

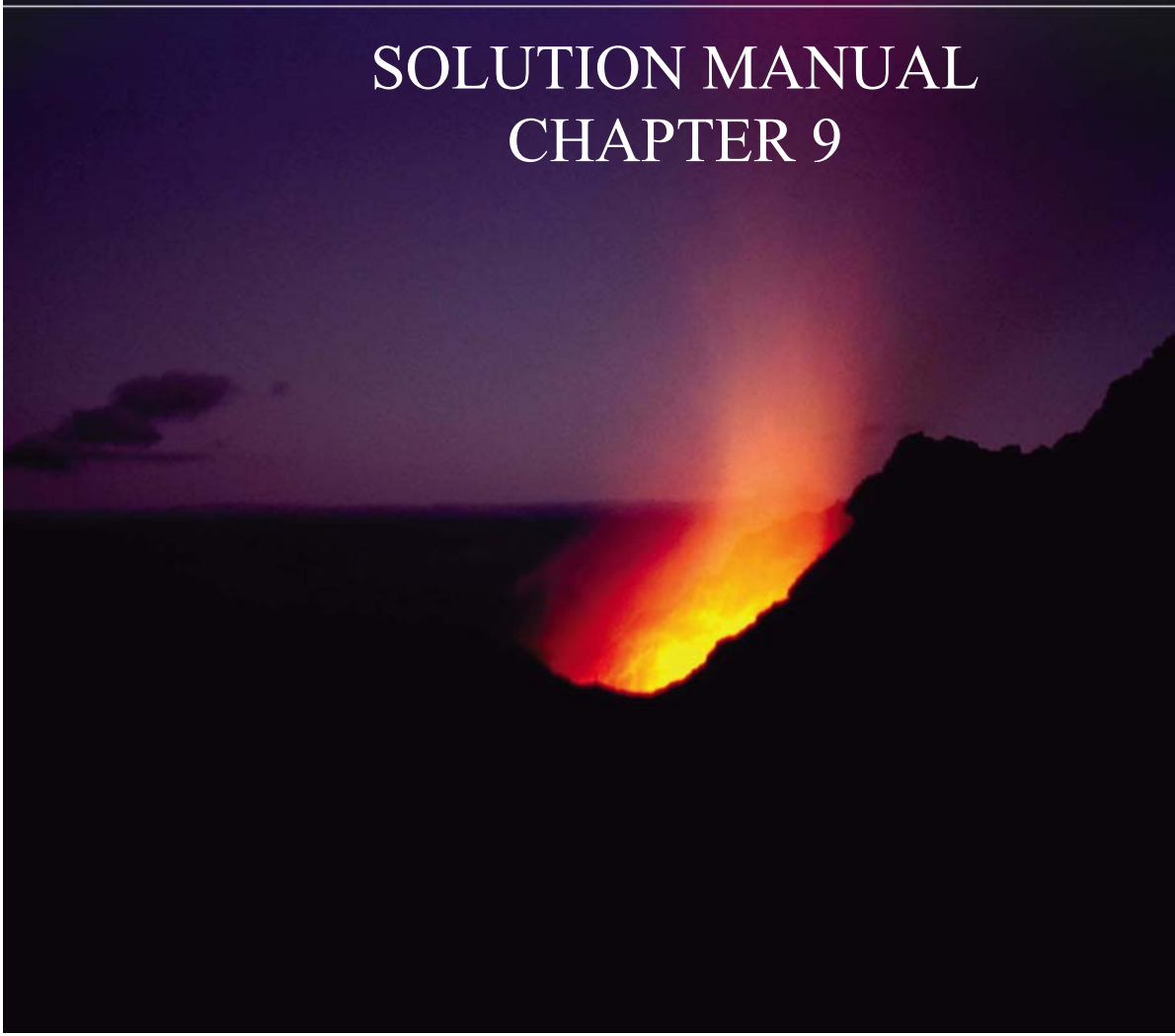


SEVENTH EDITION

# Fundamentals *of* Thermodynamics

BORGNAKKE | SONNTAG

## SOLUTION MANUAL CHAPTER 9



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## **In-Text Concept Questions**

**9.a**

A reversible adiabatic flow of liquid water in a pump has increasing P. How about T?

Solution:

Steady state single flow:  $s_e = s_i + \int_i^e \frac{dq}{T} + s_{\text{gen}} = s_i + 0 + 0$

Adiabatic ( $dq = 0$ ) means integral vanishes and reversible means  $s_{\text{gen}} = 0$ , so  $s$  is constant. Properties for liquid (incompressible) gives Eq.8.19

$$ds = \frac{C}{T} dT$$

then constant  $s$  gives constant  $T$ .

**9.b**

A reversible adiabatic flow of air in a compressor has increasing P. How about T?

Solution:

$$\text{Steady state single flow: } s_e = s_i + \int_i^e \frac{dq}{T} + s_{\text{gen}} = s_i + 0 + 0$$

so s is constant. Properties for an ideal gas gives Eq.8.23 and for constant specific heat we get Eq.8.29. A higher P means a higher T, which is also the case for a variable specific heat, recall Eq.8.28 for the standard entropy.

## 9.c

A compressor receives R-134a at  $-10^{\circ}\text{C}$ , 200 kPa with an exit of 1200 kPa,  $50^{\circ}\text{C}$ . What can you say about the process?

Solution:

Properties for R-134a are found in Table B.5

$$\text{Inlet state: } s_i = 1.7328 \text{ kJ/kg K}$$

$$\text{Exit state: } s_e = 1.7237 \text{ kJ/kg K}$$

$$\text{Steady state single flow: } s_e = s_i + \int_i^e \frac{dq}{T} + s_{\text{gen}}$$

Since  $s$  decreases slightly and the generation term can only be positive, it must be that the heat transfer is negative (out) so the integral gives a contribution that is smaller than  $-s_{\text{gen}}$ .

**9.d**

A flow of water at some velocity out of a nozzle is used to wash a car. The water then falls to the ground. What happens to the water state in terms of  $\mathbf{V}$ ,  $T$  and  $s$ ?

let us follow the water flow. It starts out with kinetic and potential energy of some magnitude at a compressed liquid state  $P$ ,  $T$ . As the water splashes onto the car it loses its kinetic energy (it turns in to internal energy so  $T$  goes up by a very small amount). As it drops to the ground it then loses all the potential energy which goes into internal energy. Both of these processes are irreversible so  $s$  goes up.

If the water has a temperature different from the ambient then there will also be some heat transfer to or from the water which will affect both  $T$  and  $s$ .

9.e

In a steady state single flow  $s$  is either constant or it increases. Is that true?

Solution:

No.

Steady state single flow: 
$$s_e = s_i + \int_i^e \frac{dq}{T} + s_{\text{gen}}$$

Entropy can only go up or stay constant due to  $s_{\text{gen}}$ , but it can go up or down due to the heat transfer which can be positive or negative. So if the heat transfer is large enough it can overpower any entropy generation and drive  $s$  up or down.



**9.f**

If a flow device has the same inlet and exit pressure, can shaft work be done?

The reversible work is given by Eq.9.14

$$w = -\int v \, dP + (\mathbf{V}_i^2 - \mathbf{V}_e^2) + g (Z_i - Z_e)$$

For a constant pressure the first term drops out but the other two remains. Kinetic energy changes can give work out (windmill) and potential energy changes can give work out (a dam).

**9.g**

A polytropic flow process with  $n = 0$  might be which device?

As the polytropic process is  $Pv^n = C$ , then  $n = 0$  is a constant pressure process. This can be a pipe flow, a heat exchanger flow (heater or cooler) or a boiler.

## Concept Problems

**9.1**

If we follow a mass element going through a reversible adiabatic flow process what can we say about the change of state?

Following a mass (this is a control mass)

$$du = dq - dw = 0 - Pdv = -Pdv; \quad \text{compression/expansion changes } u$$

$$ds = dq/T + ds_{\text{gen}} = 0 + 0 \Rightarrow \quad s = \text{constant, isentropic process.}$$

## 9.2

Which process will make the statement in concept question e) on page 267 true?

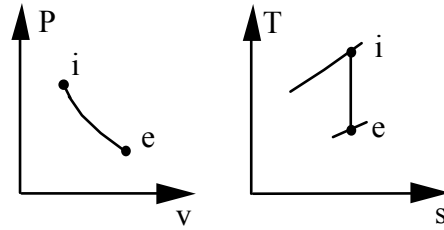
Solution:

If the process is said to be adiabatic then:

Steady state adiabatic single flow:  $s_e = s_i + s_{\text{gen}} \geq s_i$

## 9.3

A reversible process in a steady flow with negligible kinetic and potential energy changes is shown in the diagrams. Indicate the change  $h_e - h_i$  and transfers  $w$  and  $q$  as positive, zero or negative



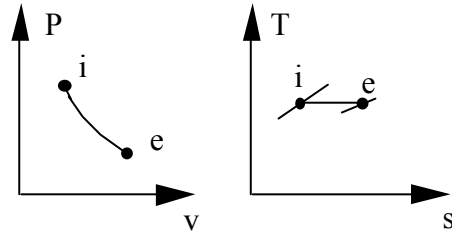
$$dw = -v dP > 0 \quad P \text{ drops so work is positive out.}$$

$$dq = T ds = 0 \quad s \text{ is constant, and process reversible so adiabatic.}$$

$$h_e - h_i = q - w = 0 - w < 0 \quad \text{so enthalpy drops}$$

## 9.4

A reversible process in a steady flow of air with negligible kinetic and potential energy changes is shown in the diagrams. Indicate the change  $h_e - h_i$  and transfers  $w$  and  $q$  as positive, zero or negative



$dw = -v dP > 0$  P drops so work is positive out.

$dq = T ds > 0$  s is increasing and process reversible so  $q$  is positive.

$h_e - h_i = 0$  as they are functions of  $T$  and thus the same.

## 9.5

A reversible steady isobaric flow has 1 kW of heat added with negligible changes in KE and PE, what is the work transfer?

P = C: Shaft work Eq. 9.14:

$$dw = -v dP + \Delta KE + \Delta PE - T ds_{\text{gen}} = 0 + 0 + 0 - 0 = \mathbf{0}$$



## 9.6

An air compressor has a significant heat transfer out. See example 9.4 for how high  $T$  becomes if there is no heat transfer. Is that good, or should it be insulated?

That depends on the use of the compressed air. If there is no need for the high  $T$ , say it is used for compressed air tools, then the heat transfer will lower  $T$  and result in lower specific volume reducing the work. For those applications the compressor may have fins mounted on its surface to promote the heat transfer. In very high pressure compression it is done in stages between which is a heat exchanger called an intercooler.



This is a small compressor driven by an electric motor. Used to charge air into car tires.

## 9.7

Friction in a pipe flow causes a slight pressure decrease and a slight temperature increase. How does that affect entropy?

Solution:

The friction converts flow work (P drops) into internal energy (T up if single phase). This is an irreversible process and s increases.

If liquid: Eq. 8.19:  $ds = \frac{C}{T} dT$  so s follows T

If ideal gas Eq. 8.23:  $ds = C_p \frac{dT}{T} - R \frac{dP}{P}$  (both terms increase)

**9.8**

To increase the work out of a turbine for a given inlet and exit pressure how should the inlet state be changed?

$$w = - \int v \, dP + \dots \quad \text{Eq.9.13}$$

For a given change in pressure boosting  $v$  will result in larger work term. So for **larger inlet T** we get a larger  $v$  and thus larger work. That is why we increase  $T$  by combustion in a gasturbine before the turbine section.

## 9.9

An irreversible adiabatic flow of liquid water in a pump has higher P. How about T?

Solution:

Steady state single flow: 
$$s_e = s_i + \int_i^e \frac{dq}{T} + s_{\text{gen}} = s_i + 0 + s_{\text{gen}}$$

so s is increasing. Properties for liquid (incompressible) gives Eq.8.19 where an increase in s gives an increase in T.

**9.10**

The shaft work in a pump to increase the pressure is small compared to the shaft work in an air compressor for the same pressure increase. Why?

The reversible work is given by Eq. 9.13 or 9.14 if reversible and no kinetic or potential energy changes

$$w = -\int v \, dP$$

The liquid has a very small value for  $v$  compared to a large value for a gas.

## 9.11

Liquid water is sprayed into the hot gases before they enter the turbine section of a large gasturbine power plant. It is claimed that the larger mass flow rate produces more work. Is that the reason?

No. More mass through the turbine does give more work, but the added mass is only a few percent. As the liquid vaporizes the specific volume increases dramatically which gives a much larger volume flow through the turbine and that gives more work output.

$$\dot{W} = \dot{m}w = -\dot{m} \int v \, dP = -\int \dot{m}v \, dP = -\int \dot{V} \, dP$$

This should be seen relative to the small work required to bring the liquid water up to the higher turbine inlet pressure from the source of water (presumably atmospheric pressure).

**9.12**

A tank contains air at 400 kPa, 300 K and a valve opens up for flow out to the outside which is at 100 kPa, 300 K. What happens to the air temperature inside?

As mass flows out of the tank the pressure will drop, the air that remains basically goes through a simple (adiabatic if process is fast enough) expansion process so the temperature also drops. If the flow rate out is very small and the process thus extremely slow, enough heat transfer may take place to keep the temperature constant.

## **Steady state reversible processes single flow**



## 9.13

An evaporator has R-410a at  $-20^{\circ}\text{C}$  and quality 20% flowing in with the exit flow being saturated vapor at  $-20^{\circ}\text{C}$ . Consider the heating to be a reversible process and find the specific heat transfer from the entropy equation.

$$\text{Entropy Eq.9.8: } s_e = s_i + \int dq/T + s_{\text{gen}} = s_i + q/T + 0$$

$$q = T (s_e - s_i) = T (s_g - s_i)$$

$$\text{Inlet: } s_i = 0.1154 + x_i 0.9625 = 0.3079 \text{ kJ/kg-K}$$

$$\text{Exit: } s_g = 1.0779 \text{ kJ/kg-K}$$

$$q = (273.15 - 20) (1.0779 - 0.3079) = \mathbf{194.926 \text{ kJ/kg}}$$

Remark: It fits with  $h_e - h_i = (1 - x_i) h_{fg} = 0.8 \times 243.65 = 194.92 \text{ kJ/kg}$

## 9.14

A reversible isothermal expander (a turbine with heat transfer) has an inlet flow of carbon dioxide at 3 MPa, 40°C and an exit flow at 1 MPa, 40°C. Find the specific heat transfer from the entropy equation and the specific work from the energy equation assuming ideal gas.

$$\text{Energy Eq.6.13: } 0 = h_i - h_e + q - w$$

$$\text{Entropy Eq.9.8: } 0 = s_i - s_e + \int dq/T + s_{\text{gen}} = s_i - s_e + q/T + 0$$

$$q = T (s_e - s_i) = T(C_{p0} \ln \frac{T_e}{T_i} - R \ln \frac{P_e}{P_i}) = -RT \ln \frac{P_e}{P_i}$$

$$= -0.1889 \text{ kJ/kg-K} \times 313.15 \text{ K} \times \ln \frac{1}{3} = \mathbf{64.99 \text{ kJ/kg}}$$

$$w = h_i - h_e + q = q = \mathbf{64.99 \text{ kJ/kg}}$$

**9.15**

Solve the previous Problem using Table B.3

$$\text{Energy Eq.6.13: } 0 = h_i - h_e + q - w$$

$$\text{Entropy Eq.9.8: } 0 = s_i - s_e + \int dq/T + s_{\text{gen}} = s_i - s_e + q/T + 0$$

$$\text{Inlet state: } h_i = 378.55 \text{ kJ/kg, } s_i = 1.4104 \text{ kJ/kg-K}$$

$$\text{Exit state: } h_e = 398.05 \text{ kJ/kg, } s_e = 1.6633 \text{ kJ/kg-K}$$

$$q = T (s_e - s_i) = 313.15 (1.6633 - 1.4104) = \mathbf{79.2 \text{ kJ/kg}}$$

$$w = h_i - h_e + q = 378.55 - 398.05 + 79.2 = \mathbf{59.7 \text{ kJ/kg}}$$

Remark: When it is not an ideal gas  $h$  is a fct. of both  $T$  and  $P$ .

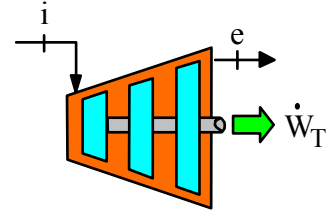
## 9.16

A first stage in a turbine receives steam at 10 MPa, 800°C with an exit pressure of 800 kPa. Assume the stage is adiabatic and neglect kinetic energies. Find the exit temperature and the specific work.

Solution:

C.V. Stage 1 of turbine.

The stage is adiabatic so  $q = 0$  and we will assume reversible so  $s_{\text{gen}} = 0$



Energy Eq.6.13:  $w_T = h_i - h_e$

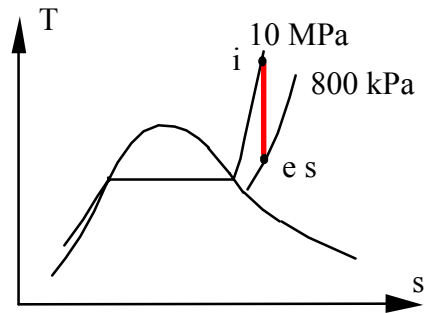
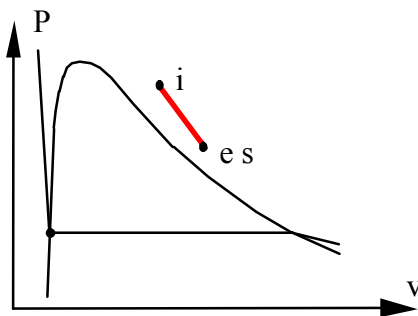
Entropy Eq.9.8:  $s_e = s_i + \int dq/T + s_{\text{gen}} = s_i + 0 + 0$

Inlet state: B.1.3:  $h_i = 4114.9 \text{ kJ/kg}$ ,  $s_i = 7.4077 \text{ kJ/kg K}$

Exit state: 800 kPa,  $s = s_i$

Table B.1.3  $\Rightarrow$   $T \cong 349.7^\circ\text{C}$ ,  $h_e = 3161 \text{ kJ/kg}$

$w_T = 4114.9 - 3161 = 953.9 \text{ kJ/kg}$



## 9.17

Steam enters a turbine at 3 MPa, 450°C, expands in a reversible adiabatic process and exhausts at 10 kPa. Changes in kinetic and potential energies between the inlet and the exit of the turbine are small. The power output of the turbine is 800 kW. What is the mass flow rate of steam through the turbine?

