

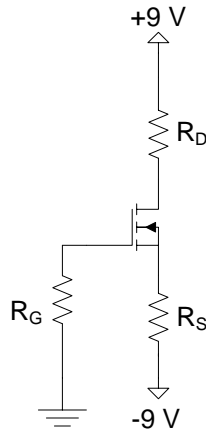
**American University of Beirut**  
Department of Electrical and Computer Engineering

EECE 310 – Electronics

Fall 2007 – 2008

Homework 8

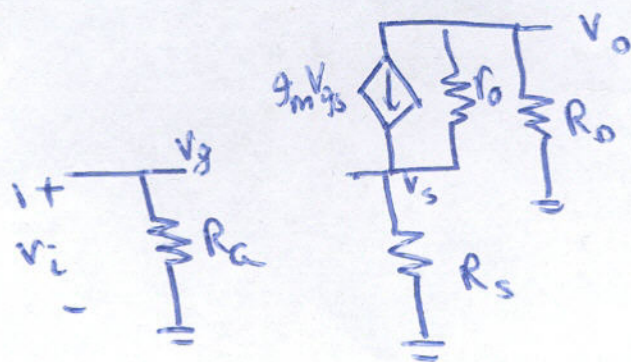
The MOSFET in the circuit has  $V_t = 1.2\text{ V}$ ,  $k'(W/L) = 1\text{ mA/V}^2$ , and  $V_A = 1/\lambda = 50\text{ V}$ .



- Find the values of  $R_S$ ,  $R_D$ , and  $R_G$  so that  $I_D = 1\text{ mA}$ , the largest possible value of  $R_D$  is used while a maximum signal swing at the drain of  $\pm 2.5\text{ V}$  is possible keeping the MOSFET in the saturation region, and the input resistance at the gate is  $5\text{ M}\Omega$ .
- Find the values of  $g_m$  and  $r_o$  at the bias point.
- If the amplifier input (at the gate) is connected to a source with a  $100\text{ K}\Omega$  source resistance, and to a load (at the drain) with a resistance of  $50\text{ K}\Omega$ , find the voltage gain and the power gain from *signal* source to output.
- To increase the voltage gain, a capacitor is used to bypass part of the resistor  $R_S$ . How much of  $R_S$  should be bypassed to obtain a voltage gain of  $-5$  from signal source to load?
- Verify your results using PSpice. Assume that the source and load are coupled to the MOSFET gate and drain, respectively, using capacitors. All capacitor values are  $500\mu\text{F}$ , including the one used to partially bypass  $R_S$ . Use a sinusoidal source with a frequency of  $1\text{ KHz}$ . Show the voltage gain, current gain, and maximum voltage swing at the drain.

# Home Work 8:

a) Small signal model:



$$R_{in} = \frac{V_i}{I_i} = R_G$$

$$\Rightarrow R_G = 5M\Omega$$

MOSFET in Saturation  $\Rightarrow V_{DS} > V_{GS} - V_t$

$$\Rightarrow V_D > V_G - V_t$$

$$\Rightarrow V_D > 0 - 1.2V$$

$$\Rightarrow V_{D \min} = -1.2V$$

$$V_{D \min} = V_D - \Delta V_{D \max} = V_D - 2.5 \Rightarrow V_D = 1.3V$$

$$R_D = \frac{V_{DD} - V_D}{I_D} = \frac{9 - 1.3}{1mA} = 7.7K\Omega$$

$$V_{DS} = V_D - R_S I_S + 9 = 10.3 - R_S$$

$$I_D = \frac{1}{2} K_n \frac{W}{L} (V_G - V_S - V_t)^2 (1 + \lambda V_{DS}) \Rightarrow 2 = (9 - R_S - 1.2)^2 \left(1 + \frac{10.3 - R_S}{50}\right)$$

$$\Rightarrow (7.8 - R_S)^2 (1.206 - 0.02R_S) = 2 \quad (R_S \text{ in } K\Omega)$$

$$\Rightarrow R_S = 60.26K\Omega \quad \text{rejected since } V_{GS} = -V_S = 9 - R_S = -51.26 < 1.2 = V_t$$

$$\text{OR } R_S = 9.2K\Omega \quad \text{rejected since } V_{GS} = -V_S = 9 - R_S = -0.2 < 1.2$$

