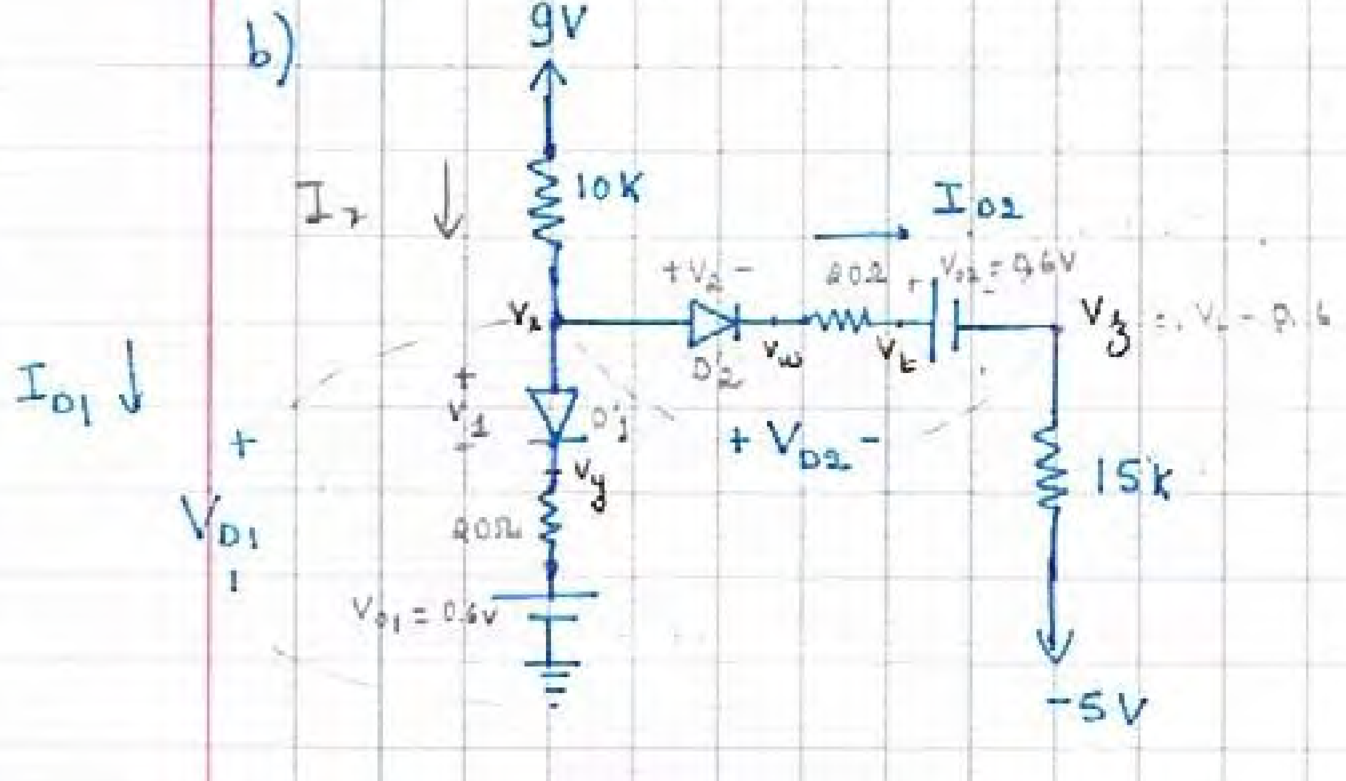
The assumptions are correct.

D_1 : $I_{D1} = 0.55\,mA$ $\Rightarrow D_1 : (0.55\,mA, 0.611V)$.
$V_{D1} = V_x = 0.611\,V$

D_2 : $I_{D2} = 0\,mA$
$V_{D2} = V_{D2} + V_2 = 0.6 - 6.211 = -5.611\,V$

$\Rightarrow D_2 : (0\,mA, -5.611\,V)$.

b)



Assume D_1' and D_2' are on.

We have $\begin{cases} V_1 = 0V \\ V_2 = 0V \end{cases}$ and we have to get $\begin{cases} I_1 > 0 \\ I_2 > 0 \end{cases}$

Since $V_1 = 0$, $V_x = V_y$

Since $V_2 = 0$, $V_x = V_w$

KCL : $I_x = I_{D1} + I_{D2}$

$$\frac{9 - V_x}{10K} = \frac{V_x - 0.6}{20} + \frac{V_x - V_t}{20}$$

$$\boxed{\left(\frac{1}{20} + \frac{1}{20} + \frac{1}{10K}\right) V_x - \frac{1}{20} V_t = \frac{9}{10K} + \frac{0.6}{20}} \quad (1)$$

$$I_{D2} = \frac{V_x - V_t}{20} = \frac{V_z + 5}{15K} \quad , \quad V_z = V_t - 0.6$$

$$\frac{V_x}{20} - \frac{V_t}{20} = \frac{V_t}{15K} + \frac{4.4}{15K}$$

$$\boxed{\frac{1}{20} V_x - \left(\frac{1}{15K} + \frac{1}{20}\right) V_t = \frac{4.4}{15k}} \quad (2)$$

Solving the 2 equations (1) & (2) gives:

$$V_x = 0.6101\,V$$
$$V_t = 0.6034\,V$$

$$I_{D1} = \frac{V_x - 0.6}{20} = \frac{0.6101 - 0.6}{20} = 0.505\,mA \ (>0)$$

$$I_{D2} = \frac{V_x - V_t}{20} = \frac{0.6101 - 0.6034}{20} = 0.335\,mA \ (>0)$$

The two assumptions are correct.

