EECE 310 - Homework 2

Problem 1

a) $$I_D = I_S\left(e^{\frac{V_D}{nV_T}} - 1\right)$$

$I_D = 0.33\ mA$; $V_D = 0.777\ V$; $n = 1.2$

$$V_T = \frac{KT}{q}$$ where $K = 1.38 \times 10^{-23}\ J/K$
$T = 37 + 273 = 310°K$
$q = 1.6 \times 10^{-19}\ C$

$$V_T = \frac{1.38 \times 10^{-23} \times 310}{1.6 \times 10^{-19}} = 26.74\ mV.$$

$$I_S = \frac{I_D}{e^{\frac{V_D}{nV_T}} - 1} = \frac{0.33}{e^{\frac{0.777}{1.2 \times 0.02674}} - 1} = 1.005 \times 10^{-11}\ mA$$

b) $$I_D = I_S\left(e^{\frac{V_D}{nV_T}} - 1\right) \simeq I_S e^{\frac{V_D}{nV_T}} \quad (I_{DO} \ggg I_S)$$

$$\begin{cases} I_{D_1} = I_S\left(e^{\frac{V_{D1}}{nV_T}}\right) \\ I_{D2} = I_S\left(e^{\frac{V_{D2}}{nV_T}}\right) \end{cases}$$

$$\frac{I_{D2}}{I_{D_1}} = e^{\frac{1}{nV_T}(V_{D2} - V_{D1})}$$

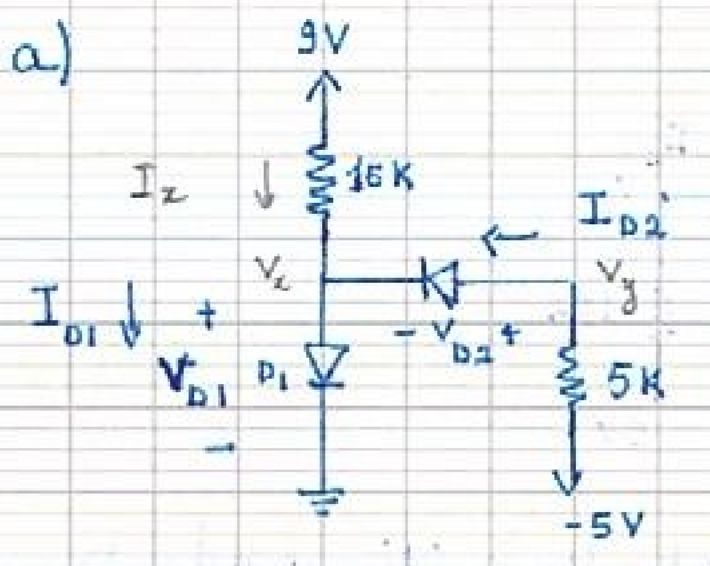
$$\ln\left(\frac{I_{D2}}{I_{D1}}\right) = \frac{1}{nV_T}(V_{D2} - V_{D1})$$

$$V_{D2} - V_{D1} = nV_T \ln\left(\frac{I_{D2}}{I_{D1}}\right) = 1.2 \times 0.02674 \times \ln(1000)$$
$$= 0.22\ V$$

The change in diode voltage is 0.22 V.

Problem 2

a)



Let's assume that D_1 is conducting (ON) and D_2 is not conducting (OFF).

Thus we have $\begin{cases} V_{D1} = 0\,V \\ I_{D2} = 0\,A \end{cases}$ and we have to get $\begin{cases} I_{D1} > 0 \\ V_{D2} < 0 \end{cases}$

Since $V_{D1} = 0V$, $V_x = 0\,V$.