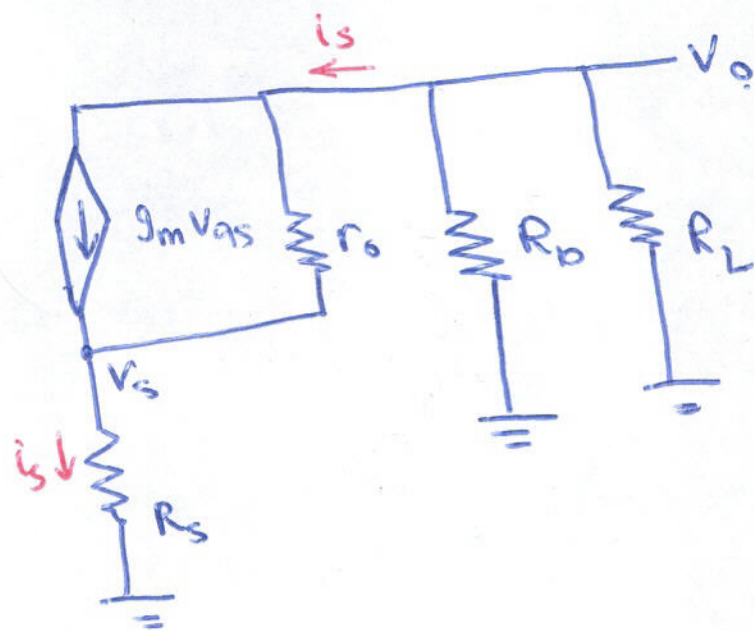
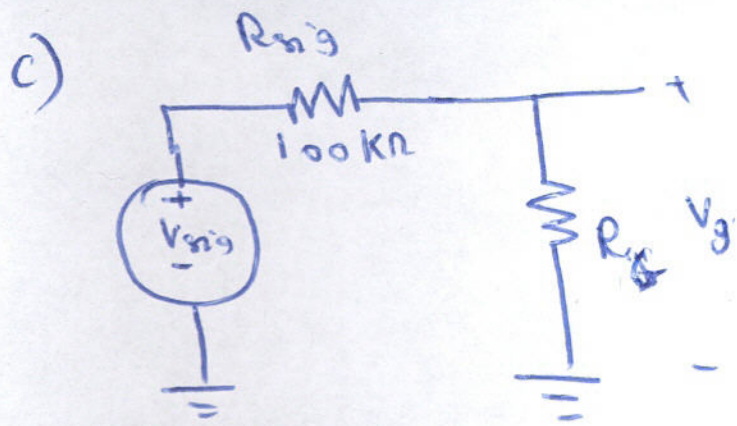
$$\text{OR } R_S = 6.44K\Omega \quad \text{accepted } V_{GS} = -V_S = 9 - R_S = 2.56 > 1.2V$$

$$\therefore R_G = 5M\Omega ; R_D = 7.7K\Omega ; R_S = 6.44K\Omega$$

$$b) g_m = \frac{2I_D}{V_{ov}} = \frac{2 \times 1 \times 10^{-3}}{V_G - V_S - V_t} = \frac{2mA}{9 - R_S - 1.2} = \frac{2mA}{1.36} = 1.47mA/V$$

$$r_o = \frac{V_{DS} + 1/\lambda}{I_D} = \frac{1.3 + 2.56 + 50}{1 \times 10^{-3}} = 53.86K\Omega$$





$$\bullet V_o = -i_s (R_D \parallel R_L)$$

$$\bullet V_s = i_s R_s = \frac{-V_o R_s}{R_D \parallel R_L}$$

$$\bullet i_s = g_m V_{gs} + \frac{V_o - V_s}{r_o}$$

$$\circ \circ \frac{-V_o}{R_D \parallel R_L} = g_m V_{gs} + \frac{V_o}{r_o} + \frac{V_o R_s}{r_o \cdot R_D \parallel R_L} = g_m V_g + g_m \frac{V_o R_s}{R_D \parallel R_L} + \frac{V_o}{r_o} + \frac{V_o R_s}{r_o \cdot R_D \parallel R_L}$$

$$\Rightarrow V_o \left( \frac{1}{r_o} + \frac{1 + g_m R_s}{R_D \parallel R_L} + \frac{R_s}{r_o \cdot R_D \parallel R_L} \right) = -g_m V_g$$

$$\Rightarrow \frac{V_o}{V_{gs}} = \frac{-g_m r_o \cdot R_D \parallel R_L}{r_o + r_o g_m R_s + R_D \parallel R_L + R_s} = -g_m \frac{r_o (R_D \parallel R_L)}{r_o (1 + g_m R_s) + R_D \parallel R_L + R_s}$$



$$\frac{V_{gs}}{V_{sig}} = \frac{R_c}{R_c + R_{sig}}$$

$$A_v = \frac{V_o}{V_{sig}} = -\frac{R_c}{R_c + R_{sig}} \cdot g_m \frac{r_o (R_D \parallel R_L)}{r_o (1 + g_m R_s) + R_D \parallel R_L + R_s}$$

Note: If we neglect  $r_o \Rightarrow r_o \rightarrow \infty \Rightarrow A_v \rightarrow \frac{-R_c}{R_c + R_{sig}} \cdot g_m \cdot \frac{R_D \parallel R_L}{1 + g_m R_s}$ .

$$A_v = \frac{-5M}{(5 + 0.1)M} \cdot 1.47 \times 10^{-3} \cdot \frac{(53.86K)(7.7K \parallel 50K)}{53.86K(1 + 1.47 \times 6.44) + (7.7K \parallel 50K) + 6.44K} = -0.897 V/V$$



$$\text{input current } i_1 = \frac{V_{sig}}{5.1M}$$

$$\text{output current } i_o = \frac{V_o}{50K}$$

$$\frac{i_o}{i_1} = \frac{V_o}{V_{sig}} \times \frac{5.1M}{50k} = A_v \times \frac{5.1M}{50k}$$

$$A_p = A_i A_v = A_v^2 \times \frac{5.1M}{50k} = 82.07 W/W$$

$$\text{OR } A_p = \frac{P_o}{P_i} = \frac{\frac{V_o}{\sqrt{2}} \frac{i_o}{\sqrt{2}}}{\frac{V_i}{\sqrt{2}} \frac{i_i}{\sqrt{2}}} = \frac{\frac{V_o^2}{2 \times 50k}}{\frac{V_i^2}{2 \times 5.1M}} = \frac{V_o^2}{V_i^2} \times \frac{5.1M}{50k} = A_v^2 \times \frac{5.1M}{50k} = 82.07 W/W$$

d) Part of  $R_s$  by passed will be short circuited.