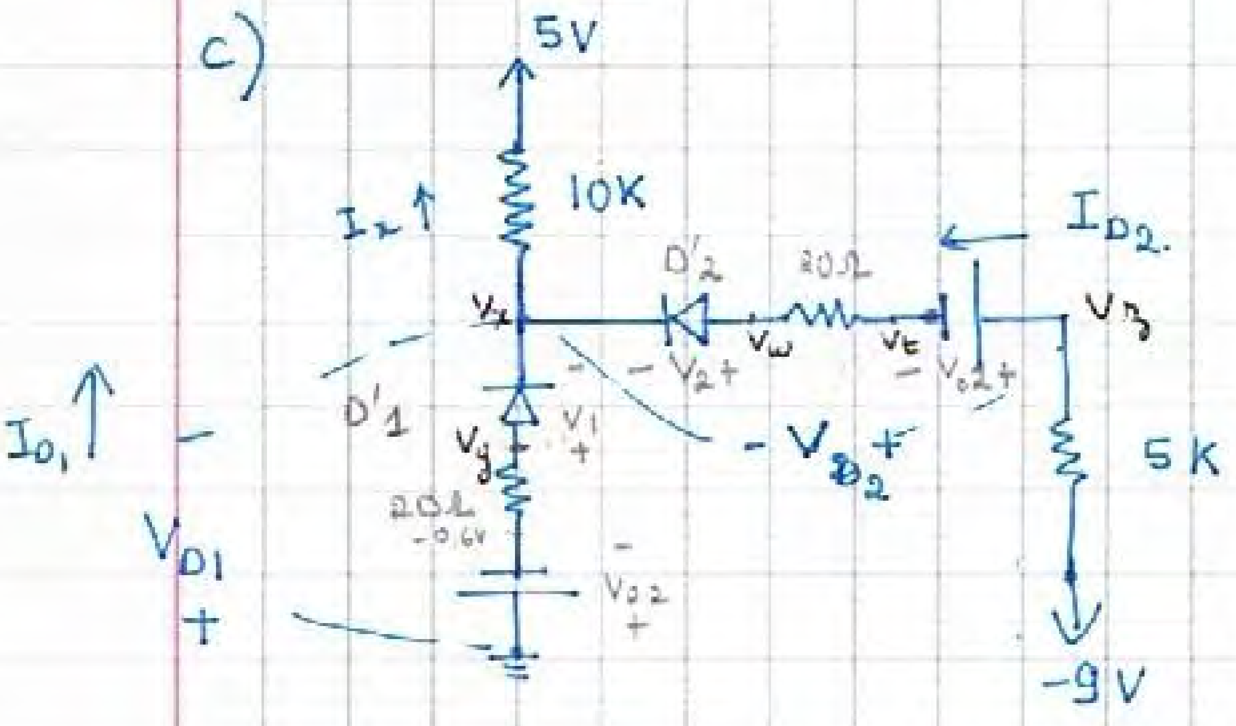
D_1 : $V_{D1} = V_x = 0.6101\,V$

$I_{D1} = 0.505\,mA \rightarrow D_1 : (0.505\,mA, 0.6101\,V)$.

D_2 : $V_{D2} = V_x - V_t + 0.6 = 0.6071\,V$

$I_{D2} = 0.335\,mA \rightarrow D_2 : (0.335\,mA, 0.6071\,V)$

c)



Assume D'_1 and D'_2 are OFF.

Thus we have $\begin{cases} V_1 = 0V \\ V_2 = 0V \end{cases}$ and we have to get $\begin{cases} I_{D1} > 0 \\ I_{D2} > 0 \end{cases}$