Solution:

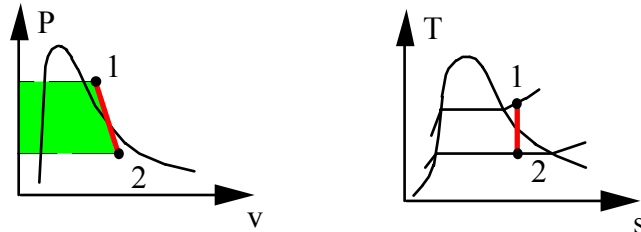
C.V. Turbine, Steady single inlet and exit flows. Adiabatic:  $\dot{Q} = 0$ .

Continuity Eq.6.11:  $\dot{m}_i = \dot{m}_e = \dot{m}$ ,

Energy Eq.6.12:  $\dot{m}h_i = \dot{m}h_e + \dot{W}_T$ ,

Entropy Eq.9.8:  $\dot{m}s_i + \dot{\theta} = \dot{m}s_e$  (Reversible  $\dot{S}_{gen} = 0$ )

Explanation for the work term is in Sect. 9.3, Eq.9.14



Inlet state: Table B.1.3  $h_i = 3344$  kJ/kg,  $s_i = 7.0833$  kJ/kg K

Exit state:  $P_e$ ,  $s_e = s_i \Rightarrow$  Table B.1.2 saturated as  $s_e < s_g$

$$x_e = (7.0833 - 0.6492)/7.501 = 0.8578,$$

$$h_e = 191.81 + 0.8578 \times 2392.82 = 2244.4 \text{ kJ/kg}$$

$$\dot{m} = \dot{W}_T/w_T = \dot{W}_T/(h_i - h_e) = \frac{800}{3344 - 2244.4} \frac{\text{kW}}{\text{kJ/kg}} = \mathbf{0.728 \text{ kg/s}}$$

## 9.18

A reversible adiabatic compressor receives 0.05 kg/s saturated vapor R-410a at 200 kPa and has an exit pressure of 800 kPa. Neglect kinetic energies and find the exit temperature and the minimum power needed to drive the unit.

Solution:

C.V. Compressor, Steady single inlet and exit flows. Adiabatic:  $\dot{Q} = 0$ .

Continuity Eq.6.11:  $\dot{m}_i = \dot{m}_e = \dot{m}$ ,

Energy Eq.6.12:  $\dot{m}h_i = \dot{m}h_e + \dot{W}_C$ ,

Entropy Eq.9.8:  $\dot{m}s_i + \dot{\theta} = \dot{m}s_e$  (Reversible  $\dot{S}_{gen} = 0$ )

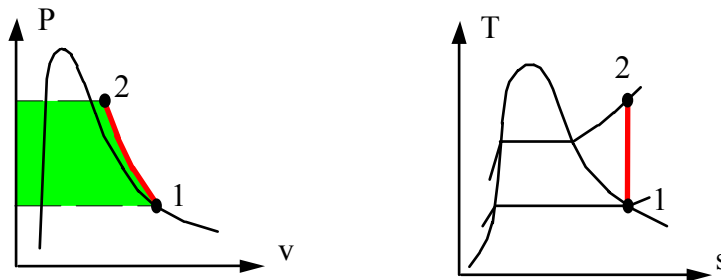
Inlet state: B 4.2.:  $h_i = 264.27$  kJ/kg,  $s_i = 1.1192$  kJ/kg K

Exit state:  $P_e$ ,  $s_e = s_i \Rightarrow$  Table B.4.2  $h_e = 302.65$  kJ/kg,  $T_e \cong 22.7^\circ\text{C}$

$$-w_c = h_e - h_i = 302.65 - 264.27 = 38.38 \text{ kJ/kg}$$

$$-\dot{W}_c = \text{Power In} = -w_c \dot{m} = 38.38 \times 0.05 = \mathbf{1.92 \text{ kW}}$$

Explanation for the work term is in Sect. 9.3, Eq.9.18



## 9.19

In a heat pump that uses R-134a as the working fluid, the R-134a enters the compressor at 150 kPa,  $-10^{\circ}\text{C}$  at a rate of 0.1 kg/s. In the compressor the R-134a is compressed in an adiabatic process to 1 MPa. Calculate the power input required to the compressor, assuming the process to be reversible.

Solution:

C.V. Compressor, Steady single inlet and exit flows. Adiabatic:  $\dot{Q} = 0$ .

Continuity Eq.6.11:  $\dot{m}_1 = \dot{m}_2 = \dot{m}$ ,

Energy Eq.6.12:  $\dot{m}h_1 = \dot{m}h_2 + \dot{W}_C$ ,

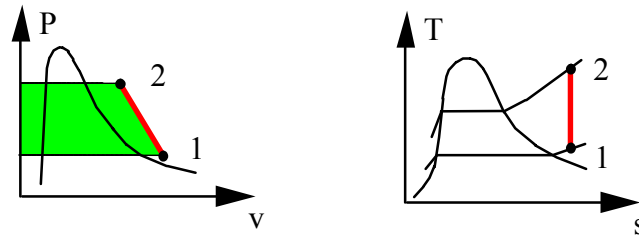
Entropy Eq.9.8:  $\dot{m}s_1 + 0 = \dot{m}s_2$  (Reversible  $\dot{S}_{\text{gen}} = 0$ )

Inlet state: Table B.5.2  $h_1 = 393.84$  kJ/kg,  $s_1 = 1.7606$  kJ/kg K

Exit state:  $P_2 = 1$  MPa &  $s_2 = s_1 \Rightarrow h_2 = 434.9$  kJ/kg

$$\dot{W}_c = \dot{m}w_c = \dot{m}(h_1 - h_2) = 0.1 \times (393.84 - 434.9) = \mathbf{-4.1 \text{ kW}}$$

Explanation for the  
work term is in  
Sect. 9.3  
Eq.9.14



## 9.20

A compressor in a commercial refrigerator receives R-410a at  $-25^{\circ}\text{C}$  and  $x = 1$ . The exit is at 2000 kPa and the process assumed reversible and adiabatic. Neglect kinetic energies and find the exit temperature and the specific work.

CV Compressor.  $q = 0$ .

Energy Eq.6.13:  $w_C = h_i - h_e$

Entropy Eq.9.8:  $s_e = s_i + \int dq/T + s_{\text{gen}} = s_i + 0 + 0$

Inlet state: B.4.1:  $h_i = 269.77 \text{ kJ/kg}$ ,  $s_i = 1.0893 \text{ kJ/kg K}$

Exit state: 1000 kPa,  $s = s_i$

Table B.4.2  $\Rightarrow$   $T \cong 60.4^{\circ}\text{C}$ ,  $h_e = 321.13 \text{ kJ/kg}$

$w_C = 269.77 - 321.13 = -51.4 \text{ kJ/kg}$





## 9.21

A boiler section boils 3 kg/s saturated liquid water at 2000 kPa to saturated vapor in a reversible constant pressure process. Assume you do not know that there is no work. Prove that there is no shaftwork using the first and second laws of thermodynamics.

Solution:

C.V. Boiler. Steady, single inlet and single exit flows.

$$\text{Energy Eq.6.13: } h_i + q = w + h_e;$$

$$\text{Entropy Eq.9.8: } s_i + q/T = s_e$$

States: Table B.1.2,  $T = T_{\text{sat}} = 212.42^\circ\text{C} = 485.57 \text{ K}$

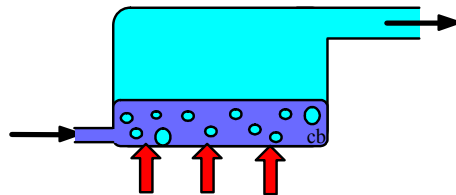
$$h_i = h_f = 908.77 \text{ kJ/kg}, \quad s_i = 2.4473 \text{ kJ/kg K}$$

$$h_e = h_g = 2799.51 \text{ kJ/kg}, \quad s_e = 6.3408 \text{ kJ/kg K}$$

$$q = T(s_e - s_i) = 485.57(6.3408 - 2.4473) = \mathbf{1890.6 \text{ kJ/kg}}$$

$$w = h_i + q - h_e = 908.77 + 1890.6 - 2799.51 = \mathbf{-0.1 \text{ kJ/kg}}$$

It should be zero (non-zero due to round off in values of  $s$ ,  $h$  and  $T_{\text{sat}}$ ).



Often it is a long pipe  
and not a chamber

## 9.22

Atmospheric air at  $-45^\circ\text{C}$ , 60 kPa enters the front diffuser of a jet engine with a velocity of 900 km/h and frontal area of  $1\text{ m}^2$ . After the adiabatic diffuser the velocity is 20 m/s. Find the diffuser exit temperature and the maximum pressure possible.

Solution:

C.V. Diffuser, Steady single inlet and exit flow, no work or heat transfer.

$$\text{Energy Eq.6.13: } h_i + V_i^2/2 = h_e + V_e^2/2, \quad \text{and} \quad h_e - h_i = C_p(T_e - T_i)$$

$$\text{Entropy Eq.9.8: } s_i + \int dq/T + s_{\text{gen}} = s_i + 0 + 0 = s_e \quad (\text{Reversible, adiabatic})$$

$$\text{Heat capacity and ratio of specific heats from Table A.5: } C_{P0} = 1.004 \frac{\text{kJ}}{\text{kg K}},$$

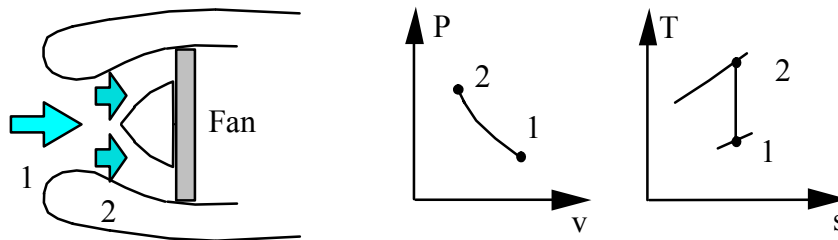
$k = 1.4$ , the energy equation then gives:

$$1.004 [T_e - (-45)] = 0.5 [(900 \times 1000 / 3600)^2 - 20^2] / 1000 = 31.05 \text{ kJ/kg}$$

$$\Rightarrow T_e = -14.05^\circ\text{C} = \mathbf{259.1 \text{ K}}$$

Constant  $s$  for an ideal gas is expressed in Eq.8.23:

$$P_e = P_i (T_e/T_i)^{\frac{k}{k-1}} = 60 (259.1/228.1)^{3.5} = \mathbf{93.6 \text{ kPa}}$$



## 9.23

A compressor is surrounded by cold R-134a so it works as an isothermal compressor. The inlet state is 0°C, 100 kPa and the exit state is saturated vapor. Find the specific heat transfer and specific work.

Solution:

C.V. Compressor. Steady, single inlet and single exit flows.

$$\text{Energy Eq.6.13: } h_i + q = w + h_e;$$

$$\text{Entropy Eq.9.8: } s_i + q/T = s_e$$

$$\text{Inlet state: Table B.5.2, } h_i = 403.4 \text{ kJ/kg, } s_i = 1.8281 \text{ kJ/kg K}$$

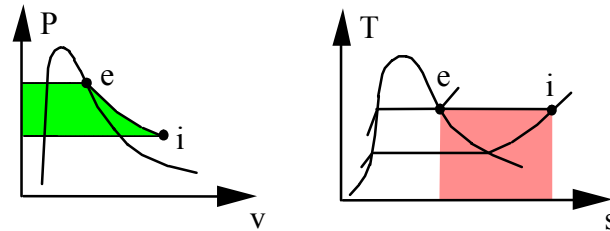
$$\text{Exit state: Table B.5.1, } h_e = 398.36 \text{ kJ/kg, } s_e = 1.7262 \text{ kJ/kg K}$$

$$q = T(s_e - s_i) = 273.15(1.7262 - 1.8281) = -27.83 \text{ kJ/kg}$$

$$w = 403.4 + (-27.83) - 398.36 = -22.8 \text{ kJ/kg}$$

Explanation for the work term is in Sect. 9.3

Eqs. 9.13 and 9.14



## 9.24

Consider the design of a nozzle in which nitrogen gas flowing in a pipe at 500 kPa, 200°C, and at a velocity of 10 m/s, is to be expanded to produce a velocity of 300 m/s. Determine the exit pressure and cross-sectional area of the nozzle if the mass flow rate is 0.15 kg/s, and the expansion is reversible and adiabatic.

Solution:

C.V. Nozzle. Steady flow, no work out and no heat transfer.

$$\text{Energy Eq.6.13: } h_i + \mathbf{V}_i^2/2 = h_e + \mathbf{V}_e^2/2$$

$$\text{Entropy Eq.9.8: } s_i + \int dq/T + s_{\text{gen}} = s_i + 0 + 0 = s_e$$

Properties Ideal gas Table A.5:

$$C_{P0} = 1.042 \frac{\text{kJ}}{\text{kg K}}, \quad R = 0.2968 \frac{\text{kJ}}{\text{kg K}}, \quad k = 1.40$$

$$h_e - h_i = C_{P0}(T_e - T_i) = 1.042(T_e - 473.2) = (10^2 - 300^2)/(2 \times 1000)$$

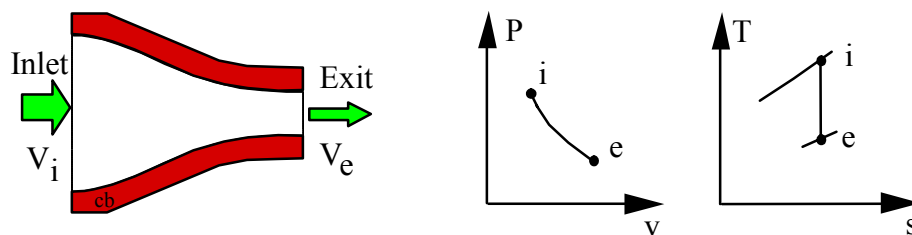
$$\text{Solving for exit T: } T_e = 430 \text{ K,}$$

Process:  $s_i = s_e \Rightarrow$  For ideal gas expressed in Eq.8.23

$$P_e = P_i(T_e/T_i)^{k/(k-1)} = 500 \left( \frac{430}{473.2} \right)^{3.5} = \mathbf{357.6 \text{ kPa}}$$

$$v_e = RT_e/P_e = (0.2968 \times 430)/357.6 = 0.35689 \text{ m}^3/\text{kg}$$

$$A_e = \dot{m}v_e/V_e = \frac{0.15 \times 0.35689}{300} = \mathbf{1.78 \times 10^{-4} \text{ m}^2}$$



## 9.25

The exit nozzle in a jet engine receives air at 1200 K, 150 kPa with negligible kinetic energy. The exit pressure is 80 kPa and the process is reversible and adiabatic. Use constant heat capacity at 300 K to find the exit velocity.

Solution:

C.V. Nozzle, Steady single inlet and exit flow, no work or heat transfer.

$$\text{Energy Eq.6.13: } h_i = h_e + \mathbf{V}_e^2/2 \quad (Z_i = Z_e)$$

$$\text{Entropy Eq.9.8: } s_e = s_i + \int dq/T + s_{\text{gen}} = s_i + 0 + 0$$

Use constant specific heat from Table A.5,  $C_{P0} = 1.004 \frac{\text{kJ}}{\text{kg K}}$ ,  $k = 1.4$

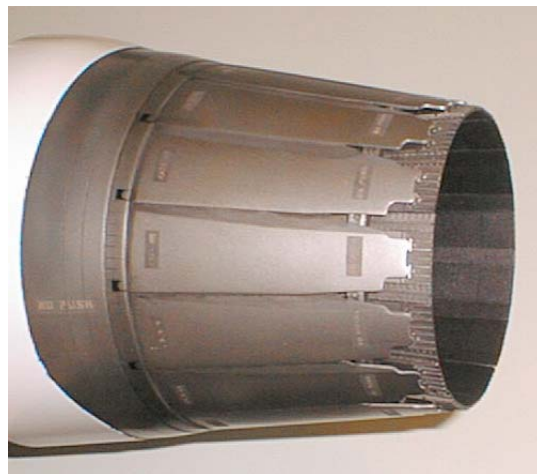
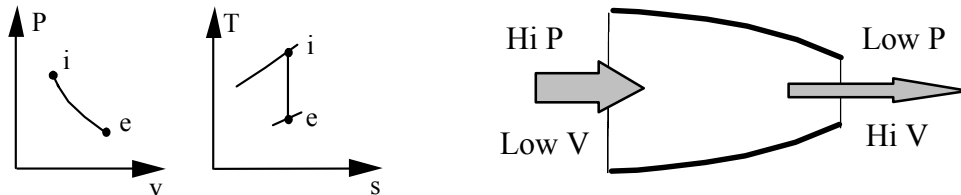
The isentropic process ( $s_e = s_i$ ) gives Eq.8.23

$$\Rightarrow T_e = T_i \left( P_e/P_i \right)^{\frac{k-1}{k}} = 1200 (80/150)^{0.2857} = 1002.7 \text{ K}$$

The energy equation becomes

$$\mathbf{V}_e^2/2 = h_i - h_e \cong C_p(T_i - T_e)$$

$$\mathbf{V}_e = \sqrt{2 C_p(T_i - T_e)} = \sqrt{2 \times 1.004(1200 - 1002.7) \times 1000} = \mathbf{629.4 \text{ m/s}}$$



## 9.26

Do the previous problem using the air tables in A.7

The exit nozzle in a jet engine receives air at 1200 K, 150 kPa with negligible kinetic energy. The exit pressure is 80 kPa and the process is reversible and adiabatic. Use constant heat capacity at 300 K to find the exit velocity.