$I_x = \dfrac{9 - V_x}{15K} = \dfrac{9}{15K} = 0.6\ mA$

Since $I_{D2} = 0\,A$, $I_{D1} = I_x = 0.6\,mA$; $(I_{D1} > 0)$

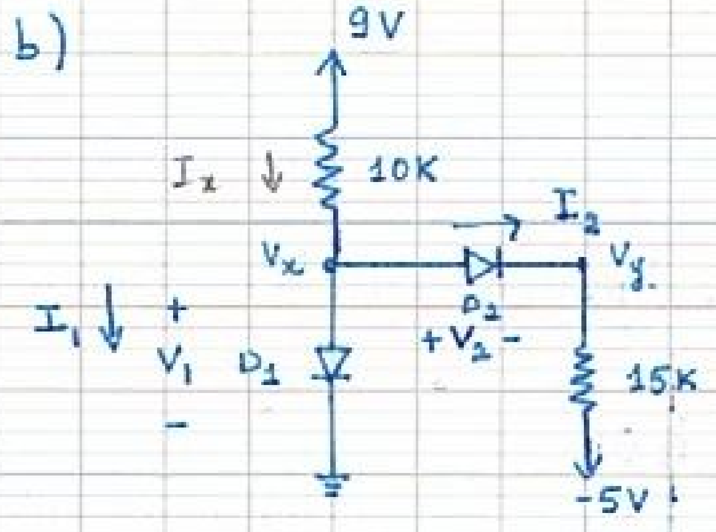
$V_{\substack{\text{across 5K} \\ \text{resistor}}} = 0\,V \iff V_y = -5\,V$, then $V_{D2} = V_y - V_x = -5V < 0$

The assumptions are correct,

$D_1 : (0.6\ mA, 0V)$

$D_2 : (0\,A, -5V)$

b)



Let's assume that D_1 is ON and D_2 is ON

Thus, we have $\begin{cases} V_1 = 0V \\ V_2 = 0V \end{cases}$ and we have to get $\begin{cases} I_1 > 0 \\ I_2 > 0 \end{cases}$

Since $V_1 = 0\,V$; $V_x = 0\,V$

Since $V_2 = 0\,V$, $V_y = 0\,V$ (because $V_2 = V_x - V_y$)

Thus $I_2 = \dfrac{V_y - (-5)}{15K} = \dfrac{5}{15K} = \dfrac{1}{3}\,mA = 0.33\,mA$ $(I_2 > 0)$

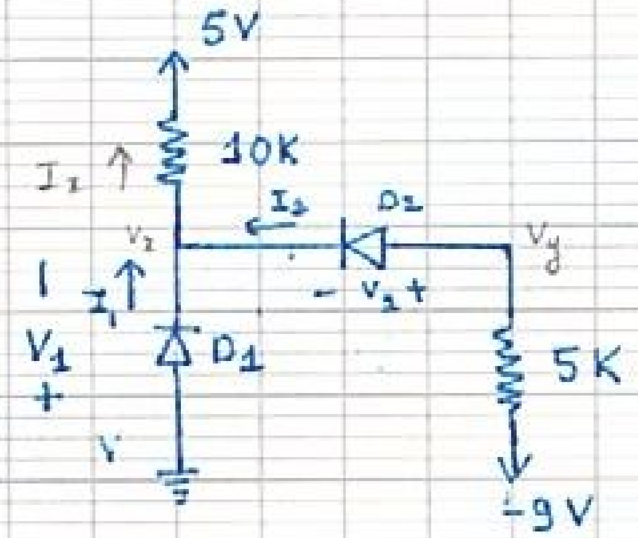
$I_x = \dfrac{9 - V_x}{10K} = \dfrac{9}{10K} = 0.9\,mA$

$I_1 = I_x - I_2 = 0.9 - 0.33 = 0.57\,mA$ $(I_1 > 0)$

Thus D_1 is ON : $(0.57mA;\ 0V)$

and D_2 is ON : $(0.33\,mA;\ 0V)$.

c)



Let's assume that D_1 is OFF and D_2 is OFF.

We have : $\begin{cases} I_1 = 0\,A \\ I_2 = 0\,A \end{cases}$ and we have to get $\begin{cases} V_{D1} < 0 \\ V_{D2} < 0 \end{cases}$

Since $I_1 = 0A$ and $I_2 = 0A$, $I_x = I_1 + I_2 = 0A$.

Thus $V_{\text{across the 10K resistor}} = 0V \rightarrow V_x = 5V$.

$V_1 = 0 - V_x = -5V$. ($V_1 < 0$)

$I_2 = 0A$, thus the voltage across the 5K resistor $= 0V$

$\rightarrow V_y = -9V$.

$V_2 = V_y - V_x = -9 - 5 = -14V$ ($V_2 < 0$).

