American University of Beirut
Department of Electrical and Computer Engineering

EECE 310 – Electronics Fall 2007 – 2008

Homework 9

Consider a CMOS inverter with the following circuit parameters:

The N-channel MOSFET has L = 0.5 μm, W = 2 μm, V_t = 0.7 V, and k' = 200 μA/V^2.

The P-channel MOSFET has L = 0.5 μm, W = 5 μm, V_t = –0.8 V, and k' = 120 μA/V^2.

The supply voltage is V_{DD} = 2.5 V.

a) Find the values of V_{OL} and V_{OH} for this inverter.

b) Find the value of V_{OL} when a 1 KΩ resistor is connected from the output to V_{DD}. The input is at V_{DD}. What is the total circuit power dissipation?

c) Find the value of V_{OH} when a 1 KΩ resistor is connected from the output to ground. The input is at 0 V. What is the total circuit power dissipation?

d) Find the values of V_{IL} and V_{IH}. Calculate the noise margins.

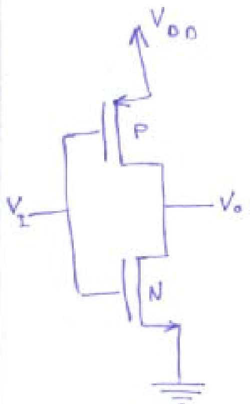
e) Find the values of V_{IL} and V_{IH} when channel length modulation is *not* negligible. Assume in this part that λ_n = 0.2 V^{-1} and $|\lambda_p|$ = 0.15 V^{-1}.

f) A 0.1 pF capacitor is now connected from output to ground.

1. The input switches from 0 to V_{DD} at t = 0. Find the time instant at which the output reaches 0.2V_{DD}.

2. The input switches back to 0 at t = 1 μsec. Find the time instant at which the output reaches 0.8V_{DD}.

Home work 9

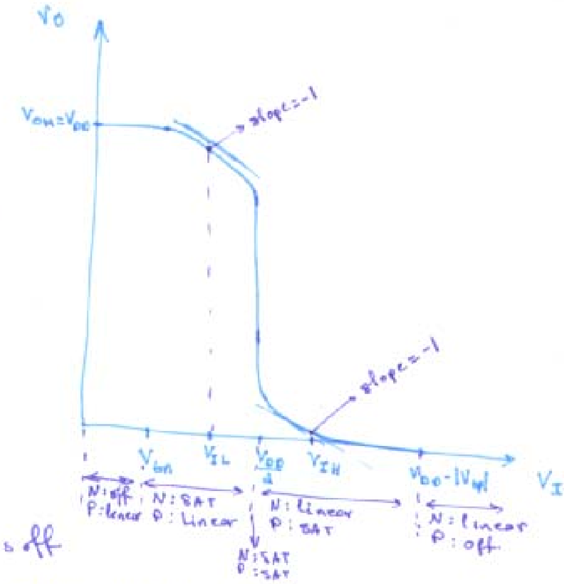


$V_{GSp} = V_I - V_{DD}$

$V_{DSp} = V_o - V_{DD}$

$V_{GSn} = V_I$

$V_{DSn} = V_o$



a) . For $V_I < V_{tn} \Rightarrow I_{DN} = I_{DP} = 0\,mA$ and N mos is off

$I_{DP} = 0\,mA \Rightarrow$ Pmos in linear region $\Rightarrow 0 = \frac{1}{2} K_p' \left(\frac{W}{L}\right)_p \left(2(V_{GSp} - V_{tp}) V_{DSp} - V_{DSp}^2\right)$

$\Rightarrow V_{DSp} = 0V \Rightarrow V_o - V_{DD} = 0 \Rightarrow V_o = V_{DD} = 3V.$