Since $I_{D2} = 0A$, $V_z = -9V \Rightarrow V_t = -9 - 0.6V$
$= -9.6V$

$V_w = V_t = -9.6V$

Since $I_{D1} + I_{D2} = 0 + 0 = 0$, $I_x \; (= I_1 + I_2) = 0A$
thus $V_x = 5V$

$V_2 = V_w - V_x = -9.6 - 5 = -14.6V \; (<0)$

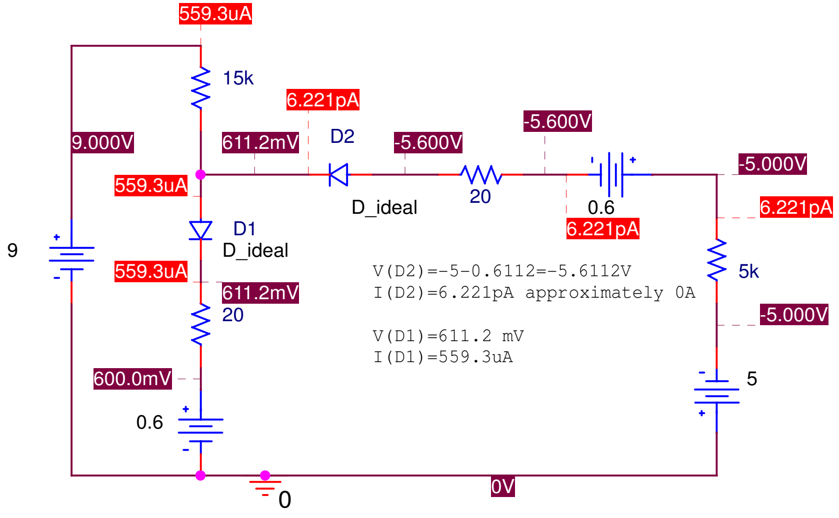
$V_y = -0.6V$; $V_1 = V_y - V_x = -0.6 - 5 = -5.6V \; (<0)$

The two assumptions are correct.

D_1: $V_{D1} = -V_x = -5V$
$I_{D1} = 0A \rightarrow D_1 : (0; -5V)$

D_2: $V_{D2} = 0.6 - 14.6 = -14V$
$I_{D2} = 0A \rightarrow D_2 : (0; -14V)$

a)



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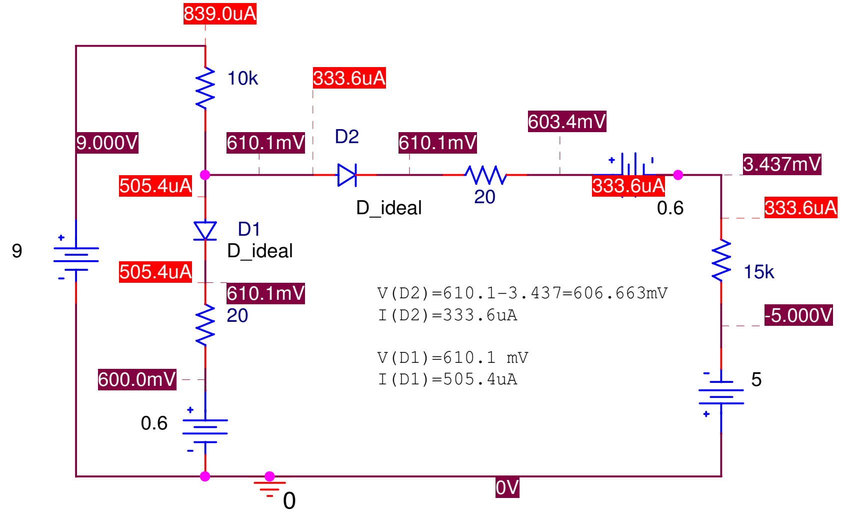

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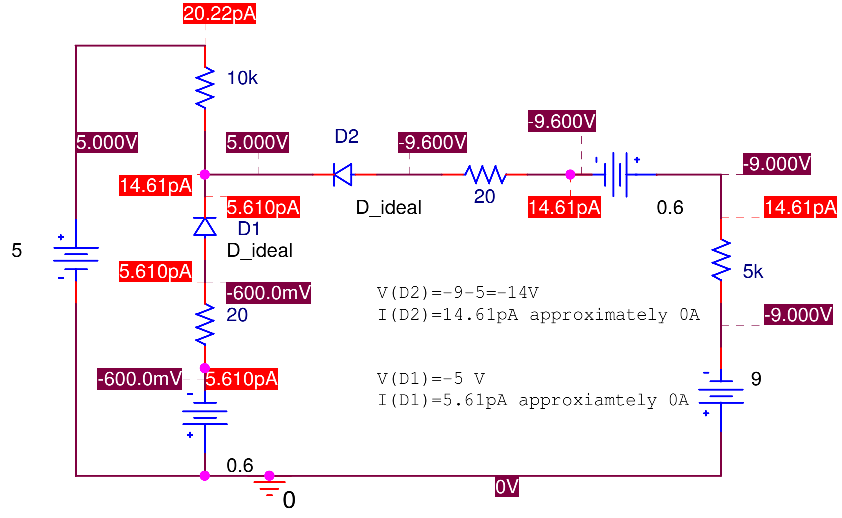

V(D2)=-9-5=-14V
I(D2)=14.61pA approximately 0A

V(D1)=-5 V
I(D1)=5.61pA approxiamtely 0A

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