$$A_v = -5 \Rightarrow \frac{-R_a}{R_a + R_{sig}} \cdot g_m \frac{r_o (R_o \parallel R_L)}{r_o (1 + g_m R_s) + R_o \parallel R_L + R_s} = -5$$

$$\Rightarrow \frac{-5}{5.1} \times \frac{1.47 \times 10^{-3} \times 53.86k \times (7.7k \parallel 50k)}{53.86k (1 + 1.47 \times 10^{-3} R_s) + (7.7k \parallel 50k) + R_s} = -5$$

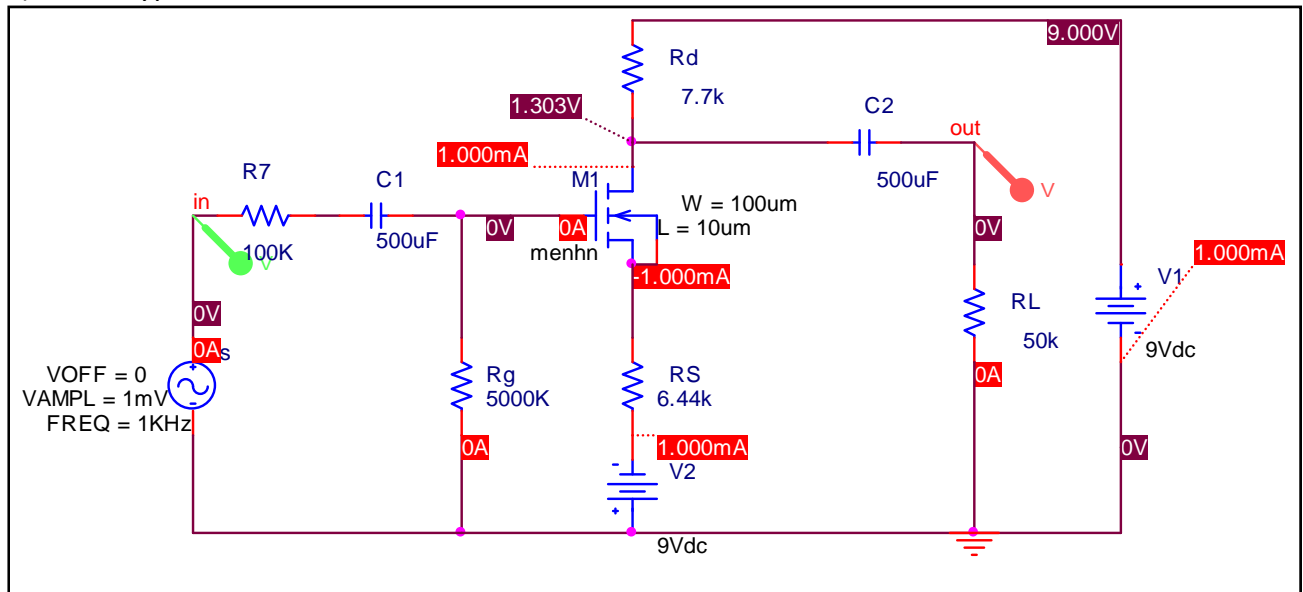
$$\Rightarrow 53.86k + 53.86 \times 1.47 R_s + (7.7k \parallel 50k) + R_s = \frac{1.47 \times 10^{-3} \times 53.86k \times (7.7k \parallel 50k)}{5.1}$$

