American University of Beirut

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

EECE 310 – Electronics I   Fall 2007 – 2008

Homework 4

1. Consider the waveform shown in Figure 1.

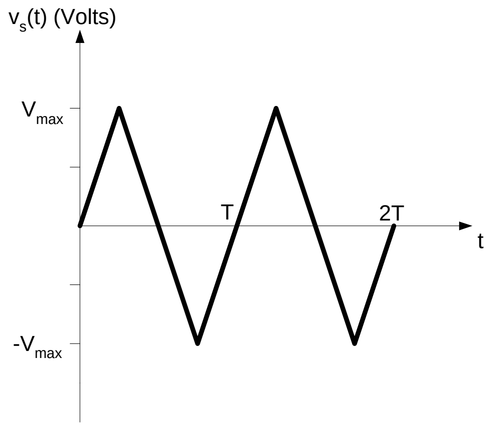


Figure 1

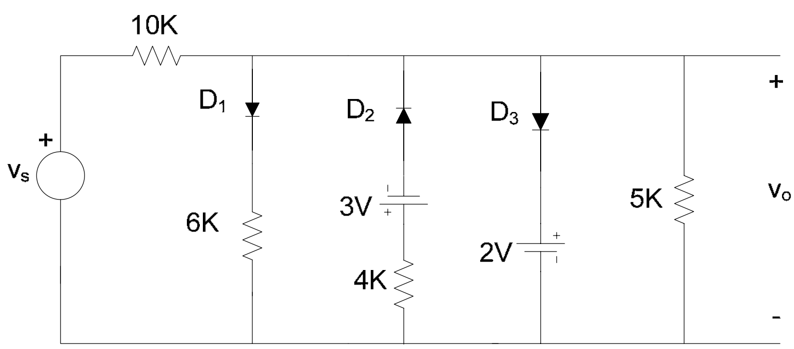
Assume that V_{max} = 12 V, and T = 1/50 sec. This voltage is the input to a half-wave rectifier with a capacitor filter. The diode drops 0.8 V when conducting, and the load is a 1000 Ω resistor.

a. Calculate the value of the capacitor if the ripple voltage is at most 1 V.
b. For what fraction of the cycle does the diode conduct?
c. Find the average diode current.
d. Find the peak diode current.
e. Find the PIV for the diode.

2. Repeat Problem 1 for the case in which the rectifier is a full-wave rectifier using a 4-diode bridge.

3.

a. Find and plot the transfer characteristics of the circuit shown below. Assume that when conducting in the forward direction, diodes drop 1 V.



b. Plot the output voltage when the source v_s is as given in Figure 1, with V_{max} = 15 V.