$\therefore V_{OH} = V_{DD} = 2.5\,V$

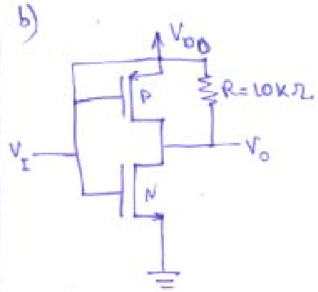
For $V_I = V_{DD} \Rightarrow V_{GSp} = 0V \Rightarrow V_{GSp} > V_{tp} \Rightarrow$ Pmos is off. $\Rightarrow I_{DP} = I_{Dn} = 0\,mA$

$I_{Dn} = 0\,mA \Rightarrow$ Nmos in linear region $\Rightarrow 0 = \frac{1}{2} K_n \left(\frac{W}{L}\right)_n \left(2(V_{GSn} - V_{tn}) V_{DSn} - V_{DSn}^2\right)$

$\Rightarrow V_{DSn} = 0V \Rightarrow V_o = 0V$

$\therefore V_{OL} = 0V.$

b)



. For $V_I = V_{DD} \Rightarrow V_{GSp} = 0V \Rightarrow$ Pmos is off $\Rightarrow I_{DP} = 0\,mA.$

. $I_{Dn} = \frac{V_{DD} - V_{OL}}{1\,K\Omega}$; $V_{GSn} = 2.5V$; V_{OL} = small $\Rightarrow$ Nmos in linear region

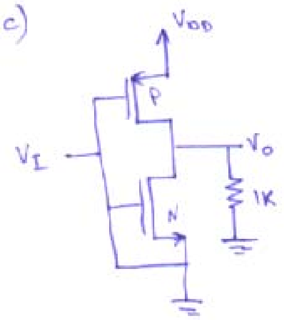
$\frac{1}{2} K_n' \left(\frac{W}{L}\right)_n \left(2(V_{GSn} - V_{tn}) V_{OL} - V_{OL}^2\right) = \frac{V_{DD} - V_{OL}}{1K} \Rightarrow \frac{1}{2} \times 200\mu \times \frac{2}{0.5} \left(2(2.5 - 0.7) V_{OL} - V_{OL}^2\right) = 2.5m - \frac{V_{OL}}{1K}$

$\Rightarrow 0.4 \times 2 \times (1.8) V_{OL} - 0.4 V_{OL}^2 = 2.5 - V_{OL} \Rightarrow 0.4 V_{OL}^2 - 2.44 V_{OL} + 2.5 = 0 \Rightarrow V_{OL} = 1.3V$

Total power dissipation in the circuit $= V_{DD} \times \text{current in } V_{DD} = V_{DD} \times (I_{in} + I_{Dp} + I_{Dn}) = V_{DD} \times I_{Dn}$

$$= V_{DD}\left(\frac{V_{DD} - V_{OL}}{1K}\right) = \frac{V_{DD}(2.5 - 1.3)}{1K} = 3\,mW$$

c)



For $V_I = 0 \Rightarrow V_{GSn} < V_{tn} \Rightarrow$ NMos is off $\Rightarrow I_{Dn} = 0\,mA$; Pmos in linear region

$\cdot\, I_{Dp} = \frac{V_{OH}}{1K}$

$$\frac{1}{2} K_p' \left(\frac{W}{L}\right)_p \left(2(V_{GSp} - V_{tp})(V_{OH} - 2.5) - (V_{OH} - 2.5)^2\right) = \frac{V_{OH}}{1K}$$

$$\frac{1}{2}\, 120\mu\, \frac{5}{0.5}\left(2(-2.5 + 0.8)(V_{OH} - 2.5) - (V_{OH} - 2.5)^2\right) = \frac{V_{OH}}{1K}$$

$$1.2(-1.7)(V_{OH} - 2.5) - 0.6(V_{OH} - 2.5)^2 = V_{OH}$$

$$-2.04V_{OH} + 5.1 - 0.6V_{OH}^2 + 3V_{OH} - 3.75 = V_{OH}$$

$$\Rightarrow 0.6V_{OH}^2 + 0.04V_{OH} - 1.35 = 0$$

$$\Rightarrow V_{OH} = 1.47V.$$

Total power dissipated in the circuit $=$ Power supplied in DC source $= V_{DD} \times I = V_{DD} \times I_{Dp}$

$$= 2.5 \times \frac{1.47}{1K} = 3.675\,mW$$

d) For V_{IL} : P_{mos} is in linear region and N_{mos} in SAT.