$$\Rightarrow 80.1742 R_s = 103.585k - 53.8k - (7.7k \parallel 50k) = 43.053k$$

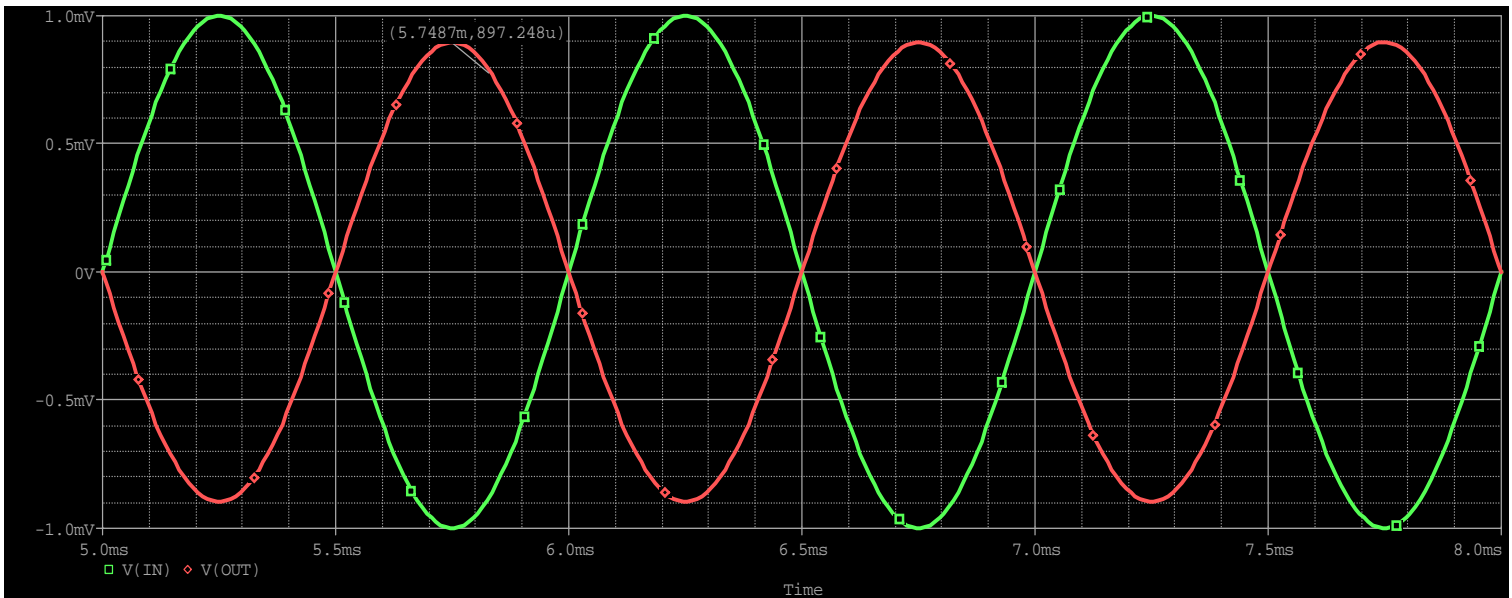
$$\Rightarrow R_s = 537 \Omega$$

$$\text{Part of } R_s \text{ by passed} = 6.44k - 537 = 5.903k \Omega$$

e) For unbypassed  $R_s$

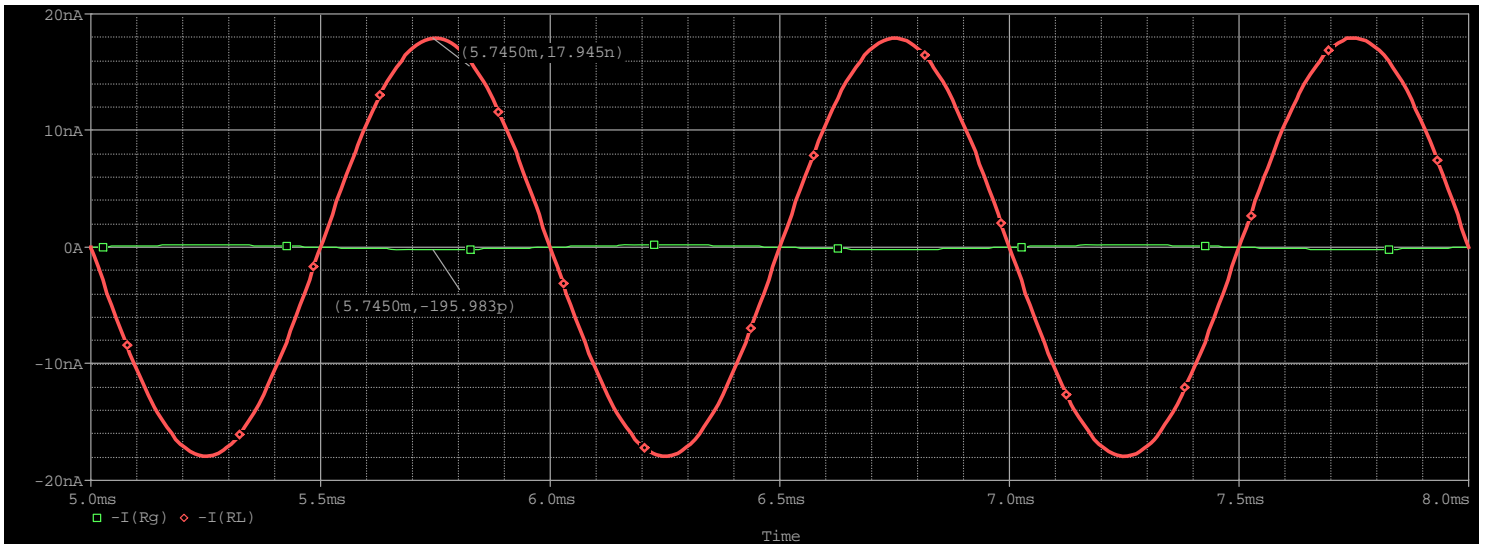


$V_o$ : red,  $V_i$  in green.