Solution:

C.V. Nozzle, Steady single inlet and exit flow, no work or heat transfer.

$$\text{Energy Eq.6.13: } h_i = h_e + V_e^2/2 \quad (Z_i = Z_e)$$

$$\text{Entropy Eq.9.8: } s_e = s_i + \int dq/T + s_{\text{gen}} = s_i + 0 + 0$$

$$\text{Process: } q = 0, \quad s_{\text{gen}} = 0 \quad \text{as used above leads to } s_e = s_i$$

$$\text{Inlet state: } h_i = 1277.8 \text{ kJ/kg}, \quad s_{Ti}^{\circ} = 8.3460 \text{ kJ/kg K}$$

The constant  $s$  is rewritten from Eq.8.19 as

$$s_{Te}^{\circ} = s_{Ti}^{\circ} + R \ln(P_e / P_i) = 8.3460 + 0.287 \ln(80/150) = 8.1656$$

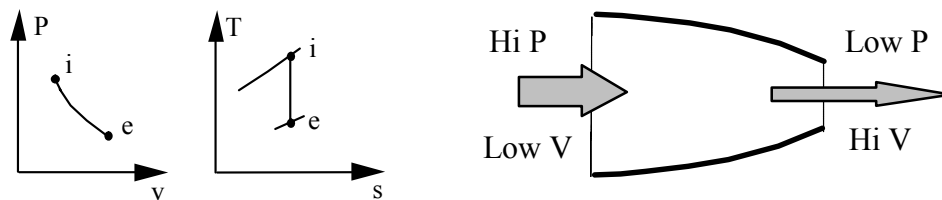
Interpolate in A.7  $\Rightarrow$

$$T_e = 1000 + 50 \frac{8.1656 - 8.1349}{8.1908 - 8.1349} = 1027.46 \text{ K}$$

$$h_e = 1046.2 + (1103.5 - 1046.3) \times \frac{8.1656 - 8.1349}{8.1908 - 8.1349} = 1077.7$$

From the energy equation we have  $V_e^2/2 = h_i - h_e$ , so then

$$V_e = \sqrt{2(h_i - h_e)} = \sqrt{2(1277.8 - 1077.7) \times 1000} = \mathbf{632.6 \text{ m/s}}$$



## 9.27

A flow of 2 kg/s saturated vapor R-410a at 500 kPa is heated at constant pressure to 60°C. The heat is supplied by a heat pump that receives heat from the ambient at 300 K and work input, shown in Fig. P9.27. Assume everything is reversible and find the rate of work input.

Solution:

C.V. Heat exchanger

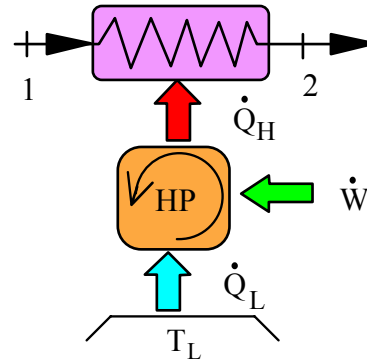
$$\text{Continuity Eq.: } \dot{m}_1 = \dot{m}_2 ;$$

$$\text{Energy Eq.: } \dot{m}_1 h_1 + \dot{Q}_H = \dot{m}_1 h_2$$

Table B.4.2:

$$h_1 = 274.33 \text{ kJ/kg}, \quad s_1 = 1.0647 \text{ kJ/kg K}$$

$$h_2 = 342.32 \text{ kJ/kg}, \quad s_2 = 1.2959 \text{ kJ/kg K}$$



Notice we can find  $\dot{Q}_H$  but the temperature  $T_H$  is not constant making it difficult to evaluate the COP of the heat pump.

C.V. Total setup and assume everything is reversible and steady state.

$$\text{Energy Eq.: } \dot{m}_1 h_1 + \dot{Q}_L + \dot{W}_{\text{in}} = \dot{m}_1 h_2$$

$$\text{Entropy Eq.: } \dot{m}_1 s_1 + \dot{Q}_L / T_L + 0 = \dot{m}_1 s_2 \quad (T_L \text{ is constant, } s_{\text{gen}} = 0)$$

$$\dot{Q}_L = \dot{m}_1 T_L [s_2 - s_1] = 2 \times 300 [1.2959 - 1.0647] = 138.72 \text{ kW}$$

$$\dot{W}_{\text{in}} = \dot{m}_1 [h_2 - h_1] - \dot{Q}_L = 2 (342.32 - 274.33) - 138.72 = -2.74 \text{ kW}$$

## 9.28

A compressor brings a hydrogen gas flow at 280 K, 100 kPa up to a pressure of 1000 kPa in a reversible process. How hot is the exit flow and what is the specific work input?

CV Compressor. Assume  $q = 0$ .

$$\text{Energy Eq.6.13: } w_C = h_i - h_e \approx C_p (T_i - T_e)$$

$$\text{Entropy Eq.9.8: } s_e = s_i + \int dq/T + s_{\text{gen}} = s_i + 0 + 0$$

$$T_e = T_i (P_e/P_i)^{(k-1)/k} = 280 (1000/100)^{(1.409-1)/1.409} = \mathbf{546.3 \text{ K}}$$

$$w_C = 14.209 \text{ kJ/kg-K} \times (280 - 546.3) \text{ K} = \mathbf{-3783.9 \text{ kJ/kg}}$$



Small hydrogen compressor.



## 9.29

A diffuser is a steady-state device in which a fluid flowing at high velocity is decelerated such that the pressure increases in the process. Air at 120 kPa, 30°C enters a diffuser with velocity 200 m/s and exits with a velocity of 20 m/s. Assuming the process is reversible and adiabatic what are the exit pressure and temperature of the air?

Solution:

C.V. Diffuser, Steady single inlet and exit flow, no work or heat transfer.

$$\text{Energy Eq.6.13: } h_i + \mathbf{V}_i^2/2 = h_e + \mathbf{V}_e^2/2, \quad \Rightarrow \quad h_e - h_i = C_{P0}(T_e - T_i)$$

$$\text{Entropy Eq.9.8: } s_i + \int dq/T + s_{\text{gen}} = s_i + 0 + 0 = s_e \quad (\text{Reversible, adiabatic})$$

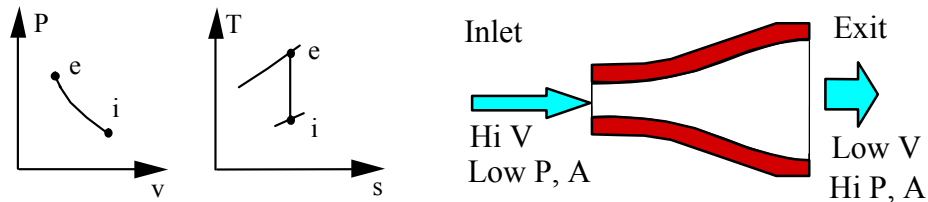
Use constant specific heat from Table A.5,  $C_{P0} = 1.004 \frac{\text{kJ}}{\text{kg K}}$ ,  $k = 1.4$

Energy equation then gives:

$$C_{P0}(T_e - T_i) = 1.004(T_e - 303.2) = (200^2 - 20^2)/(2 \times 1000) \quad \Rightarrow \quad T_e = \mathbf{322.9 \text{ K}}$$

The isentropic process ( $s_e = s_i$ ) gives Eq.8.32

$$P_e = P_i (T_e/T_i)^{\frac{k}{k-1}} = 120(322.9/303.2)^{3.5} = \mathbf{149.6 \text{ kPa}}$$

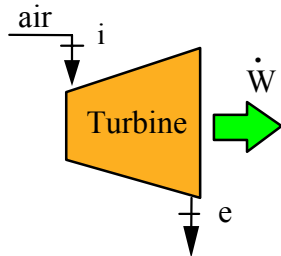


## 9.30

Air enters a turbine at 800 kPa, 1200 K, and expands in a reversible adiabatic process to 100 kPa. Calculate the exit temperature and the work output per kilogram of air, using

- The ideal gas tables, Table A.7
- Constant specific heat, value at 300 K from table A.5

Solution:



C.V. Air turbine.

Adiabatic:  $q = 0$ , reversible:  $s_{\text{gen}} = 0$

Energy Eq.6.13:  $w_T = h_i - h_e$ ,

Entropy Eq.9.8:  $s_e = s_i$

a) Table A.7:  $h_i = 1277.8 \text{ kJ/kg}$ ,  $s_{T_i}^0 = 8.34596 \text{ kJ/kg K}$

The constant  $s$  process is written from Eq.8.28 as

$$\Rightarrow s_{T_e}^0 = s_{T_i}^0 + R \ln\left(\frac{P_e}{P_i}\right) = 8.34596 + 0.287 \ln\left(\frac{100}{800}\right) = 7.7492 \text{ kJ/kg K}$$

Interpolate in A.7.1  $\Rightarrow T_e = \mathbf{706 \text{ K}}$ ,  $h_e = 719.9 \text{ kJ/kg}$

$$w = h_i - h_e = \mathbf{557.9 \text{ kJ/kg}}$$

b) Table A.5:  $C_{P0} = 1.004 \text{ kJ/kg K}$ ,  $R = 0.287 \text{ kJ/kg K}$ ,  $k = 1.4$ , then from Eq.8.32

$$T_e = T_i (P_e/P_i)^{\frac{k-1}{k}} = 1200 \left(\frac{100}{800}\right)^{0.286} = \mathbf{662.1 \text{ K}}$$

$$w = C_{P0}(T_i - T_e) = 1.004(1200 - 662.1) = \mathbf{539.8 \text{ kJ/kg}}$$

## 9.31

A highly cooled compressor brings a hydrogen gas flow at 300 K, 100 kPa up to a pressure of 1000 kPa in an isothermal process. Find the specific work assuming a reversible process.

CV Compressor. Isothermal  $T_i = T_e$  so that ideal gas gives  $h_i = h_e$ .

$$\text{Energy Eq.6.13: } w_C = h_i + q - h_e = q$$

$$\text{Entropy Eq.9.8: } s_e = s_i + \int dq/T + s_{\text{gen}} = s_i + q/T + 0$$

$$q = T(s_e - s_i) = T [-R \ln(P_e/P_i) ]$$

$$w = q = -4.1243 \times 300 \ln(10) = \mathbf{-2849 \text{ kJ/kg}}$$

## 9.32

A compressor receives air at 290 K, 100 kPa and a shaft work of 5.5 kW from a gasoline engine. It should deliver a mass flow rate of 0.01 kg/s air to a pipeline. Find the maximum possible exit pressure of the compressor.

Solution:

C.V. Compressor, Steady single inlet and exit flows. Adiabatic:  $\dot{Q} = 0$ .

Continuity Eq.6.11:  $\dot{m}_i = \dot{m}_e = \dot{m}$ ,

Energy Eq.6.12:  $\dot{m}h_i = \dot{m}h_e + \dot{W}_C$ ,

Entropy Eq.9.8:  $\dot{m}s_i + \dot{S}_{gen} = \dot{m}s_e$  (Reversible  $\dot{S}_{gen} = 0$ )

$$\dot{W}_C = \dot{m}w_c \Rightarrow -w_c = -\dot{W}/\dot{m} = 5.5/0.01 = 550 \text{ kJ/kg}$$

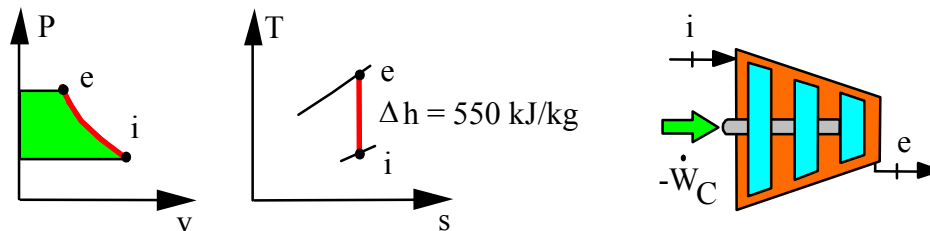
Use constant specific heat from Table A.5,  $C_{p0} = 1.004$ ,  $k = 1.4$

$$h_e = h_i + 550 \Rightarrow T_e = T_i + 550/1.004$$

$$T_e = 290 + 550/1.004 = 837.81 \text{ K}$$

$$s_i = s_e \Rightarrow P_e = P_i (T_e/T_i)^{\frac{k}{k-1}} \quad \text{Eq.8.32}$$

$$P_e = 100 \times (837.81/290)^{3.5} = \mathbf{4098 \text{ kPa}}$$



## 9.33

An expander receives 0.5 kg/s air at 2000 kPa, 300 K with an exit state of 400 kPa, 300 K. Assume the process is reversible and isothermal. Find the rates of heat transfer and work neglecting kinetic and potential energy changes.

Solution:

C.V. Expander, single steady flow.

$$\text{Energy Eq.:} \quad \dot{m}h_i + \dot{Q} = \dot{m}h_e + \dot{W}$$

$$\text{Entropy Eq.:} \quad \dot{m}s_i + \dot{Q}/T + \dot{m}s_{\text{gen}} = \dot{m}s_e$$

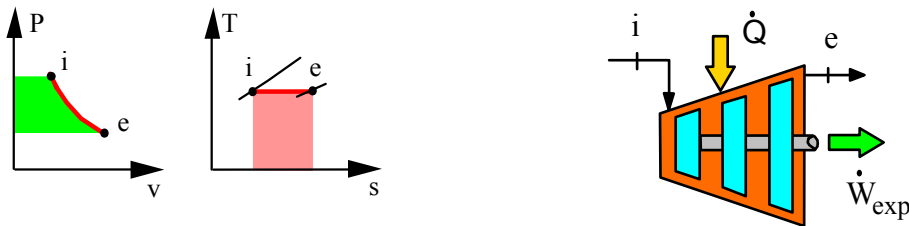
$$\text{Process:} \quad T \text{ is constant and } s_{\text{gen}} = 0$$

Ideal gas and isothermal gives a change in entropy by Eq. 8.24, so we can solve for the heat transfer

$$\begin{aligned} \dot{Q} &= T\dot{m}(s_e - s_i) = -\dot{m}RT \ln \frac{P_e}{P_i} \\ &= -0.5 \times 300 \times 0.287 \times \ln \frac{400}{2000} = \mathbf{69.3 \text{ kW}} \end{aligned}$$

From the energy equation we get

$$\dot{W} = \dot{m}(h_i - h_e) + \dot{Q} = \dot{Q} = \mathbf{69.3 \text{ kW}}$$



## 9.34

A reversible steady state device receives a flow of 1 kg/s air at 400 K, 450 kPa and the air leaves at 600 K, 100 kPa. Heat transfer of 800 kW is added from a 1000 K reservoir, 100 kW rejected at 350 K and some heat transfer takes place at 500 K. Find the heat transferred at 500 K and the rate of work produced.

Solution:

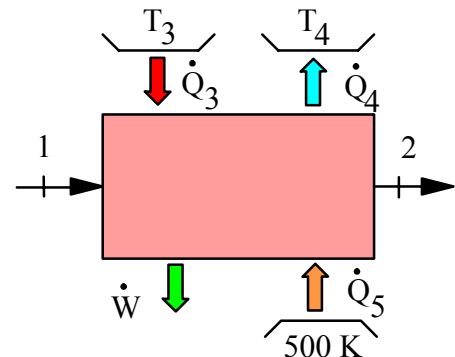
C.V. Device, single inlet and exit flows.

Energy equation, Eq.6.12:

$$\dot{m}h_1 + \dot{Q}_3 - \dot{Q}_4 + \dot{Q}_5 = \dot{m}h_2 + \dot{W}$$

Entropy equation with zero generation, Eq.9.8:

$$\dot{m}s_1 + \dot{Q}_3/T_3 - \dot{Q}_4/T_4 + \dot{Q}_5/T_5 = \dot{m}s_2$$



Solve for the unknown heat transfer using Table A.7.1 and Eq. 8.19 for change in s

$$\begin{aligned}\dot{Q}_5 &= T_5 [s_2 - s_1] \dot{m} + \frac{T_5}{T_4} \dot{Q}_4 - \frac{T_5}{T_3} \dot{Q}_3 \\ &= 500 \times 1 \left( 7.5764 - 7.1593 - 0.287 \ln \frac{100}{450} \right) + \frac{500}{350} \times 100 - \frac{500}{1000} \times 800 \\ &= 424.4 + 142.8 - 400 = 167.2 \text{ kW}\end{aligned}$$

Now the work from the energy equation is

$$\dot{W} = 1 \times (401.3 - 607.3) + 800 - 100 + 167.2 = \mathbf{661.2 \text{ kW}}$$

## 9.35

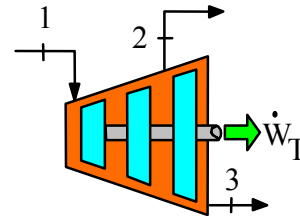
A steam turbine in a powerplant receives 5 kg/s steam at 3000 kPa, 500°C. 20% of the flow is extracted at 1000 kPa to a feedwater heater and the remainder flows out at 200 kPa. Find the two exit temperatures and the turbine power output.

C.V. Turbine. Steady flow and adiabatic  $q = 0$ .

$$\text{Continuity Eq.6.9: } \dot{m}_1 = \dot{m}_2 + \dot{m}_3 ;$$

$$\text{Energy Eq.6.10: } \dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{m}_3 h_3 + \dot{W}$$

$$\text{Entropy Eq.9.7: } \dot{m}_1 s_1 + \dot{S}_{\text{gen}} = \dot{m}_2 s_2 + \dot{m}_3 s_3$$



$$\text{State 1: } h_1 = 3456 \text{ kJ/kg, } s_1 = 7.234 \text{ kJ/kgK}$$

We also assume turbine is reversible  $\dot{S}_{\text{gen}} = 0 \Rightarrow s_1 = s_2 = s_3$

$$\text{State 2: (P,s) } T_2 = 330.6^\circ\text{C, } h_2 = 3116 \text{ kJ/kg}$$

$$\text{State 3: (P,s) } T_3 = 140.7^\circ\text{C, } h_3 = 2750 \text{ kJ/kg}$$

$$\dot{W} = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 = 5 \times 3456 - 1 \times 3116 - 4 \times 2750 = 3164 \text{ kW}$$

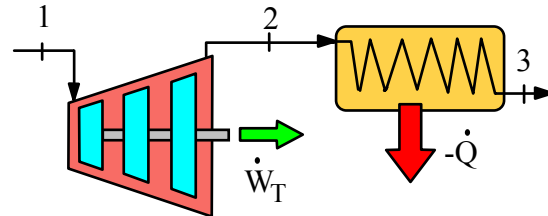
## 9.36

A small turbine delivers 150 kW and is supplied with steam at 700°C, 2 MPa. The exhaust passes through a heat exchanger where the pressure is 10 kPa and exits as saturated liquid. The turbine is reversible and adiabatic. Find the specific turbine work, and the heat transfer in the heat exchanger.