$$\Rightarrow \frac{1}{2} K_n' \left(\frac{W}{L}\right)_n (V_{GSn} - V_{tn})^2 = \frac{1}{2} K_p' \left(\frac{W}{L}\right)_p \left(2(V_{GSp} - V_{tp})(V_{DSp}) - V_{DSp}^2\right)$$

$$\Rightarrow 200 \times 2\,(V_{IL} - 0.7)^2 = 120 \times 5\left(2(V_{IL} - 2.5 + 0.8)(V_o - 2.5) - (V_o - 2.5)^2\right)$$

$$\Rightarrow 2(V_{IL} - 0.7)^2 = 3\left(2(V_{IL} - 1.7)(V_o - 2.5) - (V_o - 2.5)^2\right) \quad \text{---- (1)}$$

For V_{IL} : $\left.\frac{dV_o}{dV_I}\right|_{V_I = V_{IL}} = -1$) take derivative with respect to dV_I

$$\Rightarrow 4(V_{IL} - 0.7) = 6(V_o - 2.5) + 6(V_{IL} - 1.7)\left(\frac{dV_o}{dV_I}\right) - 6\frac{dV_o}{dV_I}(V_o - 2.5)$$

$$\Rightarrow 4V_{IL} - 2.8 = 6V_o - 15 - 6V_{IL} + 10.2 + 6V_o - 15$$

$$\Rightarrow 10V_{IL} = 12V_o - 17 \quad \Rightarrow V_o = \frac{10}{12}V_{IL} + \frac{17}{12} \quad \text{---- (2)}$$

Replace equation 2 in 1

$\Rightarrow 2(V_{IL}-0.7)^2 = 6(V_{IL}-1.7)(\frac{5}{6}V_{IL}+\frac{17}{12}-2.5) - 3(\frac{5}{6}V_{IL}+\frac{17}{12}-2.5)^2$

$\Rightarrow \frac{11}{12}V_{IL}^2 - \frac{407}{60}V_{IL} + \frac{7859}{1200} = 0$

$\Rightarrow V_{IL} = 1.14V$

$NML = V_{IL} - V_{OL} = 1.14V$

For $V_{IH} \Rightarrow$ P Mos in SAT and Nmos in linear region

$\Rightarrow \frac{1}{2}K_n'(\frac{W}{L})_n(2(V_{GSn}-V_{tn})(V_{DSn}) - V_{DSn}^2) = \frac{1}{2}K_p'(\frac{W}{L})_p(V_{GSp}-V_{tp})^2$

$\Rightarrow 200 \times 2(2(V_{IH}-0.7)(V_o) - V_o^2) = 120 \times 5(V_{IH}-2.5+0.8)^2$

$\Rightarrow 4(V_{IH}-0.7)(V_o) - 2V_o^2 = 3(V_{IH}-1.7)^2$ ----- (1)

$\left.\frac{dV_o}{dV_I}\right|_{V_I=V_{IH}} = -1$

$\Rightarrow 4V_o + 4(V_{IH}-0.7)\frac{dV_o}{dV_I} - 4\frac{dV_o}{dV_I}V_o = 6(V_{IH}-1.7)$

$\Rightarrow 4V_o - 4V_{IH} + 2.8 + 4V_o = 6V_{IH} - 10.2$

$\Rightarrow 8V_o = 10V_{IH} - 13$ --

$\Rightarrow V_o = \frac{5}{4}V_{IH} - \frac{13}{8}$ --- (2)