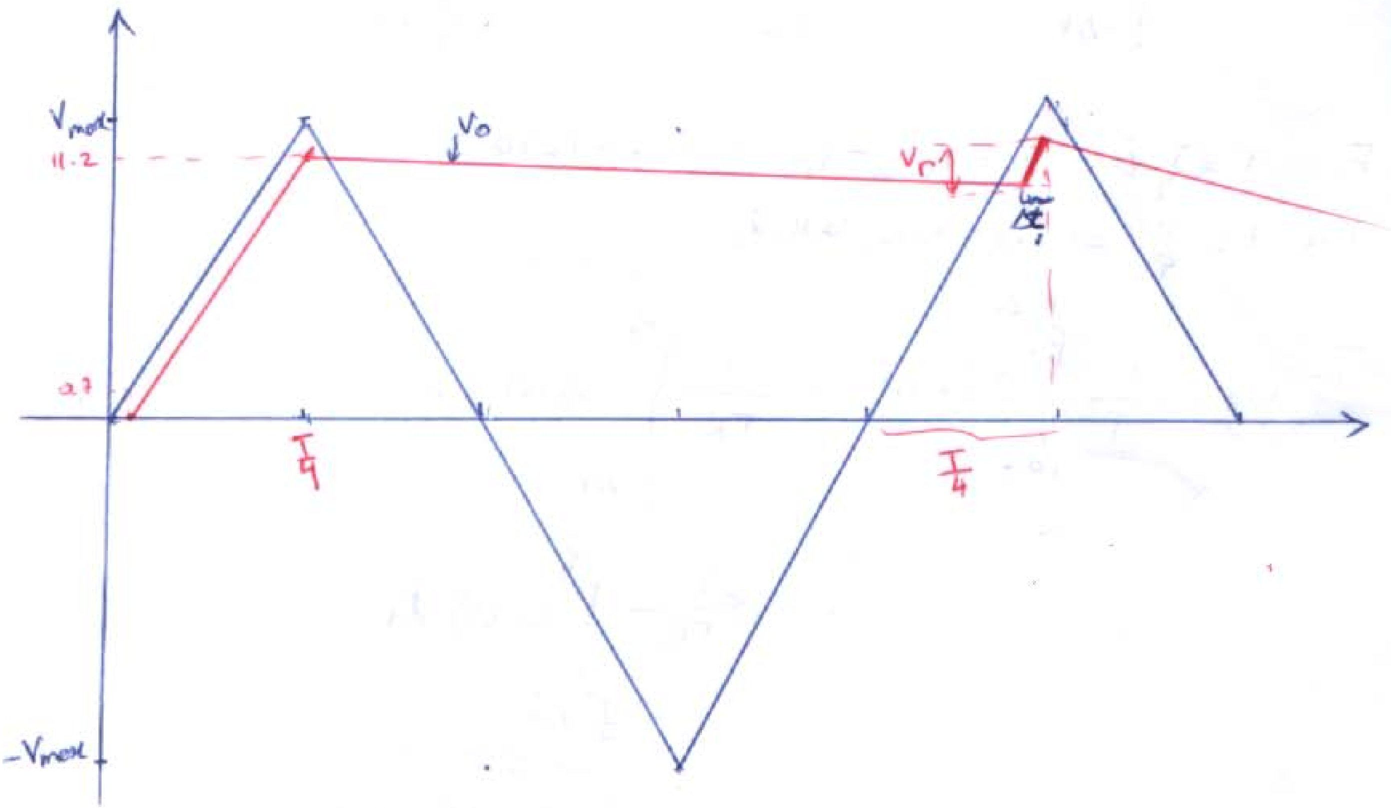
Home Work 4:

1-) a)

$$V_r = \frac{V_{max} - V_D}{RCf} = \frac{\overbrace{12 - 0.8}^{V_{omax}}}{RC} \cdot T \Rightarrow C = \frac{11.2}{50(1)(1000)} = 224\mu F$$

$V_o = V_{across\ capacitor}$.

b)



Using the slopes of the graph. with Δt_1 = conducting time.

$$\frac{V_{max}}{\frac{T}{4}} = \frac{V_r}{\Delta t_1} \Rightarrow \Delta t_1 = \frac{\frac{T}{4} V_r}{V_{max}} = \frac{\frac{1}{200} \cdot 1}{12} = 0.4167 \text{ ms}.$$

fraction of time during which diode conducts $= \frac{\Delta t_1}{T} = \frac{\frac{T}{4} V_r}{T V_{max}} = \frac{1}{48} = 0.02083$

$\Rightarrow 2.083\%$

c) $i_D(t) = i_C + i_R = \frac{C\,dv_o}{dt} + \frac{V_o(t)}{R}$

$$I_{D\,avg} = \frac{1}{T}\int_0^T i_D(t)\,dt \quad \text{but } i_D(t)=0 \text{ for } t \notin \left[\tfrac{T}{4}-\Delta t, \tfrac{T}{4}\right]$$

$$= \frac{1}{T}\int_{\frac{T}{4}-\Delta t}^{\frac{T}{4}} i_D(t)\,dt = \frac{1}{T}\int_{\frac{T}{4}-\Delta t}^{\frac{T}{4}} C\frac{dv_o}{dt}\,dt + \frac{1}{T}\int_{\frac{T}{4}-\Delta t}^{\frac{T}{4}} \frac{V_o(t)}{R}\,dt$$

For $t = \frac{T}{4}-\Delta t \Rightarrow V_o = V_{omax} - V_r = 11.2 - 1 = 10.2$

For $t = \frac{T}{4} \Rightarrow V_o = V_{omax} = 11.2$

$$\therefore I_{D\,avg} = \frac{1}{T}\int_{10.2}^{11.2} C\,dv_o(t) + \frac{1}{TR}\int_{\frac{T}{4}-\Delta t}^{\frac{T}{4}} V_o(t)\,dt.$$

$$= \frac{1}{\frac{1}{50}}\cdot C(11.2-10.2) + \underbrace{\frac{1}{TR}\int_{\frac{T}{4}-\Delta t}^{\frac{T}{4}} V_o(t)\,dt}_{\text{Area under the curve.}}$$