Solution:

Continuity Eq.6.11: Steady

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m}$$



Turbine: Energy Eq.6.13:  $w_T = h_1 - h_2$

Entropy Eq.9.8:  $s_2 = s_1 + s_{T \text{ gen}}$

Inlet state: Table B.1.3  $h_1 = 3917.45 \text{ kJ/kg}$ ,  $s_1 = 7.9487 \text{ kJ/kg K}$

Ideal turbine  $s_{T \text{ gen}} = 0$ ,  $s_2 = s_1 = 7.9487 = s_{f2} + x s_{fg2}$

State 3:  $P = 10 \text{ kPa}$ ,  $s_2 < s_g \Rightarrow$  saturated 2-phase in Table B.1.2

$$\Rightarrow x_{2,s} = (s_1 - s_{f2})/s_{fg2} = (7.9487 - 0.6492)/7.501 = 0.9731$$

$$\Rightarrow h_{2,s} = h_{f2} + x h_{fg2} = 191.8 + 0.9731 \times 2392.8 = 2520.35 \text{ kJ/kg}$$

$$w_{T,s} = h_1 - h_{2,s} = \mathbf{1397.05 \text{ kJ/kg}}$$

$$\dot{m} = \dot{W} / w_{T,s} = 150 / 1397 = 0.1074 \text{ kg/s}$$

Heat exchanger:

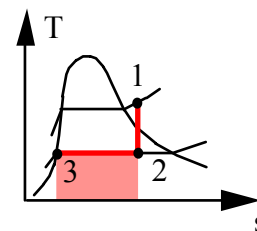
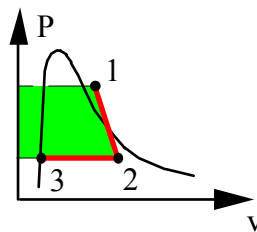
Energy Eq.6.13:  $q = h_3 - h_2$ ,

Entropy Eq.9.8:  $s_3 = s_2 + \int dq/T + s_{\text{He gen}}$

$$q = h_3 - h_{2,s} = 191.83 - 2520.35 = \mathbf{-2328.5 \text{ kJ/kg}}$$

$$\dot{Q} = \dot{m} q = 0.1074 \times (-2328.5) = \mathbf{-250 \text{ kW}}$$

Explanation for the work term is in Sect. 9.3, Eq.9.14





## 9.37

One technique for operating a steam turbine in part-load power output is to throttle the steam to a lower pressure before it enters the turbine, as shown in Fig. P9.39. The steamline conditions are 2 MPa, 400°C, and the turbine exhaust pressure is fixed at 10 kPa. Assuming the expansion inside the turbine to be reversible and adiabatic, determine

- The full-load specific work output of the turbine
- The pressure the steam must be throttled to for 80% of full-load output
- Show both processes in a  $T$ - $s$  diagram.

a) C.V Turbine. Full load reversible and adiabatic

Entropy Eq.9.8 reduces to constant  $s$  so from Table B.1.3 and B.1.2

$$s_3 = s_1 = 7.1271 = 0.6493 + x_{3a} \times 7.5009$$

$$\Rightarrow x_{3a} = 0.8636$$

$$h_{3a} = 191.83 + 0.8636 \times 2392.8 = 2258.3 \text{ kJ/kg}$$

Energy Eq.6.13 for turbine

$${}_1w_{3a} = h_1 - h_{3a} = 3247.6 - 2258.3 = \mathbf{989.3 \text{ kJ/kg}}$$

b) The energy equation for the part load operation and notice that we have constant  $h$  in the throttle process.

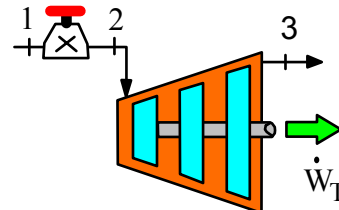
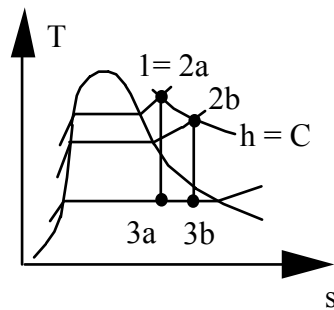
$$w_T = 0.80 \times 989.3 = 791.4 = 3247.6 - h_{3b}$$

$$h_{3b} = 2456.2 = 191.83 + x_{3b} \times 2392.8 \quad \Rightarrow \quad x_{3b} = 0.9463$$

$$s_{3b} = 0.6492 + 0.9463 \times 7.501 = 7.7474 \text{ kJ/kg}$$

$$\left. \begin{array}{l} s_{2b} = s_{3b} = 7.7474 \\ h_{2b} = h_1 = 3247.6 \end{array} \right\} \rightarrow \begin{array}{l} P_{2b} = \mathbf{510 \text{ kPa}} \\ \& T_{2b} = \mathbf{388.4^\circ\text{C}} \end{array}$$

c)



## 9.38

An adiabatic air turbine receives 1 kg/s air at 1500 K, 1.6 MPa and 2 kg/s air at 400 kPa,  $T_2$  in a setup similar to Fig. P6.76 with an exit flow at 100 kPa. What should the temperature  $T_2$  be so the whole process can be reversible?

The process is reversible if we do not generate any entropy. Physically in this problem it means that state 2 must match the state inside the turbine so we do not mix fluid at two different temperatures (we assume the pressure inside is exactly 400 kPa).

For this reason let us select the front end as C.V. and consider the flow from state 1 to the 400 kPa. This is a single flow

Entropy Eq.9.8:  $s_1 + 0/T + 0 = s_2$  ;

$$s_2 - s_1 = 0 = s_{T2}^{\circ} - s_{T1}^{\circ} - R \ln(P_2 / P_1)$$

$$s_{T2}^{\circ} = s_{T1}^{\circ} + R \ln(P_2 / P_1) = 8.61208 + 0.287 \ln \frac{400}{1600} = 8.2142 \text{ kJ/kg-K}$$

From A.7.1:  $T_2 = \mathbf{1071.8 \text{ K}}$

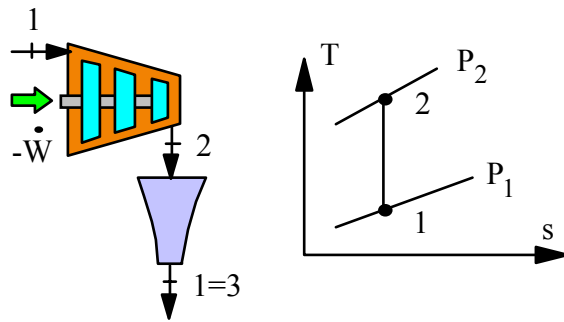
If we solve with constant specific heats we get from Eq.8.23 and  $k = 1.4$

$$T_2 = T_1 (P_2 / P_1)^{(k-1)/k} = 1500 (400/1600)^{0.2857} = 1009.4 \text{ K}$$

## 9.39

A reversible adiabatic compression of an air flow from 20°C, 100 kPa to 200 kPa is followed by an expansion down to 100 kPa in an ideal nozzle. What are the two processes? How hot does the air get? What is the exit velocity?

Solution:



Separate control volumes around compressor and nozzle. For ideal compressor we have inlet : 1 and exit : 2

Adiabatic :  $q = 0$ .  
Reversible:  $s_{\text{gen}} = 0$

$$\text{Energy Eq.6.13: } h_1 + 0 = w_C + h_2; \quad h_2 = h_3 + \frac{1}{2}V^2$$

$$\text{Entropy Eq.9.8: } s_1 + 0/T + 0 = s_2; \quad s_2 + 0/T = s_3$$

So both processes are **isentropic**.

$$-w_C = h_2 - h_1, \quad s_2 = s_1$$

Properties Table A.5 air:  $C_{P0} = 1.004 \text{ kJ/kg K}$ ,  $R = 0.287 \text{ kJ/kg K}$ ,  $k = 1.4$

Process gives constant  $s$  (isentropic) which with constant  $C_{P0}$  gives Eq.8.23

$$\Rightarrow T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = 293.15 \left( \frac{200}{100} \right)^{0.2857} = \mathbf{357.4 \text{ K}}$$

$$\Rightarrow -w_C = C_{P0}(T_2 - T_1) = 1.004 (357.4 - 293.2) = 64.457 \text{ kJ/kg}$$

The ideal nozzle then expands back down to  $P_1$  (constant  $s$ ) so state 3 equals state 1. The energy equation has no work but kinetic energy and gives:

$$\frac{1}{2}V^2 = h_2 - h_1 = -w_C = 64\,457 \text{ J/kg} \quad (\text{remember conversion to J})$$

$$\Rightarrow V_3 = \sqrt{2 \times 64\,457} = \mathbf{359 \text{ m/s}}$$

## 9.40

A turbo charger boosts the inlet air pressure to an automobile engine. It consists of an exhaust gas driven turbine directly connected to an air compressor, as shown in Fig. P9.34. For a certain engine load the conditions are given in the figure. Assume that both the turbine and the compressor are reversible and adiabatic having also the same mass flow rate. Calculate the turbine exit temperature and power output. Find also the compressor exit pressure and temperature.

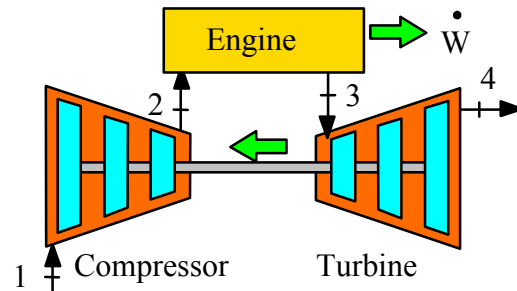
Solution:

CV: Turbine, Steady single inlet and exit flows,

Process: adiabatic:  $q = 0$ ,  
reversible:  $s_{\text{gen}} = 0$

Energy Eq. 6.13:  $w_T = h_3 - h_4$ ,

Entropy Eq. 9.8:  $s_4 = s_3$



The property relation for ideal gas gives Eq. 8.23,  $k$  from Table A.5

$$s_4 = s_3 \rightarrow T_4 = T_3(P_4/P_3)^{\frac{k-1}{k}} = 923.2 \left(\frac{100}{170}\right)^{0.286} = \mathbf{793.2 \text{ K}}$$

The energy equation is evaluated with specific heat from Table A.5

$$w_T = h_3 - h_4 = C_{p0}(T_3 - T_4) = 1.004(923.2 - 793.2) = 130.5 \text{ kJ/kg}$$

$$\dot{W}_T = \dot{m}w_T = \mathbf{13.05 \text{ kW}}$$

C.V. Compressor, steady 1 inlet and 1 exit, same flow rate as turbine.

$$\text{Energy Eq. 6.13: } -w_C = h_2 - h_1,$$

$$\text{Entropy Eq. 9.8: } s_2 = s_1$$

Express the energy equation for the shaft and compressor having the turbine power as input with the same mass flow rate so we get

$$-w_C = w_T = 130.5 \text{ kJ/kg} = C_{p0}(T_2 - T_1) = 1.004(T_2 - 303.2)$$

$$T_2 = \mathbf{433.2 \text{ K}}$$

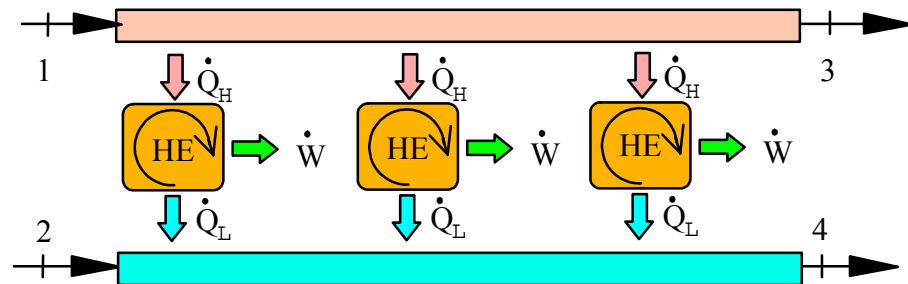
The property relation for  $s_2 = s_1$  is Eq. 8.23 and inverted as

$$P_2 = P_1(T_2/T_1)^{\frac{k}{k-1}} = 100 \left(\frac{433.2}{303.2}\right)^{3.5} = \mathbf{348.7 \text{ kPa}}$$

## 9.41

Two flows of air both at 200 kPa, one has 1 kg/s at 400 K and the other has 2 kg/s at 290 K. The two lines exchange energy through a number of ideal heat engines taking energy from the hot line and rejecting it to the colder line. The two flows then leave at the same temperature. Assume the whole setup is reversible and find the exit temperature and the total power out of the heat engines.

Solution:



C.V. Total setup

$$\text{Energy Eq.6.10: } \dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_1 h_3 + \dot{m}_2 h_4 + \dot{W}_{\text{TOT}}$$

$$\text{Entropy Eq.9.7: } \dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{S}_{\text{gen}} + \int d\dot{Q}/T = \dot{m}_1 s_3 + \dot{m}_2 s_4$$

$$\text{Process: Reversible } \dot{S}_{\text{gen}} = 0 \quad \text{Adiabatic } \dot{Q} = 0$$

Assume the exit flow has the same pressure as the inlet flow then the pressure part of the entropy cancels out and we have

$$\text{Exit same } T, P \Rightarrow h_3 = h_4 = h_e; \quad s_3 = s_4 = s_e$$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_{\text{TOT}} h_e + \dot{W}_{\text{TOT}}$$

$$\dot{m}_1 s_1 + \dot{m}_2 s_2 = \dot{m}_{\text{TOT}} s_e$$

$$s_{\text{Te}}^o = \frac{\dot{m}_1}{\dot{m}_{\text{TOT}}} s_{\text{T1}}^o + \frac{\dot{m}_2}{\dot{m}_{\text{TOT}}} s_{\text{T2}}^o = \frac{1}{3} \times 7.1593 + \frac{2}{3} \times 6.8352 = 6.9432 \text{ kJ/kgK}$$

$$\text{Table A.7: } \Rightarrow T_e \cong 323 \text{ K}; \quad h_e = 323.6 \text{ kJ/kg}$$

$$\begin{aligned} \dot{W}_{\text{TOT}} &= \dot{m}_1 (h_1 - h_e) + \dot{m}_2 (h_2 - h_e) \\ &= 1(401.3 - 323.6) + 2(290.43 - 323.6) = \mathbf{11.36 \text{ kW}} \end{aligned}$$

Note: The solution using constant heat capacity writes the entropy equation using Eq.8.16, the pressure terms cancel out so we get

$$\frac{1}{3} C_p \ln(T_e/T_1) + \frac{2}{3} C_p \ln(T_e/T_2) = 0 \quad \Rightarrow \ln T_e = (\ln T_1 + 2 \ln T_2)/3$$

## 9.42

A flow of 5 kg/s water at 100 kPa, 20°C should be delivered as steam at 1000 kPa, 350°C to some application. We have a heat source at constant 500°C. If the process should be reversible how much heat transfer should we have?

CV Around unknown device out to the source surface.

$$\text{Energy Eq.: } \dot{m}h_i + \dot{Q} = \dot{m}h_e + \dot{W}$$

$$\text{Entropy Eq.: } \dot{m}s_i + \dot{Q}/T_S + 0 = \dot{m}s_e \quad (T_S \text{ is constant, } s_{\text{gen}} = 0)$$

$$\text{Inlet state: } s_i = 0.2966 \text{ kJ/kgK, Table B.1.1}$$

$$\text{Exit state: } s_e = 7.301 \text{ kJ/kgK, Table B.1.3}$$

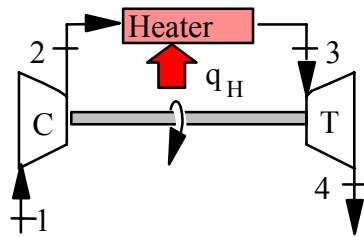
$$\dot{Q} = \dot{m} T_S (s_e - s_i) = 5 \cdot 773.15 (7.301 - 0.2966) = \mathbf{27.1 \text{ MW}}$$

The theory does not say exactly how to do it. As the pressure goes up we must have a pump or compressor and since the substance temperature is lower than the source temperature a reversible heat transfer must happen through some kind of heat engine receiving a  $Q$  from the source and delivering it to the flow extracting some work in the process.

## 9.43

A heat-powered portable air compressor consists of three components: (a) an adiabatic compressor; (b) a constant pressure heater (heat supplied from an outside source); and (c) an adiabatic turbine. Ambient air enters the compressor at 100 kPa, 300 K, and is compressed to 600 kPa. All of the power from the turbine goes into the compressor, and the turbine exhaust is the supply of compressed air. If this pressure is required to be 200 kPa, what must the temperature be at the exit of the heater?

Solution:



$$P_2 = 600 \text{ kPa}, \quad P_4 = 200 \text{ kPa}$$

Adiabatic and reversible compressor:

$$\text{Process: } q = 0 \quad \text{and} \quad s_{\text{gen}} = 0$$

$$\text{Energy Eq. 6.13: } h - w_c = h_2$$

$$\text{Entropy Eq. 9.8: } s_2 = s_1$$

For constant specific heat the isentropic relation becomes Eq. 8.32

$$T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = 300(6)^{0.2857} = 500.8 \text{ K}$$

$$-w_c = C_{P0}(T_2 - T_1) = 1.004(500.8 - 300) = 201.5 \text{ kJ/kg}$$

Adiabatic and reversible turbine:  $q = 0$  and  $s_{\text{gen}} = 0$

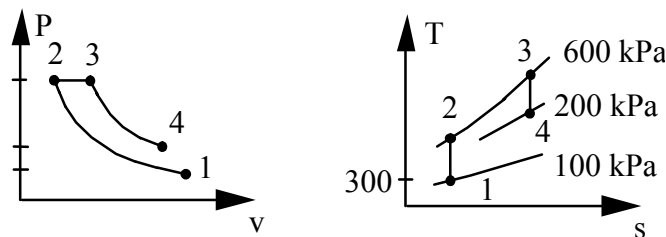
$$\text{Energy Eq. 6.13: } h_3 = w_T + h_4; \quad \text{Entropy Eq. 9.8: } s_4 = s_3$$

For constant specific heat the isentropic relation becomes Eq. 8.32

$$T_4 = T_3 \left( \frac{P_4}{P_3} \right)^{\frac{k-1}{k}} = T_3 (200/600)^{0.2857} = 0.7304 T_3$$

$$\text{Energy Eq. for shaft: } -w_c = w_T = C_{P0}(T_3 - T_4)$$

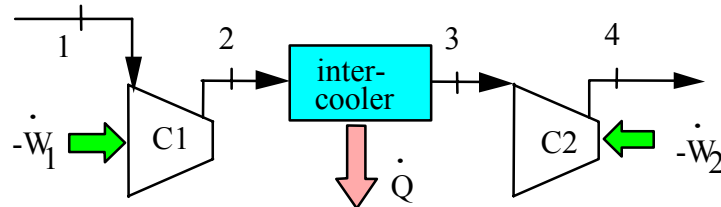
$$201.5 = 1.004 T_3 (1 - 0.7304) \Rightarrow T_3 = \mathbf{744.4 \text{ K}}$$



## 9.44

A two-stage compressor having an interstage cooler takes in air, 300 K, 100 kPa, and compresses it to 2 MPa, as shown in Fig. P9.44. The cooler then cools the air to 340 K, after which it enters the second stage, which has an exit pressure of 15.74 MPa. Both stages are adiabatic, and reversible. Find  $q$  in the cooler, total specific work, and compare this to the work required with no intercooler.