Replace equation 2 in 1

$\Rightarrow 4(V_{IH}-0.7)(\frac{5}{4}V_{IH}-\frac{13}{8}) - 2(\frac{5}{4}V_{IH}-\frac{13}{8})^2 = 3(V_{IH}-1.7)^2$

$\Rightarrow \frac{9}{8}V_{IH}^2 - \frac{333}{40}V_{IH} + \frac{7521}{800} = 0$

$\Rightarrow V_{IH} = 1.39V$

$NMH = V_{OH} - V_{IH} = 1.11V$

c) For V_{IL} : Pmos in linear region, Nmos in SAT

$\Rightarrow \frac{1}{2}K_n'(\frac{W}{L})_n(V_{GSn}-V_{tn})^2(1+\lambda_n V_{DSn}) = \frac{1}{2}K_p'(\frac{W}{L})_p(2(V_{GSp}-V_{tp})V_{DSp} - V_{DSp}^2)$

$\Rightarrow 2(V_{IL}-0.7)^2(1+0.2V_o) = 3(2(V_{IL}-2.5+0.8)(V_o-2.5) - (V_o-2.5)^2)$

$\Rightarrow 2(V_{IL}-0.7)^2(1+0.2V_o) = 6(V_{IL}-1.7)(V_o-2.5) - 3(V_o-2.5)^2$ ---- (1)

$\left.\frac{dV_o}{dV_I}\right|_{V_I=V_{IL}} = -1$

$\Rightarrow 4(V_{IL}-0.7)(1+0.2V_o) + 2(V_{IL}-0.7)^2(0.2\frac{dV_o}{dV_I}) = 6(V_o-2.5) + 6(V_{IL}-1.7)\frac{dV_o}{dV_I} - 6\frac{dV_o}{dV_I}(V_o-2.5)$

$$\Rightarrow 4(V_{IL}-0.7) + 0.8V_o(V_{IL}-0.7) - 0.4(V_{IL}-0.7)^2 = 6V_o - 15 - 6(V_{IL}-1.7) + 6V_o - 15$$

$$\Rightarrow 4(V_{IL}-0.7)(1-0.1(V_{IL}-0.7)) + 30 + 6(V_{IL}-1.7) = (12 + 0.8(V_{IL}-0.7))V_o$$

$$\Rightarrow 4V_{IL} - 2.8 - 0.4V_{IL}^2 + 0.56V_{IL} - 0.196 + 6V_{IL} - 10.2 + 30 = (0.8V_{IL} + 11.44)V_o$$

$$\Rightarrow \frac{-0.4V_{IL}^2 + 10.56V_{IL} + 16.804}{0.8V_{IL} + 11.44} = V_o \quad ---- (2)$$

Replace (2) in (1):

$$2(V_{IL}-0.7)^2\left(1 + 0.2\cdot\frac{(-0.4V_{IL}^2 + 10.56V_{IL} + 16.804)}{0.8V_{IL}+11.44}\right) = 6(V_{IL}-1.7)\left(\frac{-0.4V_{IL}^2 + 10.56V_{IL} + 16.804}{0.8V_{IL}+11.44} - 2.5\right) - 3\left(\frac{-0.4V_{IL}^2 + 10.56V_{IL} + 16.804}{0.8V_{IL}+11.44} - 2.5\right)^2.$$

$$\Rightarrow 20V_{IL}^5 - 845V_{IL}^4 - 7632V_{IL}^3 + 7192.9V_{IL}^2 - 105715.69V_{IL} + 123918.7611 = 0$$