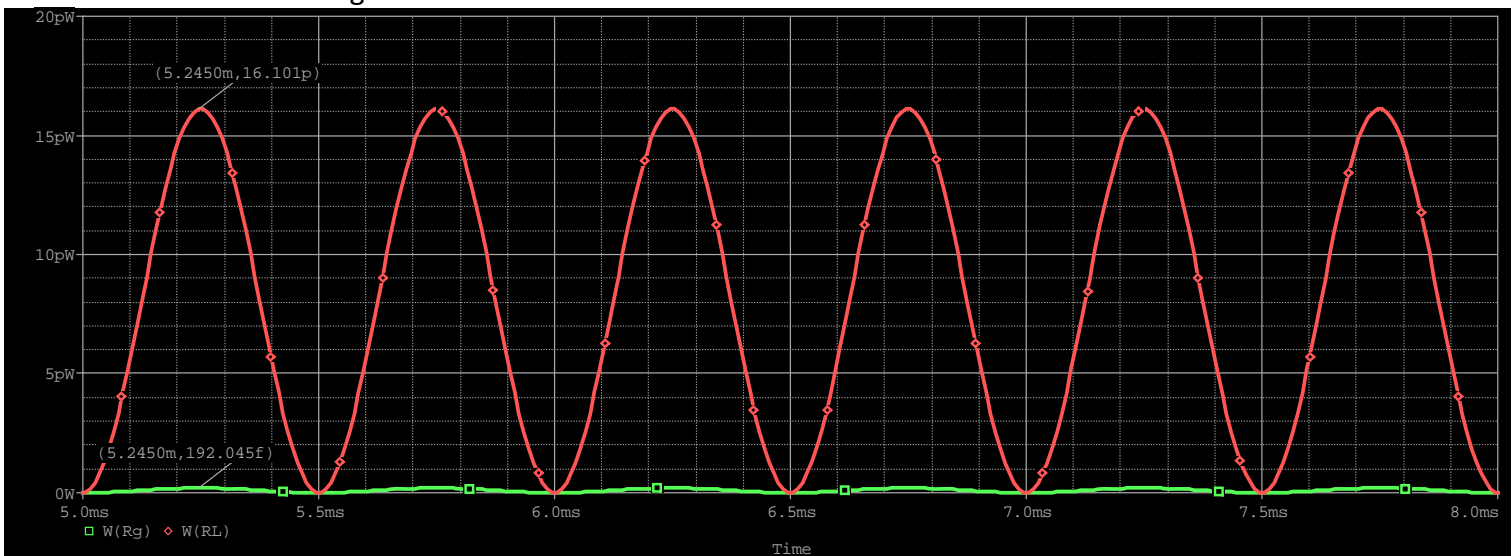
$$A_v = V_o/V_i = -0.897.248/1 = -0.897.248$$

Io in red and li in green



$$A_i = I_o / I_i = -17.945\text{n} / 195.983\text{p} = 91.564 \text{ A/A} \quad \text{1 pt}$$

Po in red and Pi in green



$$A_p = P_o / P_i = 16.1\text{p} / 192\text{f} = 83.85 \text{ W/W}$$

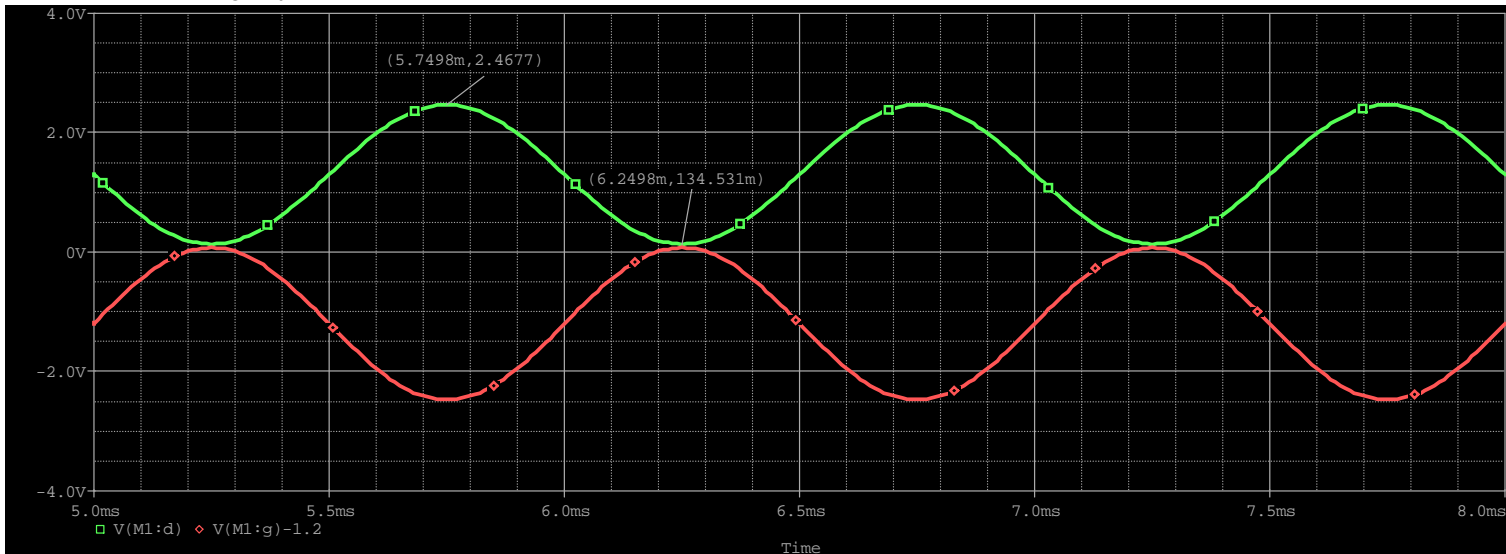
To find the maximum swing at the drain we can change  $V_s$  until  $V_D = V_G - V_t$

