Quiz 1

Problem 1 (50 points)

A large rigid vessel of unknown total volume is divided into two compartments by a partition: The first compartment of volume 0.5 m^3 initially contains saturated liquid-vapor mixture of water at 100 °C while the second compartment is evacuated. The water in the first compartment is heated while the partition is fixed (not allowed to move). In what follows, sketch all the processes on a $p - v$ and $T - v$ diagram.

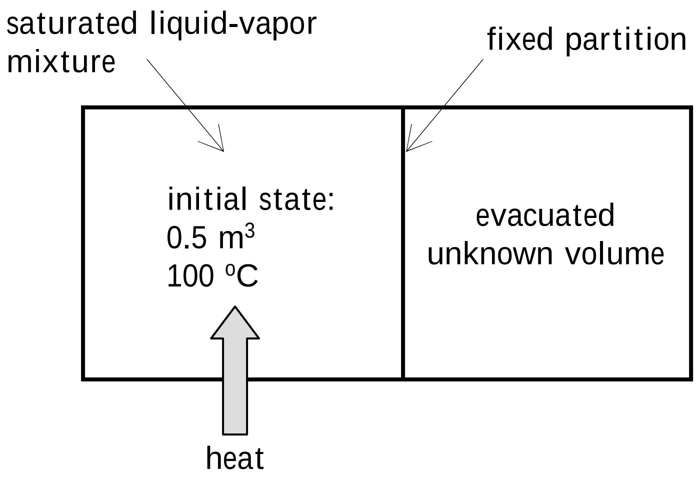

Figure 1: Schematic for problem 1.

(a) If at the end of the heating process, the final state of water is saturated liquid, what are the minimum and maximum possible values of the range of mass of the liquid water at the initial state? What is the corresponding range of the volume of liquid water at the initial state?
(b) If at the end of the heating process, the final state of water is saturated vapor, what are the minimum and maximum possible values of the range of mass of the liquid water at the initial state? What is the corresponding range of the volume of liquid water at the initial state?

If at the end of the heating process, the final state of water is compressed liquid water at 250 °C. At this point, the entire vessel is insulated and the partition separating the two compartments is removed allowing water to fill the entire vessel. At the end of this process, the water temperature dropped by 20 °C.
(c) What is the initial volume of the vacuum compartment?

Problem 1 Solution

Starting with a saturated liquid-vapor mixture (T_1, v_1, p_1), an isochoric heating process may proceed into the subcooled liquid region via a saturated liquid state or into the superheated vapor via a saturated vapor state. Whether the first scenario takes place or the second depends on whether the initial state is on the left or right of the critical point, as shown in the figure. Let v_c be the specific volume of the critical point, then

$$\text{if } v_f(T_1) < v_1 < v_c \Rightarrow \text{ scenario (a)}$$
$$\text{if } v_c < v_1 < v_g(T_1) \Rightarrow \text{ scenario (b)}$$

The ranges of water volume required in parts (a) and (b) correspond to the above ranges of the specific volume. Noting that $m_f = m(1-x) = \frac{V}{v}\left(\frac{v_g - v}{v_{fg}}\right)$, then the required ranges for the mass of liquid are obtained

$$\text{if } \frac{V}{v_c}\left(\frac{v_g - v_c}{v_{fg}}\right) < (m_f)_1 < \frac{V}{v_f(T_1)} \Rightarrow \text{ scenario (a)}$$
$$\text{if } 0 < (m_f)_1 < \frac{V}{v_c}\left(\frac{v_g - v_c}{v_{fg}}\right) \Rightarrow \text{ scenario (b)}$$

The specific volume at the critical point may be obtained by observing that at the critical point $v_f = v_g$. From the saturation tables of water, $v_c = 0.003155$ m^3/kg.

At T_1 =100°C, we have :
$v_f(T_1) = 0.0010434$ m^3/kg
$v_g(T_1) = 1.6729$ m^3/kg

we get

$$\text{if } 158.27 \text{ kg} < (m_f)_1 < 479.2 \text{ kg} \Rightarrow \text{ scenario (a)}$$
$$\text{if } 0 \text{ kg} < (m_f)_1 < 158.27 \text{ kg kg} \Rightarrow \text{ scenario (b)}$$

second part

At the end of the heating process, the water is in the subcooled compressed liquid state. The internal energy is approximated as $u_2 = u_f(T_2) = u_f(250\text{C}) = 1080.7$ kJ/kg. Also $v_2 = v_f(T_2) = 0.0012509$ m^3/kg. The mass of water is then $m = V_2/v_2 = 399.71$ kg.

Applying the first law between states 2 and 3, and noting that the vessel in insulated and that there are no changes in kinetic and potential energy, then

$$\Delta U + \Delta\,\text{K.E.} + \Delta\,\text{P.E.} = Q^{\leftarrow} + W^{\leftarrow} \Rightarrow \Delta U = 0 \Rightarrow \Delta u = 0$$

So at state 3, we have $u_3 = 1080.7$ kJ/kg and $T_3 = 250 - 20 = 230$ C. This a saturated mixture state. From the tables and at $T_3 = 230$ C, we have $u_f = 987.1$ kJ/kg and $u_{fg} = 1616.3$ kJ/kg, then $x_3 = 0.0579$. The specific volume at state 3 is then $v_3 = v_f + xv_{fg} = 0.0012084 + 0.05791(0.071577 - 0.0012084) = 0.005283$ m^3/kg. The volume at state 3 is $V_3 = mv_3 = 2.111$ m^3. So the volume of the initially evacuated compartment is $2.111 - 0.5 = 1.611$ m^3.