Area under the curve $= (V_{omax} - V_r)\,\Delta t_1 + \frac{V_r \Delta t_1}{2} = \left(V_{omax} - \frac{V_r}{2}\right)\Delta t_1$

$$\therefore I_{D\,avg} = 50(224\mu F)(1) + \frac{50}{1000}\left(V_{omax} - \frac{V_r}{2}\right)\Delta t_1$$

$$= 11.2\,mA + \frac{50}{1000}\left(11.2 - \frac{1}{2}\right)0.4167\,ms.$$

$$= 11.423\,mA.$$

d) Since $I_{D avg} = \frac{\text{Area under the curve}}{\text{period}} = \frac{I_{D peak} \times \text{conducting time}}{2} \cdot \frac{1}{T}$

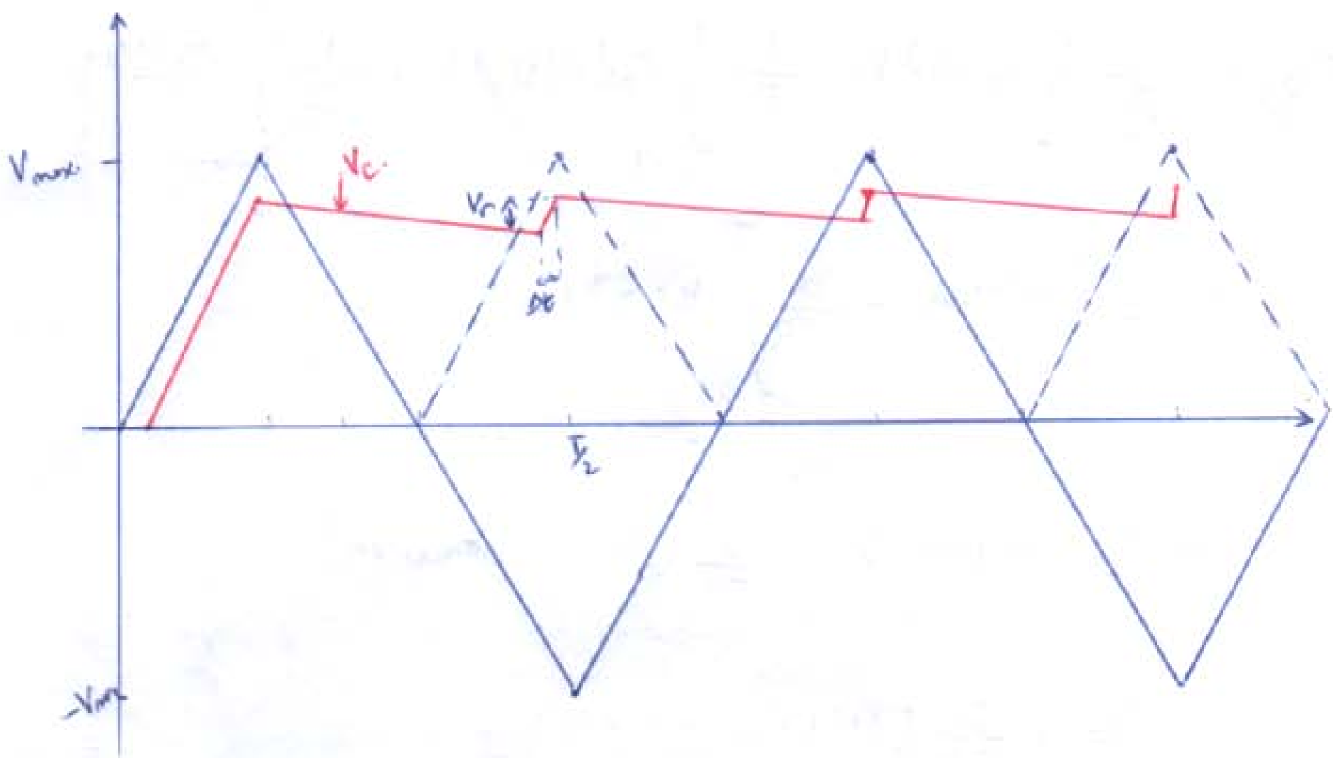
$\Rightarrow I_{D peak} = \frac{I_{D avg} \cdot 2T}{\Delta t_1} = \frac{11.423 \times 2/50}{0.4167ms} = 1.0965A$

c) PIV :