Solution:



C.V.: Stage 1 air, Steady flow

Process: adiabatic:  $q = 0$ , reversible:  $s_{\text{gen}} = 0$

Energy Eq.6.13:  $-w_{C1} = h_2 - h_1$ , Entropy Eq.9.8:  $s_2 = s_1$

Assume constant  $C_{p0} = 1.004$  from A.5 and isentropic leads to Eq.8.32

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 300(2000/100)^{0.286} = 706.7 \text{ K}$$

$$w_{C1} = h_1 - h_2 = C_{p0}(T_1 - T_2) = 1.004(300 - 706.7) = \mathbf{-408.3 \text{ kJ/kg}}$$

C.V. Intercooler, no work and no changes in kinetic or potential energy.

$$q_{23} = h_3 - h_2 = C_{p0}(T_3 - T_2) = 1.004(340 - 706.7) = \mathbf{-368.2 \text{ kJ/kg}}$$

C.V. Stage 2. Analysis the same as stage 1. So from Eq.8.32

$$T_4 = T_3(P_4/P_3)^{\frac{k-1}{k}} = 340(15.74/2)^{0.286} = 613.4 \text{ K}$$

$$w_{C2} = h_3 - h_4 = C_{p0}(T_3 - T_4) = 1.004(340 - 613.4) = \mathbf{-274.5 \text{ kJ/kg}}$$

Same flow rate through both stages so the total work is the sum of the two

$$w_{\text{comp}} = w_{C1} + w_{C2} = -408.3 - 274.5 = \mathbf{-682.8 \text{ kJ/kg}}$$

For no intercooler ( $P_2 = 15.74$  MPa) same analysis as stage 1. So Eq.8.32

$$T_2 = 300(15740/100)^{0.286} = 1274.9 \text{ K}$$

$$w_{\text{comp}} = 1.004(300 - 1274.9) = \mathbf{-978.8 \text{ kJ/kg}}$$





## 9.46

A certain industrial process requires a steady 0.5 kg/s supply of compressed air at 500 kPa, at a maximum temperature of 30°C. This air is to be supplied by installing a compressor and aftercooler. Local ambient conditions are 100 kPa, 20°C. Using an reversible compressor, determine the power required to drive the compressor and the rate of heat rejection in the aftercooler.

Solution:

Air Table A.5:  $R = 0.287 \text{ kJ/kg-K}$ ,  $C_p = 1.004 \text{ kJ/kg K}$ ,  $k = 1.4$

State 1:  $T_1 = T_0 = 20^\circ\text{C}$ ,  $P_1 = P_0 = 100 \text{ kPa}$ ,  $\dot{m} = 0.5 \text{ kg/s}$

State 2:  $P_2 = P_3 = 500 \text{ kPa}$

State 3:  $T_3 = 30^\circ\text{C}$ ,  $P_3 = 500 \text{ kPa}$

Compressor: Assume Isentropic (adiabatic  $q = 0$  and reversible  $s_{\text{gen}} = 0$ ) From entropy equation Eq.9.8 this gives constant  $s$  which is expressed for an ideal gas in Eq.8.32

$$T_2 = T_1 (P_2/P_1)^{\frac{k-1}{k}} = 293.15 (500/100)^{0.2857} = 464.6 \text{ K}$$

Energy Eq.6.13:  $q_c + h_1 = h_2 + w_c$ ;  $q_c = 0$ ,

assume constant specific heat from Table A.5

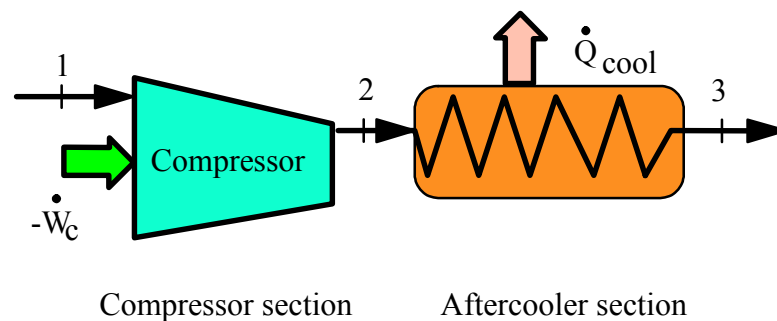
$$w_c = C_p(T_1 - T_2) = 1.004 (293.15 - 464.6) = -172.0 \text{ kJ/kg}$$

$$\dot{W}_C = \dot{m}w_c = \mathbf{-86 \text{ kW}}$$

Aftercooler Energy Eq.6.13:  $q + h_2 = h_3 + w$ ;  $w = 0$ ,

assume constant specific heat

$$\dot{Q} = \dot{m}q = \dot{m}C_p(T_3 - T_2) = 0.5 \times 1.004(303.15 - 464.6) = \mathbf{-81 \text{ kW}}$$

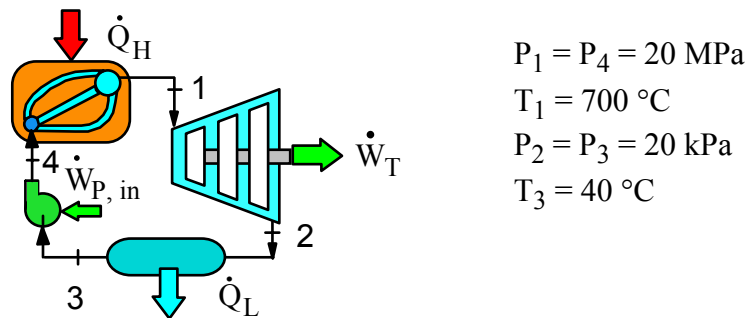


## 9.47

Consider a steam turbine power plant operating near critical pressure, as shown in Fig. P9.33. As a first approximation, it may be assumed that the turbine and the pump processes are reversible and adiabatic. Neglecting any changes in kinetic and potential energies, calculate

- The specific turbine work output and the turbine exit state
- The pump work input and enthalpy at the pump exit state
- The thermal efficiency of the cycle

Solution:



- a) State 1: (P, T) Table B.1.3  $h_1 = 3809.1 \text{ kJ/kg}$ ,  $s_1 = 6.7993 \text{ kJ/kg K}$

C.V. Turbine.

$$\text{Entropy Eq.9.8: } s_2 = s_1 = 6.7993 \text{ kJ/kg K}$$

$$\text{Table B.1.2 } s_2 = 0.8319 + x_2 \times 7.0766 \Rightarrow x_2 = 0.8433$$

$$h_2 = 251.4 + 0.8433 \times 2358.33 = \mathbf{2240.1}$$

$$\text{Energy Eq.6.13: } w_T = h_1 - h_2 = \mathbf{1569 \text{ kJ/kg}}$$

- b) State 3: (P, T) Compressed liquid, take sat. liq. Table B.1.1

$$h_3 = 167.5 \text{ kJ/kg}, \quad v_3 = 0.001008 \text{ m}^3/\text{kg}$$

Property relation  $v = \text{constant}$  gives work from Eq.9.15 as

$$w_P = -v_3(P_4 - P_3) = -0.001008(20000 - 20) = \mathbf{-20.1 \text{ kJ/kg}}$$

$$h_4 = h_3 - w_P = 167.5 + 20.1 = \mathbf{187.6 \text{ kJ/kg}}$$

- c) The heat transfer in the boiler is from energy Eq.6.13

$$q_{\text{boiler}} = h_1 - h_4 = 3809.1 - 187.6 = 3621.5 \text{ kJ/kg}$$

$$w_{\text{net}} = 1569 - 20.1 = \mathbf{1548.9 \text{ kJ/kg}}$$

$$\eta_{\text{TH}} = w_{\text{net}}/q_{\text{boiler}} = \frac{1548.9}{3621.5} = \mathbf{0.428}$$

## Transient processes

## 9.48

Air in a tank is at 300 kPa, 400 K with a volume of 2 m<sup>3</sup>. A valve on the tank is opened to let some air escape to the ambient to a final pressure inside of 200 kPa. Find the final temperature and mass assuming a reversible adiabatic process for the air remaining inside the tank.

Solution:

C.V. Total tank.

$$\text{Continuity Eq.6.15: } m_2 - m_1 = -m_{\text{ex}}$$

$$\text{Energy Eq.6.16: } m_2 u_2 - m_1 u_1 = -m_{\text{ex}} h_{\text{ex}} + {}_1Q_2 - {}_1W_2$$

$$\text{Entropy Eq.9.12: } m_2 s_2 - m_1 s_1 = -m_{\text{ex}} s_{\text{ex}} + \int dQ/T + {}_1S_2_{\text{gen}}$$

$$\text{Process: } \text{Adiabatic } {}_1Q_2 = 0; \text{ rigid tank } {}_1W_2 = 0$$

This has too many unknowns (we do not know state 2).

C.V.  $m_2$  the mass that remains in the tank. This is a control mass.

$$\text{Energy Eq.5.11: } m_2(u_2 - u_1) = {}_1Q_2 - {}_1W_2$$

$$\text{Entropy Eq.8.14: } m_2(s_2 - s_1) = \int dQ/T + {}_1S_2_{\text{gen}}$$

$$\text{Process: } \text{Adiabatic } {}_1Q_2 = 0; \text{ Reversible } {}_1S_2_{\text{gen}} = 0$$

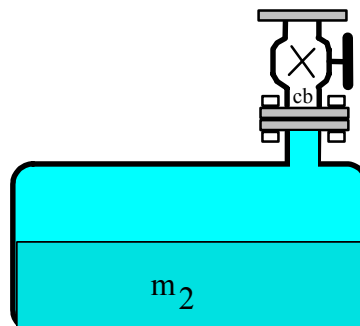
$$\Rightarrow s_2 = s_1$$

Ideal gas and process Eq.8.32

$$T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = 400(200/300)^{0.2857} = \mathbf{356.25 \text{ K}}$$

$$m_2 = \frac{P_2 V}{RT_2} = \frac{200 \times 2}{0.287 \times 356.25} = \mathbf{3.912 \text{ kg}}$$

Notice that the work term is not zero for mass  $m_2$ . The work goes into pushing the mass  $m_{\text{ex}}$  out.



## 9.49

A tank contains 1 kg of carbon dioxide at 6 MPa, 60°C and it is connected to a turbine with an exhaust at 1000 kPa. The carbon dioxide flows out of the tank and through the turbine to a final state in the tank of saturated vapor is reached. If the process is adiabatic and reversible find the final mass in the tank and the turbine work output.

C.V. The tank and turbine. This is a transient problem.

$$\text{Continuity Eq.6.15: } m_2 - m_1 = -m_{\text{ex}}$$

$$\text{Energy Eq.6.16: } m_2 u_2 - m_1 u_1 = -m_{\text{ex}} h_{\text{ex}} + {}_1Q_2 - {}_1W_2$$

$$\text{Entropy Eq.9.12: } m_2 s_2 - m_1 s_1 = -m_{\text{ex}} s_{\text{ex}} + \int dQ/T + {}_1S_2_{\text{gen}}$$

$$\text{Process: } \text{Adiabatic } {}_1Q_2 = 0; \text{ reversible } {}_1S_2_{\text{gen}} = 0$$

$$\text{State 1: } v_1 = 0.00801 \text{ m}^3/\text{kg}, \quad u_1 = 322.51 \text{ kJ/kg}, \quad s_1 = 1.2789 \text{ kJ/kg-K}$$

State 2: Sat. vapor, 1 property missing

C.V.  $m_2$  the mass that remains in the tank. This is a control mass.

$$\text{Process: } \text{Adiabatic } {}_1Q_2 = 0; \text{ Reversible } {}_1S_2_{\text{gen}} = 0$$

$$\begin{aligned} \text{Entropy Eq.8.14: } m_2(s_2 - s_1) &= \int dQ/T + {}_1S_2_{\text{gen}} = 0 + 0 \\ \Rightarrow s_2 &= s_1 \text{ this is the missing property.} \end{aligned}$$

$$\text{State 2: } T_2 = -19.19^\circ\text{C}, \quad v_2 = 0.018851 \text{ m}^3/\text{kg}, \quad u_2 = 285.87 \text{ kJ/kg}$$

State exit:  $s_{\text{ex}} = s_2 = s_1$  follows from entropy Eq. for first C.V. with the use of the continuity equation. Use 1004.5 kPa for  $-40^\circ\text{C}$ .

$$x_{\text{ex}} = (1.2789 - 0)/1.3829 = 0.924796 \Rightarrow h_{\text{ex}} = 298.17 \text{ kJ/kg}$$

Tank volume constant so  $V = m_1 v_1 = m_2 v_2$

$$m_2 = m_1 v_1 / v_2 = 1 \times 0.00801 / 0.018851 = \mathbf{0.4249 \text{ kg}}$$

From energy eq.

$$\begin{aligned} {}_1W_2 &= m_1 u_1 - m_2 u_2 - m_{\text{ex}} h_{\text{ex}} \\ &= 1 \times 322.51 - 0.4249 \times 285.87 - 0.5751 \times 298.17 \text{ [kg kJ/kg]} \\ &= \mathbf{29.57 \text{ kJ}} \end{aligned}$$

## 9.50

An underground salt mine,  $100\,000\text{ m}^3$  in volume, contains air at  $290\text{ K}$ ,  $100\text{ kPa}$ . The mine is used for energy storage so the local power plant pumps it up to  $2.1\text{ MPa}$  using outside air at  $290\text{ K}$ ,  $100\text{ kPa}$ . Assume the pump is ideal and the process is adiabatic. Find the final mass and temperature of the air and the required pump work.

Solution:

C.V. The mine volume and the pump

$$\text{Continuity Eq.6.15: } m_2 - m_1 = m_{\text{in}}$$

$$\text{Energy Eq.6.16: } m_2 u_2 - m_1 u_1 = {}_1Q_2 - {}_1W_2 + m_{\text{in}} h_{\text{in}}$$

$$\text{Entropy Eq.9.12: } m_2 s_2 - m_1 s_1 = \int dQ/T + {}_1S_2_{\text{gen}} + m_{\text{in}} s_{\text{in}}$$

$$\text{Process: Adiabatic } {}_1Q_2 = 0, \text{ Process ideal } {}_1S_2_{\text{gen}} = 0, s_1 = s_{\text{in}}$$

$$\Rightarrow m_2 s_2 = m_1 s_1 + m_{\text{in}} s_{\text{in}} = (m_1 + m_{\text{in}}) s_1 = m_2 s_1 \Rightarrow s_2 = s_1$$

$$\text{Constant } s \Rightarrow \text{Eq.8.28 } s_{T_2}^{\circ} = s_{T_1}^{\circ} + R \ln(P_2 / P_{\text{in}})$$

$$s_{T_2}^{\circ} = 6.83521 + 0.287 \ln(21) = 7.7090 \text{ kJ/kg K}$$

$$\text{A.7 } \Rightarrow T_2 = \mathbf{680\text{ K}}, u_2 = 496.94 \text{ kJ/kg}$$

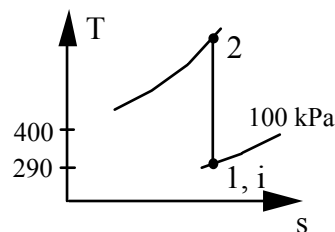
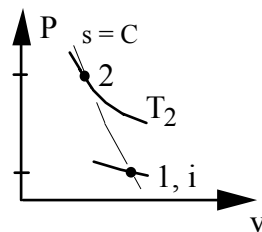
$$m_1 = P_1 V_1 / RT_1 = 100 \times 10^5 / (0.287 \times 290) = 1.20149 \times 10^5 \text{ kg}$$

$$m_2 = P_2 V_2 / RT_2 = 100 \times 21 \times 10^5 / (0.287 \times 680) = \mathbf{10.760 \times 10^5 \text{ kg}}$$

$$\Rightarrow m_{\text{in}} = 9.5585 \times 10^5 \text{ kg}$$

$${}_1W_2 = m_{\text{in}} h_{\text{in}} + m_1 u_1 - m_2 u_2$$

$$= m_{\text{in}}(290.43) + m_1(207.19) - m_2(496.94) = \mathbf{-2.322 \times 10^8 \text{ kJ}}$$



## 9.51

Air in a tank is at 300 kPa, 400 K with a volume of 2 m<sup>3</sup>. A valve on the tank is opened to let some air escape to the ambient to a final pressure inside of 200 kPa. At the same time the tank is heated so the air remaining has a constant temperature. What is the mass average value of the  $s$  leaving assuming this is an internally reversible process?

Solution:

C.V. Tank, emptying process with heat transfer.

$$\text{Continuity Eq.6.15:} \quad m_2 - m_1 = -m_e$$

$$\text{Energy Eq.6.16:} \quad m_2 u_2 - m_1 u_1 = -m_e h_e + {}_1Q_2$$

$$\text{Entropy Eq.9.12:} \quad m_2 s_2 - m_1 s_1 = -m_e s_e + {}_1Q_2/T + 0$$

$$\text{Process:} \quad T_2 = T_1 \quad \Rightarrow \quad {}_1Q_2 \text{ in at 400 K}$$

$$\text{Reversible} \quad {}_1S_2 \text{ gen} = 0$$

$$\text{State 1: Ideal gas} \quad m_1 = P_1 V / RT_1 = 300 \times 2 / 0.287 \times 400 = 5.2265 \text{ kg}$$

State 2: 200 kPa, 400 K

$$m_2 = P_2 V / RT_2 = 200 \times 2 / 0.287 \times 400 = 3.4843 \text{ kg}$$

$$\Rightarrow m_e = 1.7422 \text{ kg}$$

From the energy equation:

$$\begin{aligned} {}_1Q_2 &= m_2 u_2 - m_1 u_1 + m_e h_e \\ &= 3.4843 \times 286.49 - 5.2265 \times 286.49 + 1.7422 \times 401.3 \\ &= 1.7422(401.3 - 286.49) = 200 \text{ kJ} \end{aligned}$$

$$\begin{aligned} m_e s_e &= m_1 s_1 - m_2 s_2 + {}_1Q_2/T \\ &= 5.2265[7.15926 - 0.287 \ln(300/100)] - 3.4843[7.15926 \\ &\quad - 0.287 \ln(200/100)] + (200/400) \end{aligned}$$

$$m_e s_e = 35.770 - 24.252 + 0.5 = 12.018 \text{ kJ/K}$$

$$s_e = 12.018 / 1.7422 = 6.89817 = \mathbf{6.8982 \text{ kJ/kg K}}$$

Note that the exit state  $e$  in this process is for the air before it is throttled across the discharge valve. The throttling process from the tank pressure to ambient pressure is a highly irreversible process.