$V_{IL} = 1.1404V$

For V_{IH} : Pmos in SAT, Nmos in linear

$$\frac{1}{2}k_n'\left(\frac{W}{L}\right)_n\left(2(V_{GSn}-V_{tn})V_{DSn} - V_{DSn}^2\right) = \frac{1}{2}k_p'\left(\frac{W}{L}\right)_n(V_{GSp}-V_{tp})^2(1+\lambda_p V_{DSp})$$

$$4(V_{IH}-0.7)V_o - 2V_o^2 = 3(V_{IH}-1.7)^2(1.375 - 0.15V_o) \quad ---- (1)$$

$$\left.\frac{dV_o}{dV_I}\right|_{V_I=V_{IH}} = -1$$

$$4V_o + 4(V_{IH}-0.7)\frac{dV_o}{dV_I} - 4V_o\frac{dV_o}{dV_I} = 6(V_{IH}-1.7)(1.375-0.15V_o) + 3(V_{IH}-1.7)^2\left(-0.15\frac{dV_o}{dV_I}\right)$$

$$\Rightarrow 4V_o - 4V_{IH} + 2.8 + 4V_o = 8.25V_{IH} - 14.025 - 0.9(V_{IH}-1.7)V_o + 0.45(V_{IH}-1.7)^2$$

$$\Rightarrow V_o(8 + 0.9(V_{IH}-1.7)) = 4V_{IH} + 8.25V_{IH} - 1.53V_{IH} + 0.45V_{IH}^2 - 16.825 + 1.3005$$

$$\Rightarrow V_o = \frac{0.45V_{IH}^2 + 10.72V_{IH} - 15.5245}{0.9V_{IH} + 6.47} \quad --- (2)$$

Replace (2) in (1)

$$4(V_{IH}-0.7)\left(\frac{0.45V_{IH}^2 + 10.72V_{IH} - 15.5245}{0.9V_{IH} + 6.47}\right) - 2\left(\frac{0.45V_{IH}^2 + 10.72V_{IH} - 15.5245}{0.9V_{IH}+6.47}\right) = 3(V_{IH}-1.7)^2(1.375 - 0.5V_o)$$

$$\Rightarrow 729V_{IH}^5 + 11623.5V_{IH}^4 - 2547.9V_{IH}^3 - 5934772.97V_{IH}^2 + 3131229.9645V_{IH} - 3321757.61553 = 0$$

$\Rightarrow V_{IH} = 1.436V.$

f) 1- V_I switches from 0 to V_{DD}

$\Rightarrow$ now $V_I = V_{DD} \Rightarrow$ Pmos is off $\Rightarrow i_{DP} = 0 \Rightarrow i_{DN} = -i_C \Rightarrow i_{DN} = -C\frac{dv_o}{dt}$

$V_{GS_n} = V_I = 2.5$

$V_{DS_n} = V_o$

$\Rightarrow$ For $V_o > 2.5 - 0.7 \Rightarrow$ Nmos in SAT

For $V_o < 2.5 - 0.7 \Rightarrow$ Nmos in linear region

$$i_{DN} = -\frac{C\,dv_o}{dt} \Rightarrow dt = -\frac{C\,dv_o}{i_{DN}} \Rightarrow \Delta t = -C\int \frac{dv_o}{i_{DN}} \quad ; \Delta t = t_1 - t = t_1 - 0 = t_1$$

$$\Rightarrow t_1 = \int_{V_{DD}}^{0.2V_{DD}} -\frac{C\,dv_o}{i_{DN}} = \int_{V_{DD}}^{V_{DD}-0.7} -\frac{C\,dv_o}{i_{DN_{SAT}}} + \int_{V_{DD}-0.7}^{0.2V_{DD}} \frac{-C\,dv_o}{i_{DN_{linear}}}$$

$$= \int_{2.5}^{1.8} \frac{-C\,dv_o}{\frac{1}{2}k_n'\left(\frac{W}{L}\right)_n (V_{GS_n} - V_{tn})^2} + \int_{1.8}^{0.5} \frac{-C\,dv_o}{\frac{1}{2}k_n'\left(\frac{W}{L}\right)_n \left(2(V_{GS_n} - V_t)V_o - V_o^2\right)}$$