$V_S = V_D + V_o \Rightarrow V_D = V_S - V_o$

$PIV = |-V_{smax} - V_{omax}| = |-12 - 11.21| = 23.21V$

2)



a) $V_{cmax} = V_{smax} - V_{D1} - V_{D2}$ (by KVL)

$= 12 - 1.6 = 10.4V$

$V_r = \frac{V_{cmax}}{RCf'}$, $f' = 2f$ since the period is divided by 2. It needs less time

$\Rightarrow C = \frac{V_{cmax}}{V_r R \frac{2}{T}} = \frac{10.4}{1(1000) 2 \times 50} = 104\mu F$

b) $\frac{V_{smax}}{\frac{T}{4}} = \frac{V_r}{\Delta t} \Rightarrow \Delta t = \frac{1}{12} \times \frac{T}{4} = 0.4167\,ms.$

Fullwave rectifier $\Rightarrow$ diodes conduct twice during period. T.

$\Rightarrow$ fraction of time during which diodes conduct $= \frac{2\Delta t}{T} = 0.0467$

$\Rightarrow 4.16\%$

c) $i_o = i_c + i_R.$

$= \frac{C\,dv_c(t)}{dt} + \frac{V_c(t)}{R}$

$$I_{Davg} = \frac{1}{\frac{T}{2}} \int_0^{T/2} i_o(t)\,dt = \frac{1}{\frac{T}{2}} \int_{\frac{T}{2}-\Delta t}^{T/2} C\frac{dv_o(t)}{dt}\,dt + \frac{1}{\frac{T}{2}} \int_{\frac{T}{2}-\Delta t}^{T/2} \frac{V_o(t)}{R}\,dt$$

$$= \frac{2}{T}\int_{9.4}^{10.4} C\,dV_o(t) + \frac{2}{TR}\int_{\frac{T}{2}-\Delta t}^{T/2} V_o(t)\,dt$$

$$= \frac{2C}{T}(10.4 - 9.4) + \frac{2}{TR}(\text{Area under the curve})$$

$$= \frac{2C}{T} + \frac{2}{TR}(\Delta t)(V_{cmax} - V_r + \frac{V_r}{2})$$

$$= \frac{2C}{T} + \frac{2}{TR}\Delta t(V_{cmax} - \frac{V_r}{2})$$

$$= 100\,(104 \times 10^{-6}) + \frac{100}{1000}\cdot(0.4167 \times 10^{-3})(10.4 - \tfrac{1}{2})$$

$$= 10.8\,mA.$$

d) $I_{D\,avg} = \frac{\text{Area under the curve}}{\text{period}} = \frac{2 \times I_{D\,peak} \times \Delta t}{2T}$