## 9.52

An insulated 2 m<sup>3</sup> tank is to be charged with R-134a from a line flowing the refrigerant at 3 MPa. The tank is initially evacuated, and the valve is closed when the pressure inside the tank reaches 3 MPa. The line is supplied by an insulated compressor that takes in R-134a at 5°C, quality of 96.5 %, and compresses it to 3 MPa in a reversible process. Calculate the total work input to the compressor to charge the tank.

Solution:

C.V.: Compressor, R-134a. Steady 1 inlet and 1 exit flow, no heat transfer.

$$1^{\text{st}} \text{ Law Eq.6.13: } q_c + h_1 = h_2 + w_c$$

$$\text{Entropy Eq.9.8: } s_1 + \int dq/T + s_{\text{gen}} = s_1 + 0 = s_2$$

inlet:  $T_1 = 5^\circ\text{C}$ ,  $x_1 = 0.965$  use Table B.5.1

$$s_1 = s_f + x_1 s_{\text{fg}} = 1.0243 + 0.965 \times 0.6995 = 1.6993 \text{ kJ/kg K,}$$

$$h_1 = h_f + x_1 h_{\text{fg}} = 206.8 + 0.965 \times 194.6 = 394.6 \text{ kJ/kg}$$

exit:  $P_2 = 3 \text{ MPa}$

From the entropy eq.:  $s_2 = s_1 = 1.6993 \text{ kJ/kg K}$ ;

$$T_2 = 90^\circ\text{C}, \quad h_2 = 436.2 \text{ kJ/kg}$$

$$w_c = h_1 - h_2 = -41.6 \text{ kJ/kg}$$

C.V.: Tank;  $V_T = 2 \text{ m}^3$ ,  $P_T = 3 \text{ MPa}$

$$1^{\text{st}} \text{ Law Eq.6.16: } Q + m_i h_i = m_2 u_2 - m_1 u_1 + m_e h_e + W;$$

Process and states have:  $Q = 0$ ,  $W = 0$ ,  $m_e = 0$ ,  $m_1 = 0$ ,  $m_2 = m_i$

$$u_2 = h_i = 436.2 \text{ kJ/kg}$$

Final state:  $P_T = 3 \text{ MPa}$ ,  $u_2 = 436.2 \text{ kJ/kg}$

$$\rightarrow T_T = 101.9^\circ\text{C}, \quad v_T = 0.006783 \text{ m}^3/\text{kg}$$

$$m_T = V_T/v_T = 294.84 \text{ kg};$$

The work term is from the specific compressor work and the total mass

$$-W_c = m_T(-w_c) = \mathbf{12\,295 \text{ kJ}}$$

## 9.53

R-410a at 120°C, 4 MPa is in an insulated tank and flow is now allowed out to a turbine with a backup pressure of 800 kPa. The flow continues to a final tank pressure of 800 kPa and the process stops. If the initial mass was 1 kg how much mass is left in the tank and what is the turbine work assuming a reversible process?

Solution:

C.V. Total tank and turbine.

$$\text{Continuity Eq.6.15:} \quad m_2 - m_1 = -m_{\text{ex}}$$

$$\text{Energy Eq.6.16:} \quad m_2 u_2 - m_1 u_1 = -m_{\text{ex}} h_{\text{ex}} + {}_1Q_2 - {}_1W_2$$

$$\text{Entropy Eq.9.12:} \quad m_2 s_2 - m_1 s_1 = -m_{\text{ex}} s_{\text{ex}} + \int dQ/T + {}_1S_2_{\text{gen}}$$

$$\text{Process:} \quad \text{Adiabatic } {}_1Q_2 = 0; \quad \text{Reversible } {}_1S_2_{\text{gen}} = 0$$

This has too many unknowns (we do not know state 2 only  $P_2$ ).

C.V.  $m_2$  the mass that remains in the tank. This is a control mass.

$$\text{Entropy Eq.8.14:} \quad m_2(s_2 - s_1) = \int dQ/T + {}_1S_2_{\text{gen}}$$

$$\text{Process:} \quad \text{Adiabatic } {}_1Q_2 = 0; \quad \text{Reversible } {}_1S_2_{\text{gen}} = 0$$

$$\Rightarrow \quad s_2 = s_1$$

$$\text{State 1:} \quad v_1 = 0.00897 \text{ m}^3/\text{kg}, \quad u_1 = 331.39 \text{ kJ/kg}, \quad s_1 = 1.1529 \text{ kJ/kg-K}$$

$$\text{State 2 (P,s):} \quad T_2 = 33.23^\circ\text{C}, \quad v_2 = 0.37182 \text{ m}^3/\text{kg}, \quad u_2 = 281.29 \text{ kJ/kg}$$

State exit:  $s_{\text{ex}} = s_2 = s_1$  follows from entropy Eq. for first C.V. using the continuity eq., this is identical to state 2,  $h_{\text{ex}} = 312.85 \text{ kJ/kg}$

$$\text{Tank volume constant so } V = m_1 v_1 = m_2 v_2$$

$$m_2 = m_1 v_1 / v_2 = 1 \times 0.00897 / 0.37182 = \mathbf{0.0241 \text{ kg}}$$

From energy eq.

$$\begin{aligned} {}_1W_2 &= m_1 u_1 - m_2 u_2 - m_{\text{ex}} h_{\text{ex}} \\ &= 1 \times 331.39 - 0.0241 \times 281.29 - 0.9759 \times 312.85 \text{ [kJ/kg]} \\ &= \mathbf{19.3 \text{ kJ}} \end{aligned}$$

## **Reversible shaft work, Bernoulli equation**

## 9.54

A pump has a 2 kW motor. How much liquid water at 15°C can I pump to 250 kPa from 100 kPa?

Incompressible flow (liquid water) and we assume reversible. Then the shaftwork is from Eq.9.15

$$w = -\int v \, dP = -v \Delta P = -0.001 \, \text{m}^3/\text{kg} (250 - 100) \, \text{kPa} \\ = -0.15 \, \text{kJ/kg}$$

$$\dot{m} = \frac{\dot{W}}{-w} = \frac{2}{0.15} = \mathbf{13.3 \, \text{kg/s}}$$



## 9.55

A large storage tank contains saturated liquid nitrogen at ambient pressure, 100 kPa; it is to be pumped to 500 kPa and fed to a pipeline at the rate of 0.5 kg/s. How much power input is required for the pump, assuming it to be reversible?

Solution:

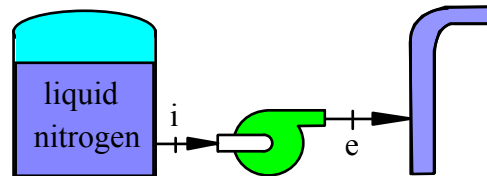
C.V. Pump, liquid is assumed to be incompressible.

Table B.6.1 at  $P_i = 101.3 \text{ kPa}$ ,  $v_{Fi} = 0.00124 \text{ m}^3/\text{kg}$

Eq.9.15

$$w_{\text{PUMP}} = -w_{\text{cv}} = \int v dP \approx v_{Fi}(P_e - P_i)$$

$$= 0.00124(500 - 101) = 0.494 \text{ kJ/kg}$$



$$\dot{W}_{\text{PUMP}} = \dot{m}w_{\text{PUMP}} = 0.5 \text{ kg/s} (0.494 \text{ kJ/kg}) = \mathbf{0.247 \text{ kW}}$$

## 9.56

A garden water hose has liquid water at 200 kPa, 15°C. How high a velocity can be generated in a small ideal nozzle? If you direct the water spray straight up how high will it go?

Solution:

Liquid water is incompressible and we will assume process is reversible.

$$\text{Bernoulli's Eq. across the nozzle Eq.9.16:} \quad v\Delta P = \Delta\left(\frac{1}{2} \mathbf{V}^2\right)$$

$$\mathbf{V} = \sqrt{2v\Delta P} = \sqrt{2 \times 0.001001 \times (200 - 101) \times 1000} = \mathbf{14.08 \text{ m/s}}$$

$$\text{Bernoulli's Eq.9.16 for the column:} \quad \Delta\left(\frac{1}{2} \mathbf{V}^2\right) = \Delta gZ$$

$$\Delta Z = \Delta\left(\frac{1}{2} \mathbf{V}^2\right)/g = v\Delta P/g = 0.001001 \times (200 - 101) \times 1000/9.807 = \mathbf{10.1 \text{ m}}$$



## 9.57

A small pump takes in water at 20°C, 100 kPa and pumps it to 2.5 MPa at a flow rate of 100 kg/min. Find the required pump power input.

Solution:

C.V. Pump. Assume reversible pump and incompressible flow.

With single steady state flow it leads to the work in Eq.9.15

$$w_p = -\int v dP = -v_i(P_e - P_i) = -0.001002(2500 - 100) = -2.4 \text{ kJ/kg}$$

$$\dot{W}_p = \dot{m}w_p = \frac{100 \text{ kg/min}}{60 \text{ sec/min}} (-2.4 \text{ kJ/kg}) = \mathbf{-4.0 \text{ kW}}$$

## 9.58

An irrigation pump takes water from a river at 10°C, 100 kPa and pumps it up to an open canal at a 100 m higher elevation. The pipe diameter in and out of the pump is 0.1 m and the motor driving the pump is 5 hp. Neglect kinetic energies and friction, find the maximum possible mass flow rate.

CV the pump. The flow is incompressible and steady flow. The pump work is the difference between the flow work in and out and from Bernoulli's eq. for the pipe that is equal to the potential energy increase since pump inlet pressure and pipe outlet pressure are the same.

$$w_p = v \Delta P = g \Delta Z = 9.81 \times 100 \text{ J/kg} = 0.981 \text{ kJ/kg}$$

The horsepower is converted from Table A.1

$$\dot{W}_{\text{motor}} = 5 \text{ hp} = 5 \times 0.746 = 3.73 \text{ kW}$$

$$\dot{m} = \dot{W}_{\text{motor}} / w_p = 3.73 / 0.981 = \mathbf{3.8 \text{ kg/s}}$$

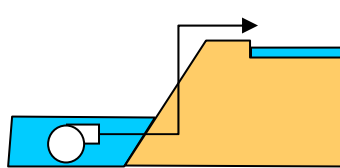
Comment:

$$\dot{m} = AV/v \quad \Rightarrow \quad v = \frac{\dot{m}v}{A} = \frac{4\dot{m}}{\rho \pi D^2} = \frac{4 \times 3.8}{997 \times \pi \times 0.1^2} = 0.485 \text{ m/s}$$

The power to generated the kinetic energy is

$$\text{Power} = \dot{m} 0.5 v^2 = 3.8 \times 0.5 \times 0.485^2 = 0.447 \text{ W}$$

This is insignificant relative to the power needed for the potential energy increase.



Pump inlet and the pipe exit both have close to atmospheric pressure.



## 9.59

Saturated R-134a at  $-10^\circ\text{C}$  is pumped/compressed to a pressure of 1.0 MPa at the rate of 0.5 kg/s in a reversible adiabatic process. Calculate the power required and the exit temperature for the two cases of inlet state of the R-134a:

- quality of 100 %.
- quality of 0 %.

Solution:

C.V.: Pump/Compressor,  $\dot{m} = 0.5 \text{ kg/s}$ , R-134a

- a) State 1: Table B.5.1,  $T_1 = -10^\circ\text{C}$ ,  $x_1 = 1.0$  Saturated vapor

$$P_1 = P_g = 202 \text{ kPa}, \quad h_1 = h_g = 392.3 \text{ kJ/kg}, \quad s_1 = s_g = 1.7319 \text{ kJ/kg-K}$$

Assume Compressor is isentropic,  $s_2 = s_1 = 1.7319 \text{ kJ/kg-K}$

$$h_2 = 425.7 \text{ kJ/kg}, \quad T_2 = 45^\circ\text{C}$$

$$\text{Energy Eq.6.13: } q_c + h_1 = h_2 + w_c; \quad q_c = 0$$

$$w_{cs} = h_1 - h_2 = -33.4 \text{ kJ/kg}; \quad \Rightarrow \quad \dot{W}_C = \dot{m}w_C = \mathbf{-16.7 \text{ kW}}$$

- b) State 1:  $T_1 = -10^\circ\text{C}$ ,  $x_1 = 0$  Saturated liquid. This is a pump.

$$P_1 = 202 \text{ kPa}, \quad h_1 = h_f = 186.72 \text{ kJ/kg}, \quad v_1 = v_f = 0.000755 \text{ m}^3/\text{kg}$$

$$\text{Energy Eq.6.13: } q_p + h_1 = h_2 + w_p; \quad q_p = 0$$

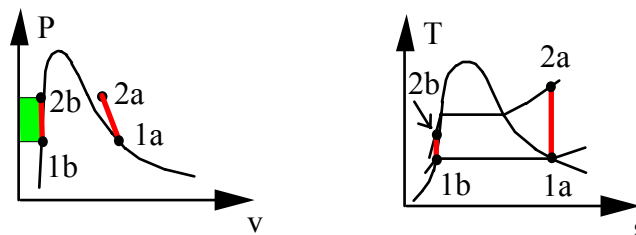
Assume Pump is isentropic and the liquid is incompressible, Eq.9.15:

$$w_{ps} = - \int v \, dP = -v_1(P_2 - P_1) = -0.6 \text{ kJ/kg}$$

$$h_2 = h_1 - w_p = 186.72 - (-0.6) = 187.3 \text{ kJ/kg}, \quad P_2 = 1 \text{ MPa}$$

Assume State 2 is approximately a saturated liquid  $\Rightarrow T_2 \cong \mathbf{-9.6^\circ\text{C}}$

$$\dot{W}_P = \dot{m}w_P = \mathbf{-0.3 \text{ kW}}$$



## 9.60

Liquid water at ambient conditions, 100 kPa, 25°C, enters a pump at the rate of 0.5 kg/s. Power input to the pump is 3 kW. Assuming the pump process to be reversible, determine the pump exit pressure and temperature.

Solution:

C.V. Pump. Steady single inlet and exit flow with no heat transfer.

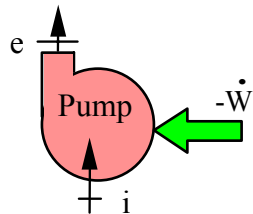
$$\text{Energy Eq.6.13: } w = h_i - h_e = \dot{W}/\dot{m} = -3/0.5 = -6.0 \text{ kJ/kg}$$

Using also incompressible media we can use Eq.9.15

$$w = -\int v dP \approx -v_i(P_e - P_i) = -0.001003(P_e - 100)$$

from which we can solve for the exit pressure

$$P_e = 100 + 6.0/0.001003 = 6082 \text{ kPa} = \mathbf{6.082 \text{ MPa}}$$



$$-\dot{W} = 3 \text{ kW}, \quad P_i = 100 \text{ kPa}$$

$$T_i = 25^\circ\text{C}, \quad \dot{m} = 0.5 \text{ kg/s}$$

$$\text{Energy Eq.: } h_e = h_i - w = 104.87 - (-6) = 110.87 \text{ kJ/kg}$$

$$\text{Use Table B.1.4 at 5 MPa} \Rightarrow \mathbf{T_e = 25.3^\circ\text{C}}$$

**Remark:**

$$\text{If we use the software we get: } \left. \begin{array}{l} s_i = 0.36736 = s_e \\ \text{At } s_e \text{ \& } P_e \end{array} \right\} \rightarrow T_e = \mathbf{25.1^\circ\text{C}}$$

## 9.61

A small water pump on ground level has an inlet pipe down into a well at a depth  $H$  with the water at 100 kPa, 15°C. The pump delivers water at 400 kPa to a building. The absolute pressure of the water must be at least twice the saturation pressure to avoid cavitation. What is the maximum depth this setup will allow?

Solution:

C.V. Pipe in well, no work, no heat transfer

From Table B.1.1

$$P_{\text{inlet pump}} \geq 2 P_{\text{sat}, 15^\circ\text{C}} = 2 \times 1.705 = 3.41 \text{ kPa}$$

Process:

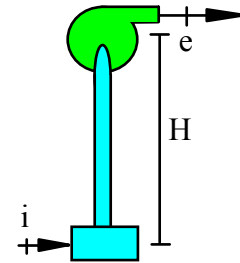
Assume  $\Delta KE \approx 0$ ,  $v \approx \text{constant}$ .  $\Rightarrow$

Bernoulli Eq.9.16:

$$v \Delta P + g H = 0 \Rightarrow$$

$$1000 \times 0.001001 (3.41 - 100) + 9.80665 \times H = 0$$

$$\Rightarrow \mathbf{H = 9.86 \text{ m}}$$



Since flow has some kinetic energy and there are losses in the pipe the height is overestimated. Also the start transient would generate a very low inlet pressure (it moves flow by suction)



This pump can bring water up about 7 m by suction.

## 9.62

A small dam has a pipe carrying liquid water at 150 kPa, 20°C with a flow rate of 2000 kg/s in a 0.5 m diameter pipe. The pipe runs to the bottom of the dam 15 m lower into a turbine with pipe diameter 0.35 m. Assume no friction or heat transfer in the pipe and find the pressure of the turbine inlet. If the turbine exhausts to 100 kPa with negligible kinetic energy what is the rate of work?

Solution:

C.V. Pipe. Steady flow no work, no heat transfer.