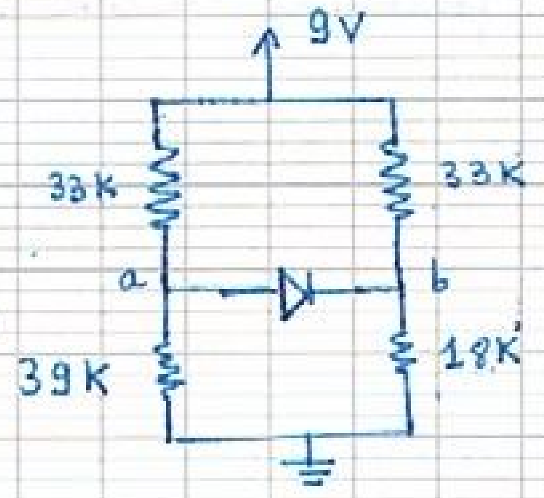
The two assumptions are correct

$D_1 : (0A, -5V)$

$D_2 : (0A, -14V)$.

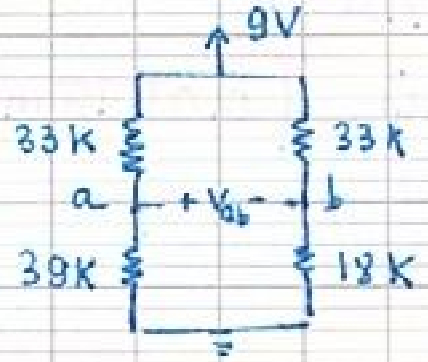
Problem 3

a)



To calculate the operating point of the diode, we must find V_{th} and R_{th} across the diode.

V_{th}: We open the circuit between the terminals a and b and we calculate V_{ab}.

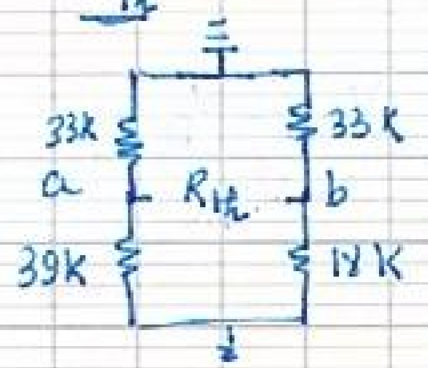


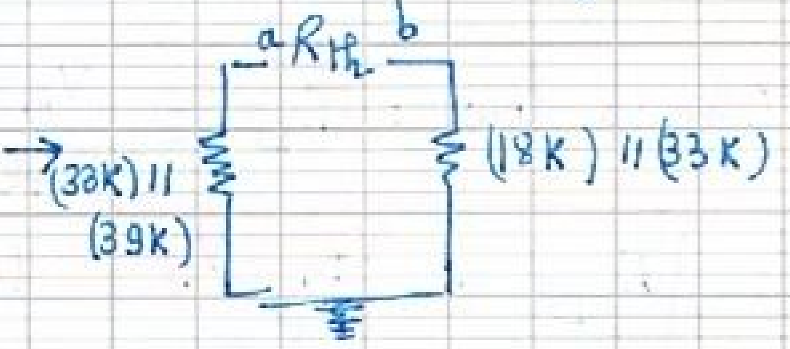
$$V_a = \frac{39K}{39K+33K} \times 9 = 4.875 \text{ V}$$

$$V_b = \frac{18K}{18K+33K} \times 9 = \frac{54}{17} \text{ V}$$

$$V_{th} = V_{ab} = V_a - V_b = 4.875 - \frac{54}{17} = \frac{231}{136} \text{ V}$$

R_{th}: We short circuit the voltage source.

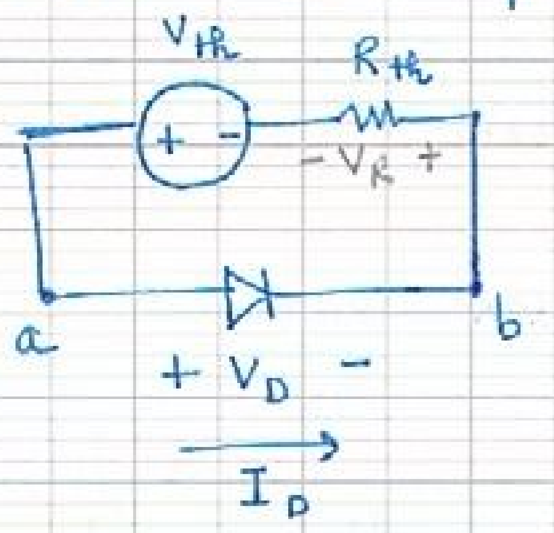




$$R_{th} = (33K) \parallel (39K) + (18K) \parallel (33K)$$

$$= \frac{4015}{136} \text{ K}\Omega$$

We obtain the following circuit:



Assume D is ON $\rightarrow V_D = 0V$ we have to get $I_D > 0$.