$\Rightarrow I_{D\,peak} = \frac{I_{D\,avg} T}{\Delta t} = 518.36\,mA$

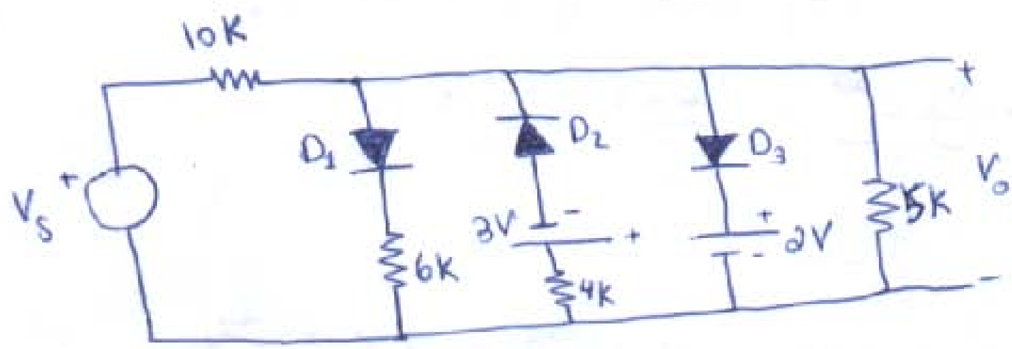
e) PIVs

$V_c + V_{D_1} + V_{D_2} = V_s$

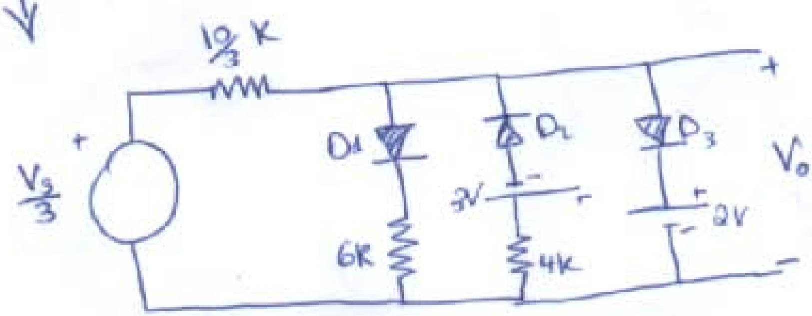
$\Rightarrow V_D = \frac{V_s - V_c}{2}$

$PIV = \left| \frac{-V_{s\,max} - V_{c\,max}}{2} \right| = \left| \frac{-12 - 10.4}{2} \right| = 11.2V.$

3)



V_{th}.



There are 8 possible cases but some are impossible for all values of V_s

	D_1	D_2	D_3
1	ON	ON	ON
2	ON	ON	OFF
3	ON	OFF	ON
4	OFF	ON	ON
5	ON	OFF	OFF
6	OFF	ON	OFF
7	OFF	OFF	ON
8	OFF	OFF	OFF

Case 1 : D1 ON ; D2 ON ; D3 : ON

KVL $\Rightarrow$. $V_{D1} + 6I_1 = V_{D3} + 2 \Rightarrow I_1 = \frac{1}{3}$ mA.

. $V_{D1} + 6I_1 = -V_{D2} - 3V - 4I_2 \Rightarrow 7 = -4I_2 \Rightarrow I_2 < 0 \Rightarrow D2$ off.

$\therefore$ This case is impossible.

Case 2 : D1 ON , D2 ON ; D3 : OFF

. $V_{D1} + 6I_1 = -V_{D2} - 3V - 4I_2 \Rightarrow 6I_1 + 4I_2 = -5$

$\Rightarrow I_1 + I_2 < 0 \Rightarrow I_1 < 0$ or $I_2 < 0 \Rightarrow$ D1 or D2 is off.

$\therefore$ This case is impossible.

Case 3 : D1 : ON ; D2 : OFF ; D3 : ON.

. $V_{D1} + 6I_1 = V_{D3} + 2 \Rightarrow I_1 = \frac{1}{3}$ mA. ; $V_0 = V_{D3} + 2 = 3V$.

$$I_{in \frac{10}{3}k} = I = \frac{\frac{V_s}{3} - V_0}{\frac{10}{3}} = \frac{\frac{V_s}{3} - 3}{\frac{10}{3}} = \frac{V_s - 9}{10}$$

$$I_3 = I - I_1 = \frac{V_s - 9}{10} - \frac{1}{3}$$

$$I_3 > 0 \Rightarrow \frac{V_s - 9}{10} > \frac{1}{3} \Rightarrow V_s > \frac{10}{3} + 9 \Rightarrow V_s > \frac{37}{3}$$

$V_{D2} + 3 + V_{D3} + 2 = 0 \Rightarrow V_{D2} = -6 < 0 \Rightarrow$ correct.

$\therefore$ D1 : ON ; D2 : OFF ; D3 : ON.

For $V_s \geq \frac{37}{3}V$; $V_0 = 3V$

Case 4 : D1 : OFF ; D2 : ON ; D3 : ON

$V_{D2} + 3 + 4I_2 = -V_{D3} - 2 \Rightarrow I_2 = \frac{-7}{4} < 0 \Rightarrow$ D2 : OFF

$\therefore$ This case is impossible.

Case 5: D1: ON ; D2: OFF; D3: OFF

$\therefore \frac{V_s}{3} = \frac{10}{3}k I_1 + V_{D1} + 6I_1 \Rightarrow \frac{V_s}{3} - 1 = \left(\frac{10}{3}+6\right) I_1 \Rightarrow I_1 = \frac{V_s}{28} - \frac{3}{28}$.

$I_1 > 0 \Rightarrow \frac{V_s}{3} - 1 > 0 \Rightarrow V_s > 3$.