States: compressed liquid B.1.1  $v_2 \approx v_1 \approx v_f = 0.001002 \text{ m}^3/\text{kg}$

Continuity Eq.6.3:  $\dot{m} = \rho AV = AV/v$

$$V_1 = \dot{m}v_1/A_1 = 2000 \times 0.001002 / \left(\frac{\pi}{4}0.5^2\right) = 10.2 \text{ m s}^{-1}$$

$$V_2 = \dot{m}v_2/A_2 = 2000 \times 0.001002 / \left(\frac{\pi}{4}0.35^2\right) = 20.83 \text{ m s}^{-1}$$

From Bernoulli Eq.9.16 for the pipe (incompressible substance):

$$v(P_2 - P_1) + \frac{1}{2}(V_2^2 - V_1^2) + g(Z_2 - Z_1) = 0 \Rightarrow$$

$$P_2 = P_1 + \left[\frac{1}{2}(V_1^2 - V_2^2) + g(Z_1 - Z_2)\right]/v$$

$$= 150 + \left[\frac{1}{2} \times 10.2^2 - \frac{1}{2} \times 20.83^2 + 9.80665 \times 15\right] / (1000 \times 0.001002)$$

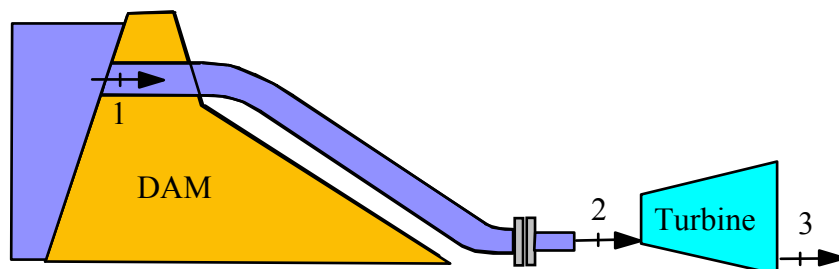
$$= 150 - 17.8 = \mathbf{132.2 \text{ kPa}}$$

Note that the pressure at the bottom should be higher due to the elevation difference but lower due to the acceleration. Now apply the energy equation Eq.9.13 for the total control volume

$$w = -\int v dP + \frac{1}{2}(V_1^2 - V_3^2) + g(Z_1 - Z_3)$$

$$= -0.001002(100 - 150) + \left[\frac{1}{2} \times 10.2^2 + 9.80665 \times 15\right] / 1000 = 0.25 \text{ kJ/kg}$$

$$\dot{W} = \dot{m}w = 2000 \times 0.25 = \mathbf{500 \text{ kW}}$$

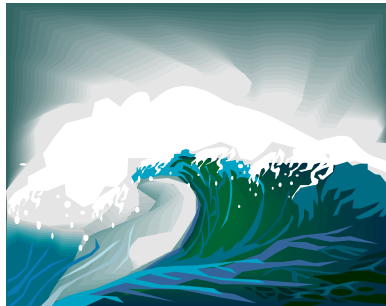


## 9.63

A wave comes rolling in to the beach at 2 m/s horizontal velocity. Neglect friction and find how high up (elevation) on the beach the wave will reach.

We will assume a steady reversible single flow at constant pressure and temperature for the incompressible liquid water. The water will flow in and up the sloped beach until it stops ( $\mathbf{V} = 0$ ) so Bernoulli Eq.9.16 leads to

$$gz_{\text{in}} + \frac{1}{2}\mathbf{V}_{\text{in}}^2 = gz_{\text{ex}} + 0$$
$$(z_{\text{ex}} - z_{\text{in}}) = \frac{1}{2g}\mathbf{V}_{\text{in}}^2 = \frac{1}{2 \times 9.807 \text{ m/s}^2} 2^2 (\text{m/s})^2 = \mathbf{0.204 \text{ m}}$$



## 9.64

A firefighter on a ladder 25 m above ground should be able to spray water an additional 10 m up with the hose nozzle of exit diameter 2.5 cm. Assume a water pump on the ground and a reversible flow (hose, nozzle included) and find the minimum required power.

Solution:

C.V.: pump + hose + water column, total height difference 35 m. Here  $\mathbf{V}$  is velocity, not volume.

$$\text{Continuity Eq.6.3, 6.11:} \quad \dot{m}_{\text{in}} = \dot{m}_{\text{ex}} = (\rho A \mathbf{V})_{\text{nozzle}}$$

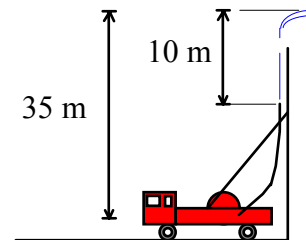
$$\text{Energy Eq.6.12:} \quad \dot{m}(-w_p) + \dot{m}(h + \mathbf{V}^2/2 + gz)_{\text{in}} = \dot{m}(h + \mathbf{V}^2/2 + gz)_{\text{ex}}$$

$$\text{Process:} \quad h_{\text{in}} \cong h_{\text{ex}}, \quad \mathbf{V}_{\text{in}} \cong \mathbf{V}_{\text{ex}} = 0, \quad z_{\text{ex}} - z_{\text{in}} = 35 \text{ m}, \quad \rho = 1/v \cong 1/v_f$$

$$-w_p = g(z_{\text{ex}} - z_{\text{in}}) = 9.81 \times (35 - 0) = 343.2 \text{ J/kg}$$

The velocity in the exit nozzle is such that it can rise 10 m. Make that column a C.V. for which Bernoulli Eq.9.16 is:

$$\begin{aligned} gz_{\text{noz}} + \frac{1}{2} \mathbf{V}_{\text{noz}}^2 &= gz_{\text{ex}} + 0 \\ \mathbf{V}_{\text{noz}} &= \sqrt{2g(z_{\text{ex}} - z_{\text{noz}})} \\ &= \sqrt{2 \times 9.81 \times 10} = 14 \text{ m/s} \end{aligned}$$



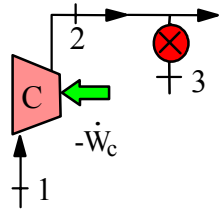
$$\dot{m} = \frac{\pi (D)^2}{4} \mathbf{V}_{\text{noz}} = (\pi/4) 0.025^2 \times 14 / 0.001 = 6.873 \text{ kg/s}$$

$$-\dot{W}_p = -\dot{m}w_p = 6.873 \text{ kg/s} \times 343.2 \text{ J/kg} = \mathbf{2.36 \text{ kW}}$$

## 9.65

A pump/compressor pumps a substance from 100 kPa, 10°C to 1 MPa in a reversible adiabatic process. The exit pipe has a small crack, so that a small amount leaks to the atmosphere at 100 kPa. If the substance is (a) water, (b) R-134a, find the temperature after compression and the temperature of the leak flow as it enters the atmosphere neglecting kinetic energies.

Solution:



C.V.: Compressor, reversible adiabatic

$$\text{Eq. 6.13: } h_1 - w_c = h_2; \quad \text{Eq. 9.8: } s_1 = s_2$$

$$\text{State 2: } P_2, \quad s_2 = s_1$$

C.V.: Crack (Steady throttling process)

$$\text{Eq. 6.13: } h_3 = h_2; \quad \text{Eq. 9.8: } s_3 = s_2 + s_{\text{gen}}$$

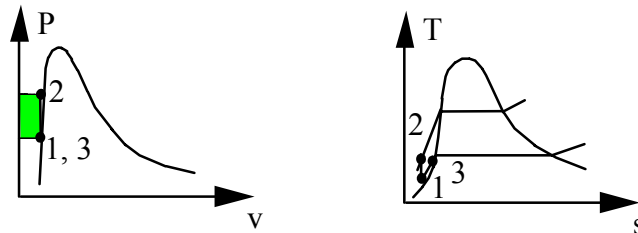
$$\text{State 3: } P_3, \quad h_3 = h_2$$

a) Water 1: compressed liquid, Table B.1.1

$$-w_c = + \int v dP = v_{f1}(P_2 - P_1) = 0.001 \times (1000 - 100) = 0.9 \text{ kJ/kg}$$

$$h_2 = h_1 - w_c = 41.99 + 0.9 = 42.89 \text{ kJ/kg} \Rightarrow T_2 = 10.2^\circ\text{C}$$

$$P_3, h_3 = h_2 \Rightarrow \text{compressed liquid at } \sim 10.2^\circ\text{C}$$



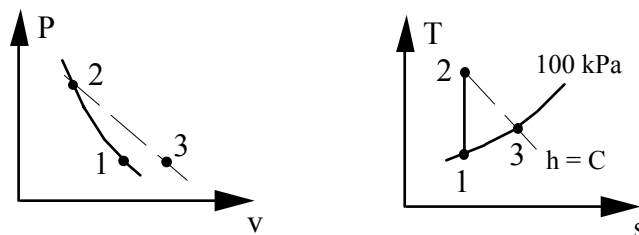
States 1 and 3 are at the same 100 kPa, and same  $v$ . You cannot separate them in the P-v fig.

b) R-134a 1: superheated vapor, Table B.5.2,  $s_1 = 1.8578 \text{ kJ/kg K}$

$$s_2 = s_1 \text{ \& } P_2 \Rightarrow T_2 = 85.3^\circ\text{C}, \quad h_2 = 468.19 \text{ kJ/kg}$$

$$-w_c = h_2 - h_1 = 468.19 - 411.67 = 56.52 \text{ kJ/kg}$$

$$P_3, h_3 = h_2 \Rightarrow T_3 = 74.4^\circ\text{C}$$



## 9.66

A small pump is driven by a 2 kW motor with liquid water at 150 kPa, 10°C entering. Find the maximum water flow rate you can get with an exit pressure of 1 MPa and negligible kinetic energies. The exit flow goes through a small hole in a spray nozzle out to the atmosphere at 100 kPa. Find the spray velocity.

Solution:

C.V. Pump. Liquid water is incompressible so work from Eq.9.15

$$\dot{W} = \dot{m}w = -\dot{m}v(P_e - P_i) \Rightarrow$$

$$\dot{m} = \dot{W} / [-v(P_e - P_i)] = -2 / [-0.001003 (1000 - 150)] = 2.35 \text{ kg/s}$$

C.V. Nozzle. No work, no heat transfer,  $v \approx \text{constant} \Rightarrow$  Bernoulli Eq.9.16

$$\frac{1}{2} \mathbf{V}_{\text{ex}}^2 = v\Delta P = 0.001 (1000 - 100) = 0.9 \text{ kJ/kg} = 900 \text{ J/kg}$$

$$\mathbf{V}_{\text{ex}} = \sqrt{2 \times 900 \text{ J/kg}} = \mathbf{42.4 \text{ m/s}}$$



## 9.67

The underwater bulb nose of a container ship has a velocity relative to the ocean water as 10 m/s. What is the pressure at the front stagnation point that is 2 m down from the water surface.

Solution:

C.V. A stream line of flow from the freestream to the wall.

$$\text{Eq.9.16:} \quad v(P_e - P_i) + \frac{1}{2}(V_e^2 - V_i^2) + g(Z_e - Z_i) = 0$$

$$\Delta P = \frac{1}{2v} V_i^2 = \frac{10^2}{0.001001 \times 2000} = 49.95 \text{ kPa}$$

$$P_i = P_o + gH/v = 101 + 9.81 \times 2 / (0.001001 \times 1000) = 120.6 \text{ kPa}$$

$$P_e = P_i + \Delta P = 120.6 + 49.95 = \mathbf{170.6 \text{ kPa}}$$



This container-ship is under construction and not loaded. The red line is the water line under normal load.

## 9.68

A speed boat has a small hole in the front of the drive with the propeller that sticks down into the water at a water depth of 0.25 m. Assume we have a stagnation point at that hole when the boat is sailing with 60 km/h, what is the total pressure there?

Solution:

C.V. A stream line of flow from the freestream to the wall.

$$\text{Eq.9.16: } v(P_e - P_i) + \frac{1}{2}(V_e^2 - V_i^2) + g(Z_e - Z_i) = 0$$

$$V_i = 60 \frac{\text{km}}{\text{h}} \times 1000 \frac{\text{m}}{\text{km}} \times \frac{1}{3600} \frac{\text{h}}{\text{s}} = 16.667 \text{ m/s}$$

$$\Delta P = \frac{1}{2} \rho V_i^2 = \frac{1000 \times 16.667^2}{2} = 138.8 \text{ kPa}$$

$$P_i = P_o + \rho g H = 101 + 1000 \times 9.81 \times 0.25 = 103.45 \text{ kPa}$$

$$P_e = P_i + \Delta P = 103.45 + 138.8 = \mathbf{242.3 \text{ kPa}}$$

Remark: This is very fast for a boat

## 9.69

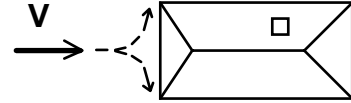
Atmospheric air at 100 kPa, 17°C blows at 60 km/h towards the side of a building. Assume the air is nearly incompressible find the pressure and the temperature at the stagnation point (zero velocity) on the wall.

Solution:

C.V. A stream line of flow from the freestream to the wall.

Eq.9.16:

$$v(P_e - P_i) + \frac{1}{2}(V_e^2 - V_i^2) + g(Z_e - Z_i) = 0$$



$$V_i = 60 \frac{\text{km}}{\text{h}} \times 1000 \frac{\text{m}}{\text{km}} \times \frac{1}{3600} \frac{\text{h}}{\text{s}} = 16.667 \text{ m/s}$$

$$v = \frac{RT_i}{P_i} = \frac{0.287 \times 290.15}{100} = 0.8323 \frac{\text{m}^3}{\text{kg}}$$

$$\Delta P = \frac{1}{2v} V_i^2 = \frac{16.667^2}{0.8323 \times 2000} = 0.17 \text{ kPa}$$

$$P_e = P_i + \Delta P = 100.17 \text{ kPa}$$

Then Eq.8.23 for an isentropic process:

$$T_e = T_i (P_e/P_i)^{0.286} = 290.15 \times 1.0005 = \mathbf{290.3 \text{ K}}$$

Very small effect due to low velocity and air is light (large specific volume)

## 9.70

You drive on the highway with 120 km/h on a day with 17°C, 100 kPa atmosphere. When you put your hand out of the window flat against the wind you feel the force from the air stagnating, i.e. it comes to relative zero velocity on your skin. Assume the air is nearly incompressible and find the air temperature and pressure right on your hand.

Solution:

Energy Eq.6.13:  $\frac{1}{2} \mathbf{V}^2 + h_o = h_{st}$

$$T_{st} = T_o + \frac{1}{2} \mathbf{V}^2 / C_p = 17 + \frac{1}{2} [(120 \times 1000) / 3600]^2 \times (1 / 1004)$$

$$= 17 + 555.5 / 1004 = \mathbf{17.6^\circ C}$$

$$v = RT_o / P_o = 0.287 \times 290 / 100 = 0.8323 \text{ m}^3/\text{kg}$$

From Bernoulli Eq.9.16:

$$v \Delta P = \frac{1}{2} \mathbf{V}^2$$

$$P_{st} = P_o + \frac{1}{2} \mathbf{V}^2 / v = 100 + 555.5 / (0.8323 \times 1000) = \mathbf{100.67 \text{ kPa}}$$



## 9.71

An air flow at 100 kPa, 290 K, 200 m/s is directed towards a wall. At the wall the flow stagnates (comes to zero velocity) without any heat transfer. Find the stagnation pressure a) assuming incompressible flow b) assume an adiabatic compression. Hint: T comes from the energy equation.

Solution:

Ideal gas:  $v = RT_o/P_o = 0.287 \times 290/100 = 0.8323 \text{ m}^3/\text{kg}$

Kinetic energy:  $\frac{1}{2} \mathbf{V}^2 = \frac{1}{2} (200^2/1000) = 4.05 \text{ kJ/kg}$

a) Reversible and incompressible gives Bernoulli Eq.9.16:

$$\Delta P = \frac{1}{2} \mathbf{V}^2/v = 4.05/0.8323$$

$$= 4.866 \text{ kPa}$$

$$P_{st} = P_o + \Delta P = 104.9 \text{ kPa}$$

b) adiabatic compression

Energy Eq.6.13:  $\frac{1}{2} \mathbf{V}^2 + h_o = h_{st}$

$$h_{st} - h_o = \frac{1}{2} \mathbf{V}^2 = C_p \Delta T$$

$$\Delta T = \frac{1}{2} \mathbf{V}^2/C_p = 4.05/1.004 = 4.03^\circ\text{C}$$

$$\Rightarrow T_{st} = 290 + 4.03 = 294 \text{ K}$$

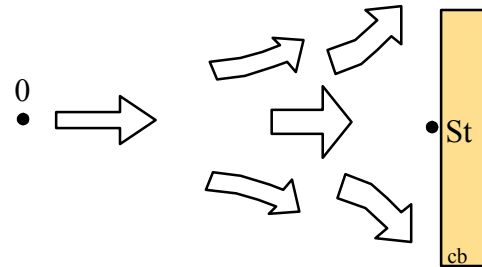
Entropy Eq.9.8 assume also reversible process:

$$s_o + s_{gen} + \int (1/T) dq = s_{st}$$

as  $dq = 0$  and  $s_{gen} = 0$  then it follows that  $s = \text{constant}$

This relation gives Eq.8.23:

$$P_{st} = P_o \left( \frac{T_{st}}{T_o} \right)^{\frac{k}{k-1}} = 100 \times (294/290)^{3.5} = \mathbf{105 \text{ kPa}}$$



## 9.72

Calculate the air temperature and pressure at the stagnation point right in front of a meteorite entering the atmosphere (-50 °C, 50 kPa) with a velocity of 2000 m/s. Do this assuming air is incompressible at the given state and repeat for air being a compressible substance going through an adiabatic compression.

Solution:

$$\text{Kinetic energy: } \frac{1}{2} \mathbf{V}^2 = \frac{1}{2} (2000)^2/1000 = 2000 \text{ kJ/kg}$$

$$\text{Ideal gas: } v_{\text{atm}} = RT/P = 0.287 \times 223/50 = 1.28 \text{ m}^3/\text{kg}$$

a) incompressible

$$\text{Energy Eq.6.13: } \Delta h = \frac{1}{2} \mathbf{V}^2 = 2000 \text{ kJ/kg}$$

If A.5  $\Delta T = \Delta h/C_p = 1992 \text{ K}$  unreasonable, too high for that  $C_p$

$$\text{Use A.7: } h_{\text{st}} = h_o + \frac{1}{2} \mathbf{V}^2 = 223.22 + 2000 = 2223.3 \text{ kJ/kg}$$

$$T_{\text{st}} = 1977 \text{ K}$$

Bernoulli (incompressible) Eq.9.17:

$$\Delta P = P_{\text{st}} - P_o = \frac{1}{2} \mathbf{V}^2/v = 2000/1.28 = 1562.5 \text{ kPa}$$

$$P_{\text{st}} = 1562.5 + 50 = 1612.5 \text{ kPa}$$

b) compressible

$T_{\text{st}} = 1977 \text{ K}$  the same energy equation.

$$\text{From A.7.1: } s_{T_{\text{st}}}^o = 8.9517 \text{ kJ/kg K}; \quad s_{T_o}^o = 6.5712 \text{ kJ/kg K}$$

Eq.8.28:

$$\begin{aligned} P_{\text{st}} &= P_o \times e^{(s_{T_{\text{st}}}^o - s_{T_o}^o)/R} \\ &= 50 \times \exp \left[ \frac{8.9517 - 6.5712}{0.287} \right] \\ &= \mathbf{200\ 075 \text{ kPa}} \end{aligned}$$

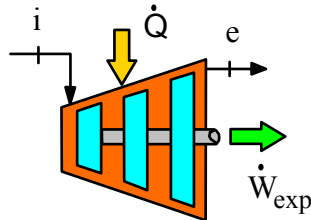


Notice that this is highly compressible,  $v$  is not constant.