**Note: The maximum swing here is less than  $\pm 2.5V$  since when we designed the circuit and calculated the value of  $R_D$ , we neglected the swing at the gate (swing at drain / Gain).**

$$V_D - 2.5 > V_G + \left(\frac{2.5}{A_v}\right) - V_t$$

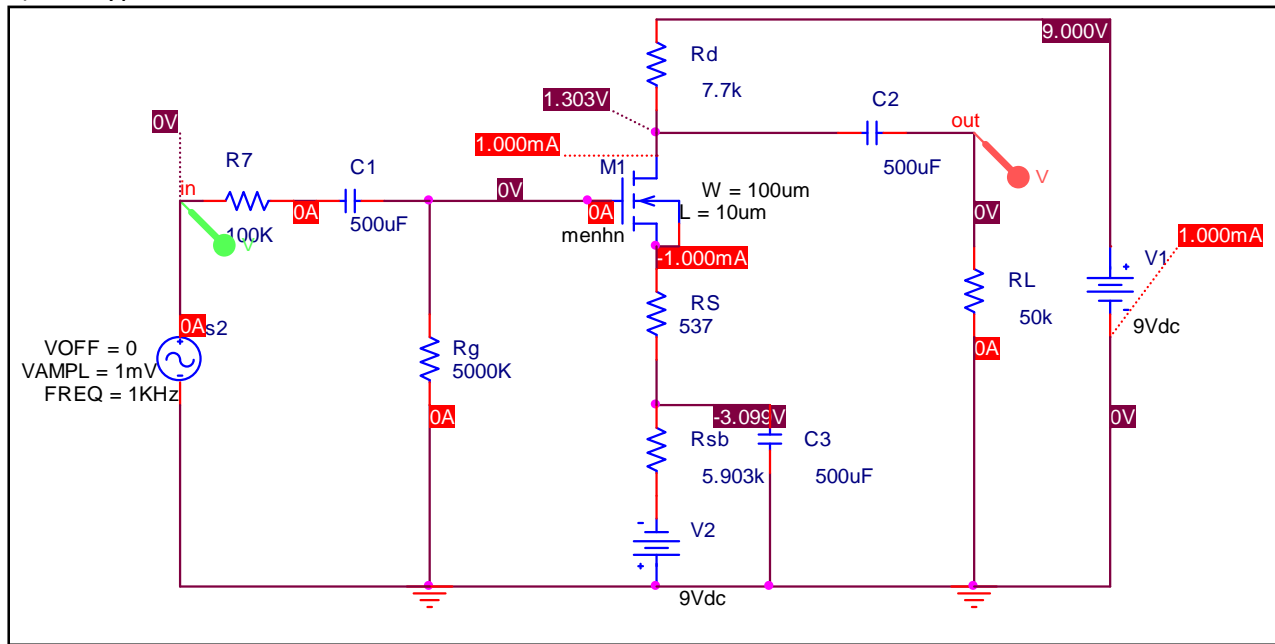
**However  $A_v$  turned out to be less than 1 and the swing of 2.5 requires  $V_g$  to change by more than 2.5V which can no longer be neglected. That is why the maximum swing at the drain here is less than 2.5V.**