$V_0 = V_{D1} + 6I_1 = 1 + 6\left(\frac{V_s}{28} - \frac{3}{28}\right) = 1 + \frac{6}{28}V_s - \frac{18}{28} = \frac{6}{28}V_s + \frac{10}{28} = \frac{3}{14}V_s + \frac{5}{14}$

$\frac{V_s}{3} = \frac{10}{3}I_1 - V_{D2} - 3 \Rightarrow V_{D2} = \frac{10}{3}I_1 - \frac{V_s}{3} - 3$.

$\Rightarrow V_{D2} = \frac{10}{3}\left(\frac{1}{28}V_s - \frac{3}{28}\right) - \frac{V_s}{3} - 3 = \frac{5}{3 \times 14}V_s - \frac{5}{14} - \frac{V_s}{3} - 3 = -\frac{3V_s}{14} - \frac{47}{14}$

$V_{D2} < 1 \Rightarrow -\frac{3V_s}{14} - \frac{47}{14} < 1 \Rightarrow -\frac{3}{14}V_s < \frac{61}{14} \Rightarrow V_s > -\frac{61}{3}$

which is true $V_s > 3$

$V_{D3} = -V_{D2} - 3 - 2 = \frac{3V_s}{14} + \frac{47}{14} - 5 = \frac{3V_s}{14} - \frac{23}{14}$

$V_{D3} < 1 \Rightarrow \frac{3V_s}{14} - \frac{23}{14} < 1 \Rightarrow \frac{3}{14}V_s < \frac{37}{14} \Rightarrow V_s < \frac{37}{3}$

$\therefore$ D1: ON ; D2: OFF ; D3: OFF

$3 < V_s < \frac{37}{3}$; $V_0 = \frac{3V_s}{14} + \frac{5}{14}$

Case 6: D1: OFF ; D2: ON ; D3: OFF

$\frac{V_s}{3} = \frac{10}{3}(I_2) - V_{D2} - 3 - 4(I_2) \Rightarrow \frac{V_s}{3} = \left(-\frac{10}{3} - 4\right) I_2 - 4$

$\Rightarrow I_2 = \frac{1}{-\left(\frac{10}{3}+4\right)} \cdot \left(\frac{V_s}{3} + 4\right)$

$I_2 > 0 \Rightarrow -\left(\frac{V_s}{3} + 4\right) > 0 \Rightarrow -\frac{V_s}{3} > -4 \Rightarrow V_s < -12V$.

$V_{D1} = \frac{10}{3}I_2 + \frac{V_s}{3} = \frac{-10/3}{\frac{10}{3}+4}\left(\frac{V_s}{3}+4\right) + \frac{V_s}{3} = \frac{-5}{11}\left(\frac{V_s}{3}+4\right) + \frac{V_s}{3}$

$= \frac{2}{11}V_s - \frac{20}{11}$

$V_{D1} < 1 \Rightarrow 2V_s - 20 < 11 \Rightarrow V_s < \frac{31}{2}$ but $V_s < -12V$

$V_{D3} = V_{D1} - 2 = \frac{2}{11}V_s - \frac{42}{11}$

$V_{D3} < 1 \Rightarrow V_s < \frac{53}{2}$ but $V_s < -12$

$\therefore \quad V_o = \frac{10}{3} I_2 + \frac{V_s}{3} = \frac{2}{11} V_s - \frac{20}{11}$

$\therefore$ D1 : OFF ; D2 : ON ; D3 : OFF

$V_s < -12V ; \quad V_o = \frac{2}{11} V_s - \frac{20}{11}$

Case 7: D1 : OFF ; D2 : OFF ; D3 : ON

$V_{D1} + 6(0) = V_{D3} + 2 = 3$

$\Rightarrow V_{D1} > 1 \Rightarrow$ D1 ON.

$\therefore$ This case is impossible.

Case 8: All are off.