Problem 2 (50 points)

Consider the piston-cylinder arrangement shown in the Figure. The cylinder contains 3.5 grams of air and the piston is tightly sealed so that air cannot escape to the environment. The piston mass is 1 kg and its diameter is 20 cm. Initially air inside the cylinder is in thermodynamic equilibrium and the piston is at an elevation of 10 cm relative to the bottom of the cylinder. Stops in the cylinder at an elevation of 15 cm relative to the cylinder bottom prevent the cylinder from upward motion beyond that elevation. 1 kJ of heat is slowly added via an electric resistance properly situated inside the bottom wall of the cylinder.

(a) What are the initial pressure and temperature?
(b) Determine the final pressure and temperature.
(c) Determine the work done by the gas during the process and the work done against atmosphere.
(d) Sketch the process on $T - v$ and $p - v$ diagrams.

Assume the following: The piston is frictionless, air may be modeled as an ideal gas, and except for heat addition via the electric resistance no other forms of heat transfer take place across the boundary. Neglect changes in potential energy and kinetic energy of the air inside the cylinder. The ideal gas constant for air is $R = 0.287$ kJ/kg °K. State any additional assumptions you make.

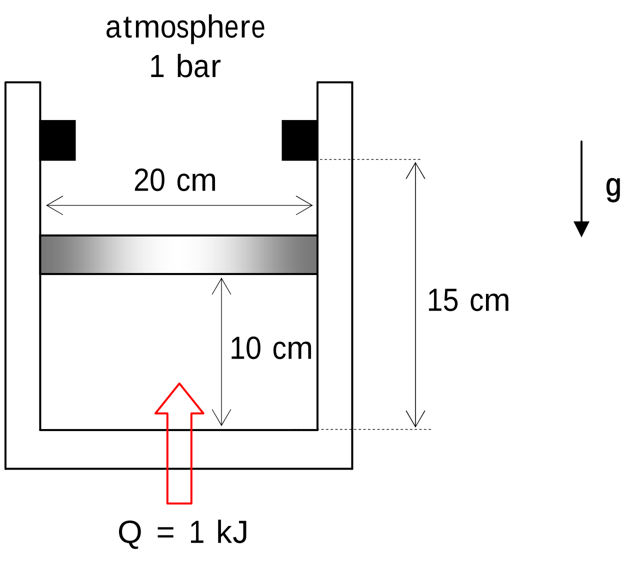

Figure 2: Schematic for problem 2.

Problem 2 Solution

(a) The initial volume of air in the control mass is $V_1 = h_1 A_p$ where the piston area is $A_p = \pi D^2/4$, where $h_1 = 10$ cm , $D = 20$ cm, so that $V_1 = 0.00314159$ m^3. The initial pressure is obtained by performing force balance on the piston, so that $p_1 = p_a + \frac{m_p g}{A_p}$ which for $m_p = 1$ kg and $g = 10$ N/kg leads to $p_1 = 101318$ pa. The initial temperature is then $T_1 = p_1 V_1/mR = 316$ K.

(b) We do not know whether the piston hits the stops or not. We assume it does not hit the stops and we see what happens. Under this assumption we apply the first law of thermodynamics for the control mass between initial and final state, with neglecting changes in kinetic and potential energy,

$$\begin{aligned} & U_2 - U_1 = Q^{\leftarrow} + W^{\leftarrow} \\ \Rightarrow \quad & mc_v(T_2 - T_1) = Q^{\leftarrow} - \int_1^2 p\, dV \end{aligned}$$

since we assume that during the process the piston does not hit the stops and since the heat addition is slow then $p = p_1$ during the process. Denoting h_2 as the piston elevation at the end of the process, and using $T_2 = p_2 V_2/(mR) = p_1 A_p h_2/(mR)$, then

$$\begin{aligned} & mc_v\left(\frac{p_1 A_p h_2}{mR} - T_1\right) = Q^{\leftarrow} - pA_p(h_2 - h_1) \\ \Rightarrow \quad & h_2 = \frac{Q^{\leftarrow} + mc_v T_1 + pA_p h_1}{pA_p(c_v/R + 1)} \end{aligned}$$

We get $h_2 = 16.13$ cm which is larger than 15 cm. So the piston hits the stops. Applying the first law again, but this time with piston hitting the stops. In this case $p_2 \neq p_1$ and $h_2 = 15$ cm and the only work done by the gas is when the piston is rising during which $p = p_1$ so that $W^{\rightarrow} = p_1(V_2 - V_1)$, then

$$\begin{aligned} & mc_v\left(\frac{p_2 V_2}{mR} - T_1\right) = Q^{\leftarrow} - p_1(V_2 - V_1) \\ \Rightarrow \quad & p_2 = \frac{mR}{V_2}\left(T_1 + \frac{Q^{\leftarrow} - p_1(V_2 - V_1)}{mc_v}\right) = 138.68 \text{ kpa} \end{aligned}$$

The final temperature is $T_2 = p_2 V_2/(mR) = 650.6$ K.

(c) The work done by the gas during the process is $W^{\rightarrow} = p_1(V_2 - V_1) = 0.159$ kJ.