$$= \frac{C}{\frac{1}{2}k_n'\left(\frac{W}{L}\right)_n}\left[\int_{1.8}^{2.5} \frac{dv_o}{(1.8)^2} + \int_{0.5}^{1.8} \frac{dv_o}{3.6V_o - V_o^2}\right]$$

$$= \frac{0.1\times10^{-12}}{100\times10^{-6}\times\frac{2}{0.5}}\left[\frac{(2.5-1.8)}{(1.8)^2} + \frac{5}{18}\ln\left(\left|\frac{V_o}{5V_o - 18}\right|\right)\Bigg|_{0.5}^{1.8}\right]$$

$$= 2.5\times10^{-10}\left[\frac{0.7}{1.8^2} + \frac{5}{18}\ln(0.2) - \frac{5}{18}\ln\left(\frac{1}{31}\right)\right]$$

$= 0.18\,\text{nsec}.$

2- $V_I = 0 \Rightarrow$ Nmos become off $\Rightarrow i_{DN} = 0 \Rightarrow i_{DP} = i_C \Rightarrow i_{D_P} = C\frac{dv_o}{dt}$

$V_{GS_p} = -V_{DD} + V_I = V_I - V_{DD}$

$V_{DS_p} = V_o - V_{DD}$

For $V_o - V_{DD} < V_I - V_{DD} - V_{tp} \Rightarrow V_o < 0.8 \Rightarrow$ Pmos in SAT

For $V_o - V_{DD} > V_I - V_{DD} - V_{tp} \Rightarrow V_o > 0.8 \Rightarrow$ Pmos in linear region

$$i_{DP} = +\frac{C\,dv_o}{dt} \Rightarrow dt = \frac{C\,dv_o}{i_{DP}} \Rightarrow \Delta t = C\int\frac{dv_o}{i_{DP}} \quad ; \Delta t = t_2 - t_1 \Rightarrow t_2 = \Delta t + 1\mu\text{sec}.$$

$$\Delta t = C\int_0^{0.8}\frac{dv_o}{i_{DP_{SAT}}} + C\int_{0.8}^{0.8V_{DD}}\frac{dv_o}{i_{DP_{linear}}} = C\int_0^{0.8}\frac{dv_o}{\frac{1}{2}k_p'\left(\frac{W}{L}\right)_p (V_{GS_p} - V_{tp})^2} + C\int_{0.8}^{2}\frac{dv_o}{\frac{1}{2}k_p'\left(\frac{W}{L}\right)_p\left(2(V_{GS_p} - V_{tp})V_{DS_p} - V_{DS_p}^2\right)}$$

$$\Delta t = \frac{C}{\frac{1}{2} k_p \left(\frac{W}{L}\right)_p} \left[\int_0^{0.8} \frac{dv_o}{(-1.7)^2} + \int_{0.8}^{2} \frac{dv_o}{-3.4(v_o - 2.5) - (v_o - 2.5)^2} \right]$$

$$= \frac{0.1 \times 10^{-12}}{\frac{1}{2} \times 120 \times 10^{-6} \times \frac{5}{0.5}} \left[\frac{0.8}{1.7^2} + \int_{0.8}^{2} \frac{-dv_o}{(v_o - 2.5)(v_o + 0.9)} \right]$$

$$= \frac{10^{-9}}{6} \left[\frac{0.8}{1.7^2} + \frac{5}{17} \ln\left(\left| \frac{10 v_o + 9}{2 v_o - 5} \right| \right) \right]_{0.8}^{2}$$

$$= \frac{10^{-9}}{6} \left[\frac{0.8}{1.7^2} + \frac{5}{17} \ln 29 - \frac{5}{17} \ln(5) \right] = 0.1323 \text{ nsec.}$$

$\Rightarrow t_2 = \Delta t + 1\mu sec = 1000.1323 \text{ nsec} \simeq 1\mu sec$