$\frac{V_s}{3} = V_{D_3} + 2V \Rightarrow V_{D3} = \frac{V_s}{3} - 2 \; ; V_{D3} < 1 \Rightarrow V_s < 9$

$\frac{V_s}{3} = V_{D1} + 6(0) \Rightarrow V_{D1} < 1 \Rightarrow V_s < 3$

$\frac{V_s}{3} = -V_{D2} - 3 \Rightarrow V_{D2} < 1 \Rightarrow \frac{V_s}{3} + 3 > 1 \Rightarrow V_s > -12.$

$V_o = \frac{V_s}{3}.$

$\therefore$ All Diode off

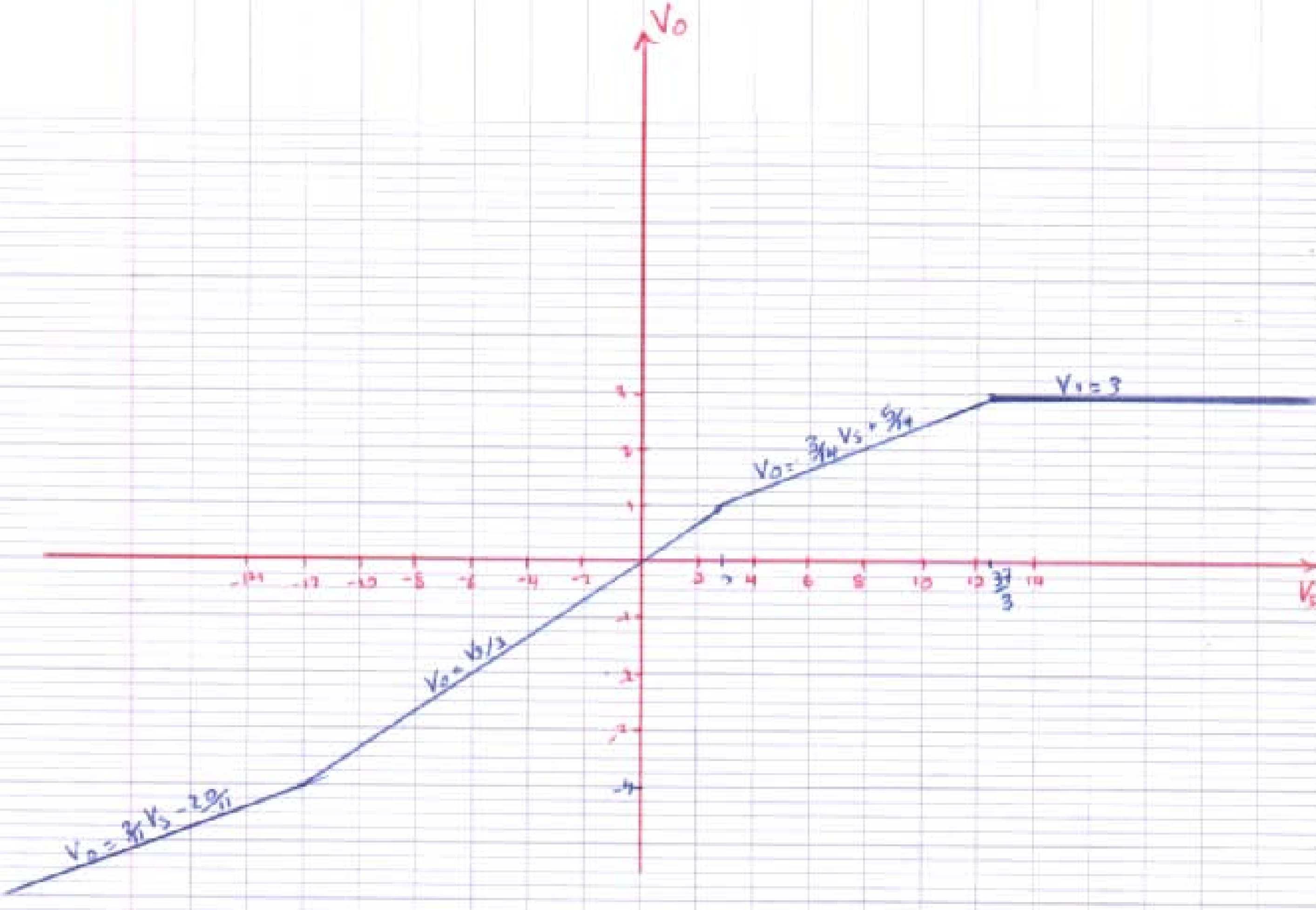
for $-12 \leq V_s \leq 3 \quad ; V_o = \frac{V_s}{3}$