Trace of  $V_G - V_t$  and  $V_D$

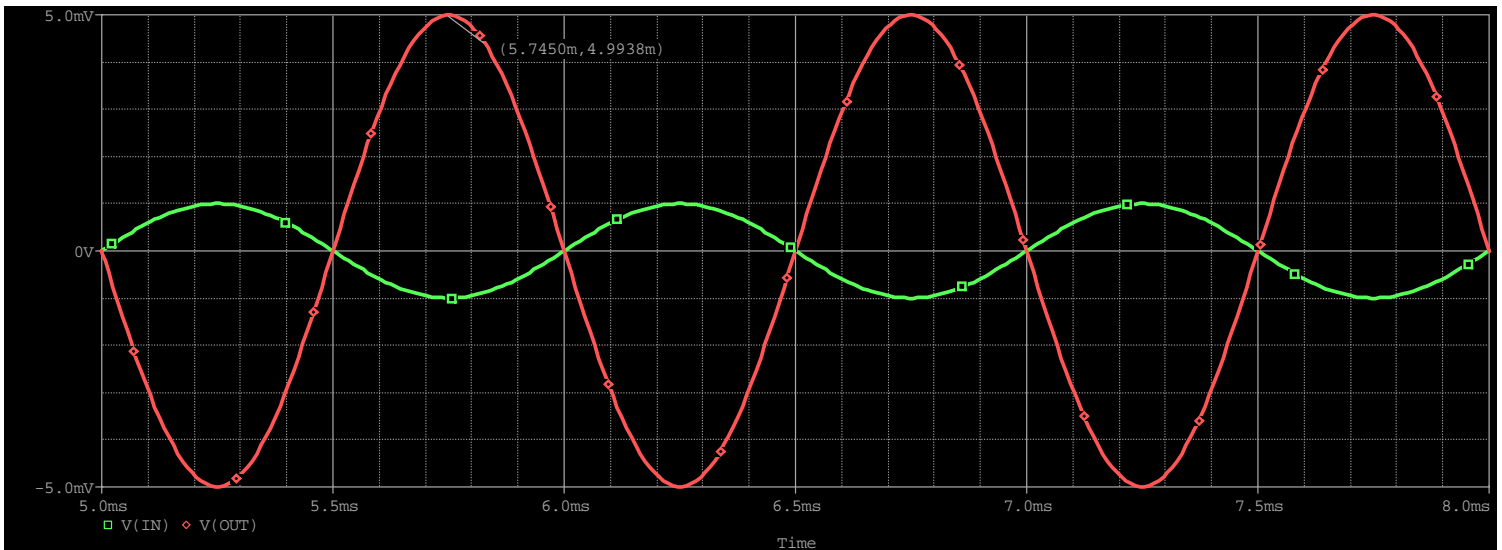


We find that for  $V_s = 1.3V$  (amplitude)  $V_D$  starts to become equal to  $V_G - V_t$  for some values. So for larger values of  $V_s$  we will have distortion. So the maximum swing will be  $\pm 1.17V$

e) For bypassed  $R_s$



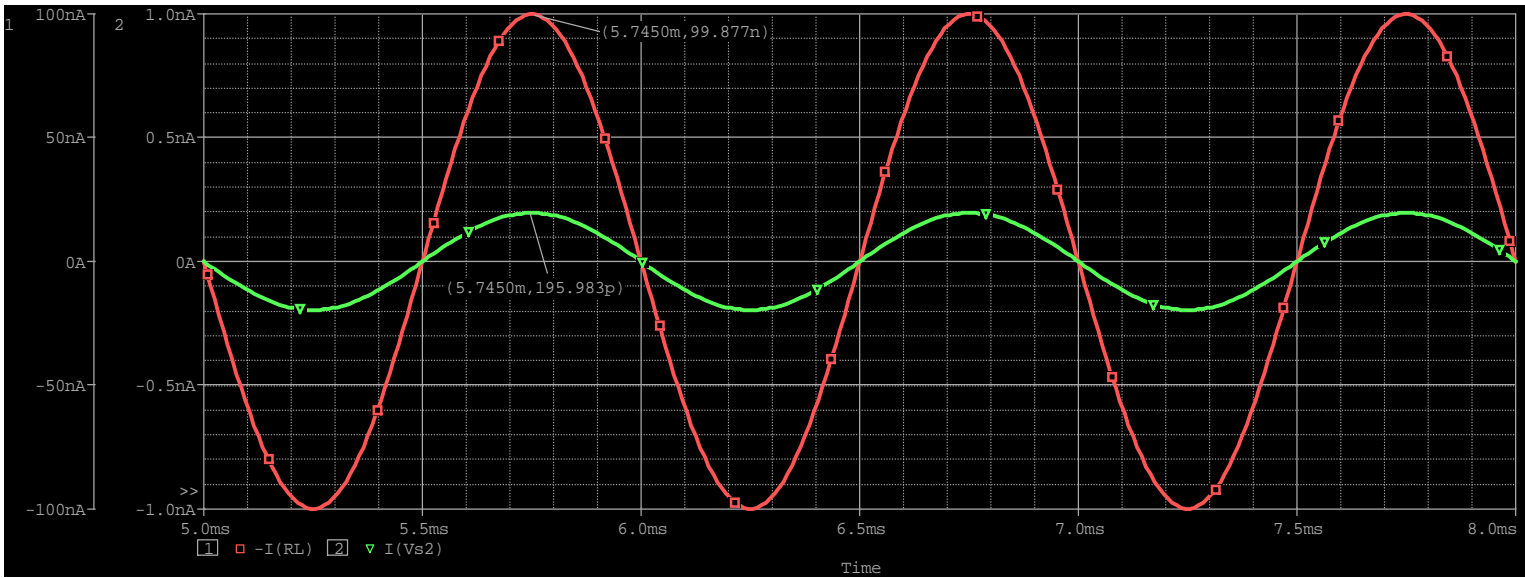
$V_o$ : red,  $V_i$  in green.