$V_{th} = V_D + V_R$ thus $V_R = V_{th}$

$$I_D = \frac{V_R}{R_{th}} = \frac{V_{th}}{R_{th}} = \frac{231/136}{(4015/136)K} = 0.0575 \text{ mA } (I_D > 0)$$

Hence the operating point is $(0.0575 \text{ mA}, 0V)$.

b)

i- Iterative method

Using KVL: $V_{Th} = V_D + V_R$

$$V_R = V_{th} - V_D$$

Ohm's law: $I_D = \frac{V_R}{R_{th}} = \frac{V_{Th} - V_D}{R_{Th}}$

$$\boxed{I_D = \frac{V_{Th} - V_D}{R_{Th}}} \quad (1)$$

Getting V_D:

$$I_D = I_S \left(e^{\frac{V_D}{nV_T}} - 1 \right)$$

$$e^{\frac{V_D}{nV_T}} = \frac{I_D}{I_s} + 1.$$

$$\boxed{V_D = nV_T \ln\left(\frac{I_D}{I_s} + 1\right)} \quad (2)$$

Replacing V_D with the above expression ~~of V_D~~ in (1):

$$I_D = \frac{V_{Th} - nV_T \ln\left(\frac{I_D}{I_s} + 1\right)}{R_{Th}}$$

$V_{Th} = \frac{231}{136}$ V; $R_{th} = \frac{4015}{136}$ KΩ; $n = 1.2$;

$I_s = 10^{-14}$ A; $V_T = \frac{KT}{q} = \frac{1.38 \times 10^{-23} \times (27+273)}{1.6 \times 10^{-19}} = 0.026$ V.

$$I_D = \frac{231/136 - 1.2 \times 0.026 \times \ln\left(\frac{I_D}{10^{-14}} + 1\right)}{4015/136}$$

$$I_D = 0.0575 - 1.0568 \times 10^{-3} \times \ln\left(\frac{I_D}{10^{-14}} + 1\right)$$

Initial guess: $I_D = 0.0575$ mA.

1st iteration: $I_D = 0.0575 - 1.0568 \times 10^{-3} \times \ln\left(\frac{0.0575 \times 10^{-3}}{10^{-14}} + 1\right)$

$I_D = 0.03375$ mA

2nd iteration: $I_D = 0.0575 - 1.0568 \times 10^{-3} \times \ln\left(\frac{0.03375 \times 10^{-3}}{10^{-14}} + 1\right)$

$I_D =$ ~~0.0340 mA~~ 0.0343 mA

3rd iteration: $I_D = 0.0575 - 1.0568 \times 10^{-3} \times \ln\left(\frac{0.0343 \times 10^{-3}}{10^{-14}} + 1\right)$

$= 0.0343$ mA.

Thus $I_D = 0.0343 \text{ mA}$

$$V_D = V_{Th} - R_{Th} I_D$$

$$= \frac{231}{136} - \frac{4015}{136} \times 0.0343$$

$$\boxed{V_D = 0.686 \text{ V}}$$

Hence the operating point is $(0.0343\text{mA}, 0.686\text{V})$

ii. <u>Load line method</u>

$$\begin{cases} I_D = \dfrac{V_{Th} - V_D}{R_{Th}} = 0.0575 - 0.0339\, V_D \\ I_D = I_s \left(e^{\frac{V_D}{nV_T}} - 1\right) \end{cases}$$

$$\begin{cases} I_D = 10^{-11} \left(e^{\frac{V_D}{0.0312}} - 1\right) \text{ (in mA)} \\ I_D = 0.0575 - 0.0339 V_D \text{ (in mA)} \end{cases}$$

We plot these two equations.

The intersection point is P (0.68 V; 0.34 mA)