$\therefore$ For $V_s \leq -12V \quad ; V_o = \frac{2}{11} V_s - \frac{20}{11}$

$-12V \leq V_s \leq 3V \quad ; V_o = \frac{V_s}{3}$

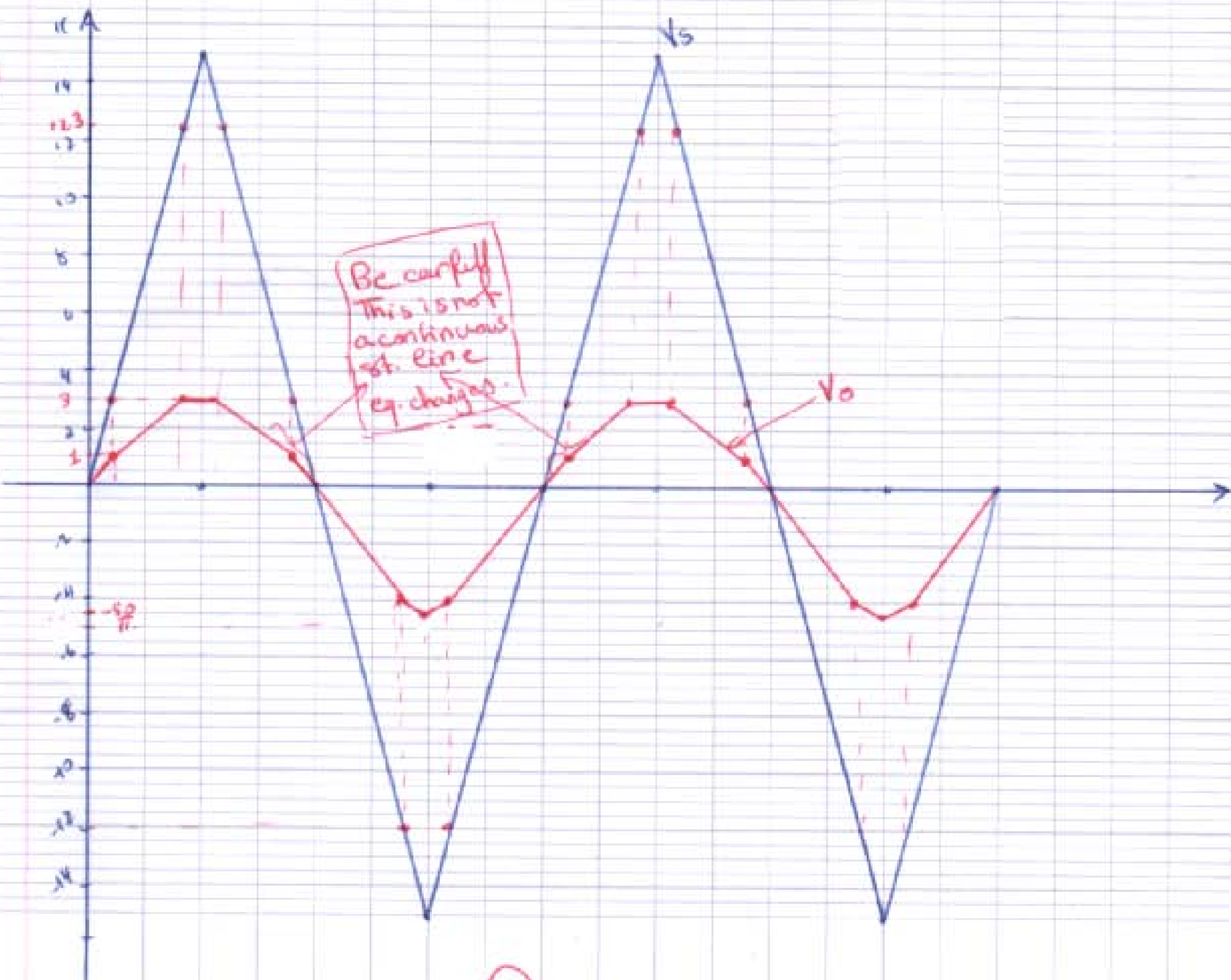
$3V \leq V_s \leq \frac{37}{3} V \quad ; V_o = \frac{3}{14} V_s + \frac{5}{14}$

$V_s \geq \frac{37}{3} V \quad ; V_o = 3V.$



Vo
Vs
Vo = 3/14 Vs + 5/14
Vo = Vs/3
Vo = 3/11 Vs − 20/11
37/3

b)



Vs
Vo
Be carfull
This is not
a continuous
st. line
eq. changes.