$$A_v = V_o/V_i = -4.993/1 = -4.993 \text{ V/V}$$

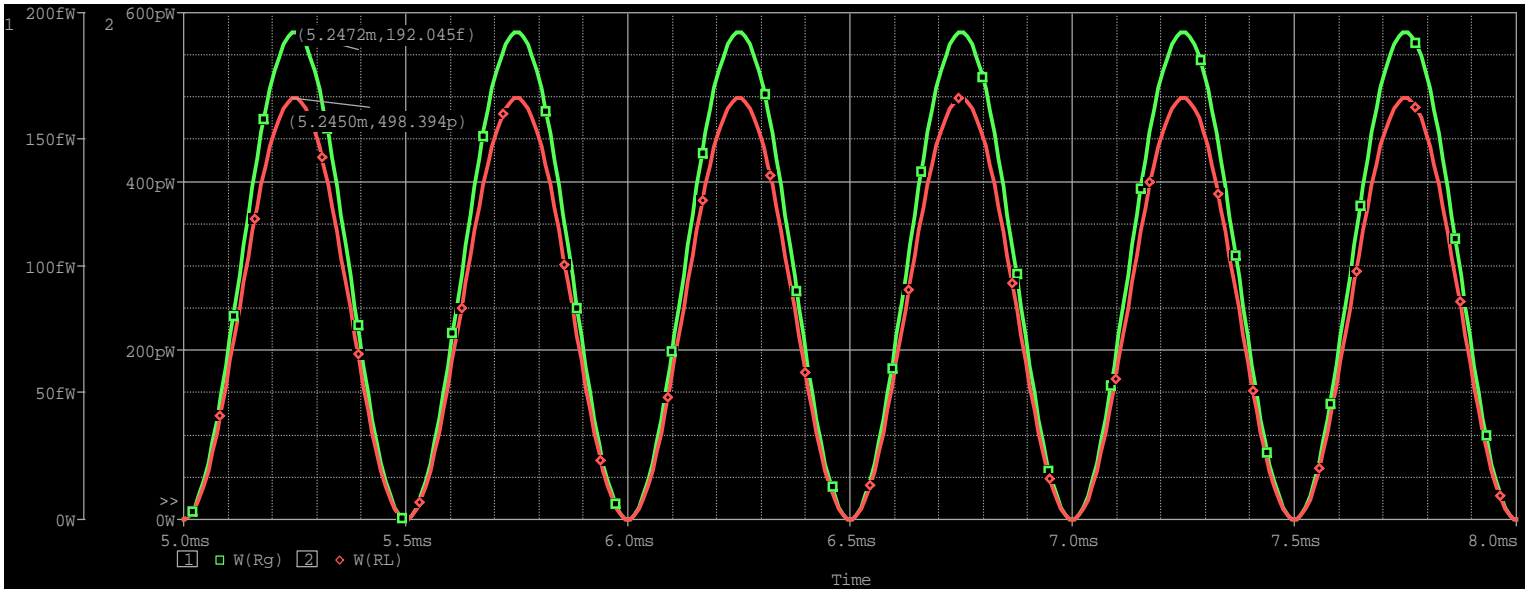


Io in red and Ii in green



$$A_i = I_o / I_i = 99.877\text{n} / 195.983\text{p} = 510 \text{ A/A}$$

Po in red and Pi in green

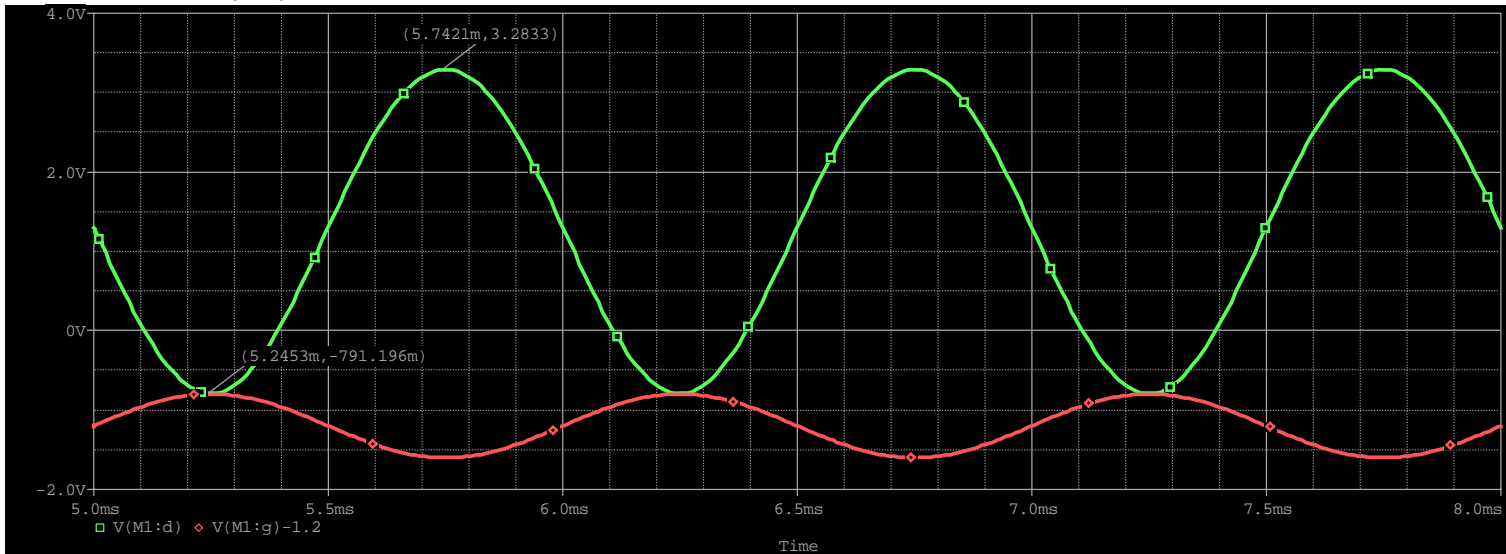


$$A_p = P_o / P_i = 498.4\text{p} / 192\text{f} = 2595 \text{ W/W}$$

To find the maximum swing at the drain we can change  $V_s$  until  $V_D = V_G - V_t$

**Note: The maximum swing here is slightly less than  $\pm 2.5V$  since now the gain is -5 and the swing at the gate can be neglected**

Trace of  $V_G - V_t$  and  $V_D$



We find that for  $V_s = 0.41V$  (amplitude)  $V_D$  starts to become equal to  $V_G - V_t$  for some values. So for larger values of  $V_s$  we will have distortion. So the maximum swing will be  $\pm 2.04V$