## 9.73

Helium gas enters a steady-flow expander at 800 kPa, 300°C, and exits at 120 kPa. The mass flow rate is 0.2 kg/s, and the expansion process can be considered as a reversible polytropic process with exponent,  $n = 1.3$ . Calculate the power output of the expander.

Solution:



CV: expander, reversible polytropic process.  
From Eq.8.37:

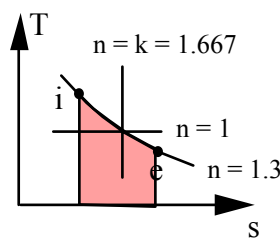
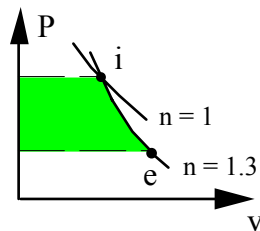
$$T_e = T_i \left( \frac{P_e}{P_i} \right)^{\frac{n-1}{n}} = 573.2 \left( \frac{120}{800} \right)^{\frac{0.3}{1.3}} = 370 \text{ K}$$

Work evaluated from Eq.9.17

$$w = - \int v dP = - \frac{nR}{n-1} (T_e - T_i) = \frac{-1.3 \times 2.07703}{0.3} (370 - 573.2)$$

$$= 1828.9 \text{ kJ/kg}$$

$$\dot{W} = \dot{m}w = 0.2 \times 1828.9 = \mathbf{365.8 \text{ kW}}$$



## 9.74

A flow of air at 100 kPa, 300 K enters a device and goes through a polytropic process with  $n = 1.3$  before it exits at 1000 K. Find the exit pressure, the specific work and heat transfer using constant specific heats.

Solution:

C.V. Steady state device, single inlet and single exit flow.

Energy Eq.6.13:  $h_1 + q = h_2 + w$  Neglect kinetic, potential energies

Entropy Eq.9.8:  $s_1 + \int dq/T + s_{\text{gen}} = s_2$

$T_e = 1000 \text{ K}; \quad T_i = 300 \text{ K}; \quad P_i = 100 \text{ kPa}$

Process Eq.8.37:  $P_e = P_i (T_e/T_i)^{\frac{n}{n-1}} = 100 (1000/300)^{\frac{1.3}{0.3}} = \mathbf{18\ 442 \text{ kPa}}$

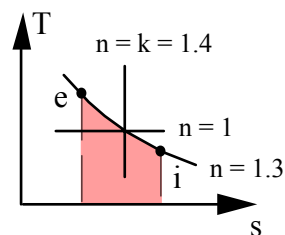
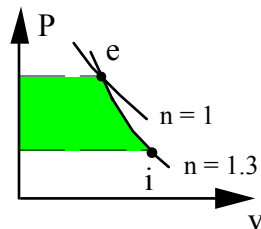
and the process leads to Eq.9.17 for the work term

$$w = \frac{n}{n-1} R (T_e - T_i) = (1.3/-0.3) \times 0.287 \text{ kJ/kg-K} \times (1000 - 300) \text{ K}$$

$$= -\mathbf{849.3 \text{ kJ/kg}}$$

$$q = h_e - h_i + w = C_p (T_e - T_i) + w = 1.004(1000 - 300) - 849.3$$

$$= -\mathbf{146.5 \text{ kJ/kg}}$$



Notice:

$dP > 0$

so  $dw < 0$

$ds < 0$

so  $dq < 0$



## 9.75

Solve the previous problem but use the air tables A.7

Air at 100 kPa, 300 K, flows through a device at steady state with the exit at 1000 K during which it went through a polytropic process with  $n = 1.3$ . Find the exit pressure, the specific work and heat transfer.

Solution:

C.V. Steady state device, single inlet and single exit flow.

Energy Eq.6.13:  $h_1 + q = h_2 + w$       Neglect kinetic, potential energies

Entropy Eq.9.8:  $s_1 + \int dq/T + s_{\text{gen}} = s_2$

$T_e = 1000 \text{ K}; \quad T_i = 300 \text{ K}; \quad P_i = 100 \text{ kPa}$

Process Eq.8.37:  $P_e = P_i (T_e/T_i)^{\frac{n}{n-1}} = 100 (1000/300)^{\frac{1.3}{0.3}} = \mathbf{18\ 442 \text{ kPa}}$

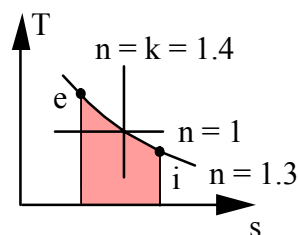
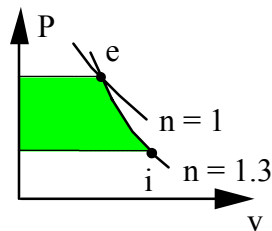
and the process leads to Eq.9.19 for the work term

$$w = \frac{n}{n-1} R (T_e - T_i) = (1.3/-0.3) \times 0.287 \text{ kJ/kg-K} \times (1000 - 300) \text{ K}$$

$$= -\mathbf{849.3 \text{ kJ/kg}}$$

$$q = h_e - h_i + w = 1046.2 - 300.5 - 849.3$$

$$= -\mathbf{103.6 \text{ kJ/kg}}$$



Notice:

$dP > 0$   
so  $dw < 0$

$ds < 0$   
so  $dq < 0$

## 9.76

A flow of 4 kg/s ammonia goes through a device in a polytropic process with an inlet state of 150 kPa,  $-20^{\circ}\text{C}$  and an exit state of 400 kPa,  $80^{\circ}\text{C}$ . Find the polytropic exponent  $n$ , the specific work and heat transfer.

Solution:

C.V. Steady state device, single inlet and single exit flow.

Energy Eq.6.13:  $h_1 + q = h_2 + w$  Neglect kinetic, potential energies

Entropy Eq.9.8:  $s_1 + \int dq/T + s_{\text{gen}} = s_2$

Process Eq.8.37:  $P_1 v_1^n = P_2 v_2^n$ :

State 1: Table B.2.2  $v_1 = 0.79774$ ,  $s_1 = 5.7465$  kJ/kg K,  $h_1 = 1422.9$  kJ/kg

State 2: Table B.2.2  $v_2 = 0.4216$ ,  $s_2 = 5.9907$  kJ/kg K,  $h_2 = 1636.7$  kJ/kg

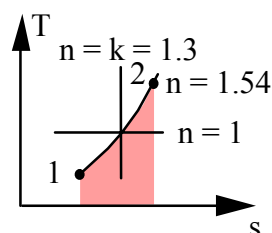
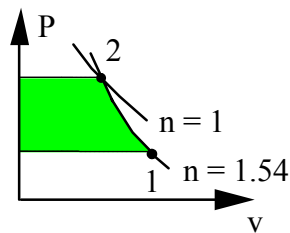
$$\ln(P_2/P_1) = n \ln(v_1/v_2) \Rightarrow 0.98083 = n \times 0.63772$$

$$n = \ln(P_2/P_1) / \ln(v_1/v_2) = \mathbf{1.538}$$

From the process and the integration of  $v \, dP$  gives Eq.9.19.

$$w_{\text{shaft}} = -\frac{n}{n-1} (P_2 v_2 - P_1 v_1) = -2.8587 (168.64 - 119.66) = \mathbf{-140.0 \text{ kJ/kg}}$$

$$q = h_2 + w - h_1 = 1636.7 - 1422.9 - 140 = \mathbf{73.8 \text{ kJ/kg}}$$



Notice:

$dP > 0$   
so  $dw < 0$

$ds > 0$   
so  $dq > 0$

## 9.77

An expansion in a gas turbine can be approximated with a polytropic process with exponent  $n = 1.25$ . The inlet air is at 1200 K, 800 kPa and the exit pressure is 125 kPa with a mass flow rate of 0.75 kg/s. Find the turbine heat transfer and power output.

Solution:

C.V. Steady state device, single inlet and single exit flow.

Energy Eq.6.13:  $h_i + q = h_e + w$  Neglect kinetic, potential energies

Entropy Eq.9.8:  $s_i + \int dq/T + s_{\text{gen}} = s_e$

Process Eq.8.37:

$$T_e = T_i (P_e / P_i)^{\frac{n-1}{n}} = 1200 (125/800)^{\frac{0.25}{1.25}} = 827.84 \text{ K}$$

so the exit enthalpy is from Table A.7.1

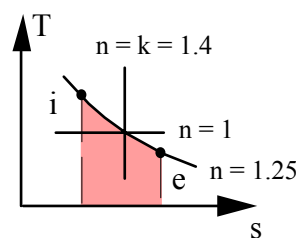
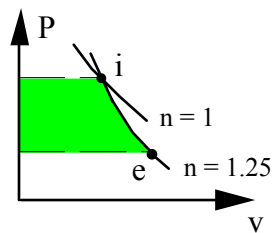
$$h_e = 822.2 + \frac{27.84}{50}(877.4 - 822.2) = 852.94 \text{ kJ/kg}$$

The process leads to Eq.9.17 for the work term

$$\begin{aligned} \dot{W} = \dot{m}w &= -\dot{m} \frac{nR}{n-1} (T_e - T_i) = -0.75 \frac{1.25 \times 0.287}{0.25} \times (827.84 - 1200) \\ &= \mathbf{400.5 \text{ kW}} \end{aligned}$$

Energy equation gives

$$\begin{aligned} \dot{Q} = \dot{m}q &= \dot{m}(h_e - h_i) + \dot{W} = 0.75(852.94 - 1277.81) + 400.5 \\ &= -318.65 + 400.5 = \mathbf{81.9 \text{ kW}} \end{aligned}$$



Notice:  
 $dP < 0$   
 so  $dw > 0$   
 $ds > 0$   
 so  $dq > 0$

Notice this process has some heat transfer in during expansion which is unusual. The typical process would have  $n = 1.5$  with a heat loss.

## **Steady state irreversible processes**

## 9.78

Consider the steam turbine in Example 6.6. Is this a reversible process?

Solution:

At the given states

$$\text{Table B.1.3: } s_i = 6.9552 \text{ kJ/kg K; } s_e = 7.3593 \text{ kJ/kg K}$$

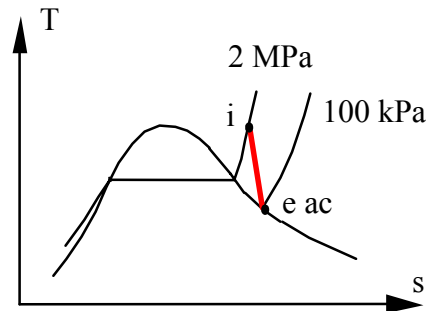
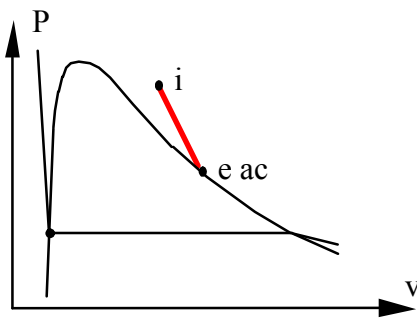
Do the second law for the turbine, Eq.9.8

$$\dot{m}_e s_e = \dot{m}_i s_i + \int d\dot{Q}/T + \dot{S}_{\text{gen}}$$

$$s_e = s_i + \int dq/T + s_{\text{gen}}$$

$$s_{\text{gen}} = s_e - s_i - \int dq/T = 7.3593 - 6.9552 - (\text{negative}) > 0$$

Entropy goes up even if q goes out. **This is an irreversible process.**



## 9.79

A large condenser in a steam power plant dumps 15 MW by condensing saturated water vapor at 45°C to saturated liquid. What is the water flow rate and the entropy generation rate with an ambient at 25°C?

Solution:

This process transfers heat over a finite temperature difference between the water inside the condenser and the outside ambient (cooling water from the sea, lake or river or atmospheric air)

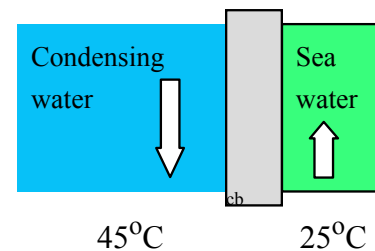
C.V. The Condensing water flow

Energy Eq.:  $0 = \dot{m} (h_g - h_f) - \dot{Q}_{\text{out}}$

$$\dot{m} = \dot{Q}_{\text{out}} / h_{fg} = \frac{15\,000}{2394.77} \frac{\text{kW}}{\text{kJ/kg}} = \mathbf{6.264 \text{ kg/s}}$$

C.V. The wall that separates the inside 45°C water from the ambient at 25°C.

Entropy Eq. 9.1 for steady state operation:



$$\frac{dS}{dt} = 0 = \sum \frac{\dot{Q}}{T} + \dot{S}_{\text{gen}} = \frac{\dot{Q}}{T_{45}} - \frac{\dot{Q}}{T_{25}} + \dot{S}_{\text{gen}}$$

$$\dot{S}_{\text{gen}} = \frac{15}{25 + 273} \frac{\text{MW}}{\text{K}} - \frac{15}{45 + 273} \frac{\text{MW}}{\text{K}} = \mathbf{3.17 \frac{\text{kW}}{\text{K}}}$$

## 9.80

The throttle process described in Example 6.5 is an irreversible process. Find the entropy generation per kg ammonia in the throttling process.

Solution:

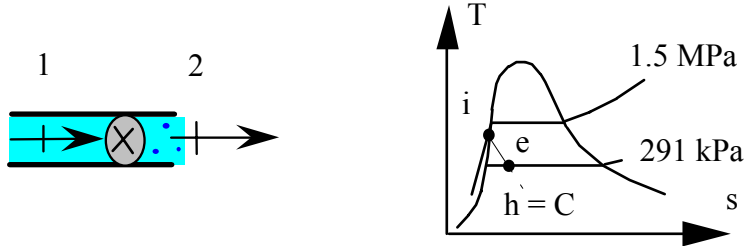
The process is adiabatic and irreversible. The consideration with the energy given in the example resulted in a constant  $h$  and two-phase exit flow.

$$\text{Table B.2.1: } s_i = 1.2792 \text{ kJ/kg K}$$

$$\begin{aligned} \text{Table B.2.1: } s_e &= s_f + x_e s_{fg} = 0.5408 + 0.1638 \times 4.9265 \\ &= 1.34776 \text{ kJ/kg K} \end{aligned}$$

We assumed no heat transfer so the entropy equation Eq.9.8 gives

$$s_{\text{gen}} = s_e - s_i - \int dq/T = 1.34776 - 1.2792 - 0 = \mathbf{0.0686 \text{ kJ/kg K}}$$



## 9.81

R-134a at  $30^\circ\text{C}$ , 800 kPa is throttled in a steady flow to a lower pressure so it comes out at  $-10^\circ\text{C}$ . What is the specific entropy generation?

Solution:

The process is adiabatic and irreversible. The consideration of the energy given in example 6.5 resulted in a constant  $h$  and two-phase exit flow.

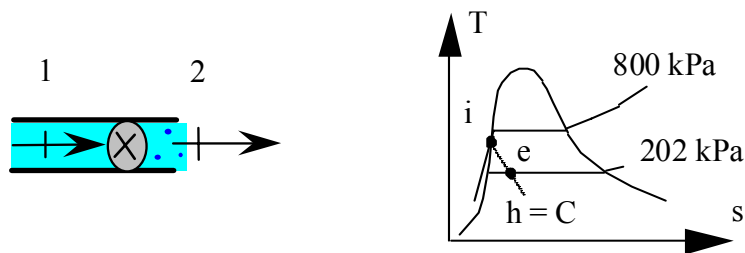
Table B.4.1:  $h_i = 241.79 \text{ kJ/kg}$ ,  $s_i = 1.143 \text{ kJ/kg K}$  (compressed liquid)

State 2:  $-10^\circ\text{C}$ ,  $h_e = h_i < h_g$  so two-phase  $x_e = (h_e - h_f)/h_{fg} = 0.267$

Table B.4.1:  $s_e = s_f + x_e s_{fg} = 0.9507 + 0.267 \times 0.7812 = 1.16 \text{ kJ/kg K}$

We assumed no heat transfer so the entropy equation Eq.9.8 gives

$$s_{\text{gen}} = s_e - s_i - \int dq/T = 1.16 - 1.143 - 0 = \mathbf{0.017 \text{ kJ/kg K}}$$





## 9.82

Analyze the steam turbine described in Problem 6.64. Is it possible?

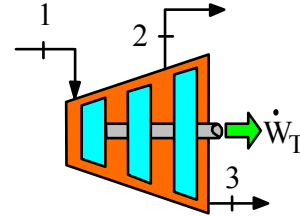
Solution:

C.V. Turbine. Steady flow and adiabatic.

$$\text{Continuity Eq. 6.9: } \dot{m}_1 = \dot{m}_2 + \dot{m}_3 ;$$

$$\text{Energy Eq. 6.10: } \dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{m}_3 h_3 + \dot{W}$$

$$\text{Entropy Eq. 9.7: } \dot{m}_1 s_1 + \dot{S}_{\text{gen}} = \dot{m}_2 s_2 + \dot{m}_3 s_3$$



States from Table B.1.3:  $s_1 = 6.6775$ ,  $s_2 = 6.9562$ ,  $s_3 = 7.14413$  kJ/kg K

$$\dot{S}_{\text{gen}} = 20 \times 6.9562 + 80 \times 7.14413 - 100 \times 6.6775 = \mathbf{42.9 \text{ kW/K} > 0}$$

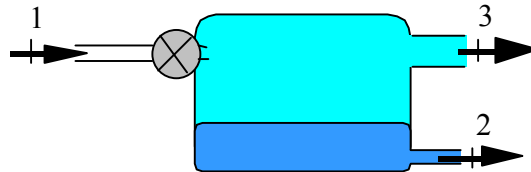
Since it is positive  $\Rightarrow$  **possible**.

Notice the entropy is increasing through turbine:  $s_1 < s_2 < s_3$

## 9.83

A geothermal supply of hot water at 500 kPa, 150°C is fed to an insulated flash evaporator at the rate of 1.5 kg/s. A stream of saturated liquid at 200 kPa is drained from the bottom of the chamber and a stream of saturated vapor at 200 kPa is drawn from the top and fed to a turbine. Find the rate of entropy generation in the flash evaporator.

Solution:



Two-phase out of the valve. The liquid drops to the bottom.

$$\text{Continuity Eq.6.9:} \quad \dot{m}_1 = \dot{m}_2 + \dot{m}_3$$

$$\text{Energy Eq.6.10:} \quad \dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{m}_3 h_3$$

$$\text{Entropy Eq.9.7:} \quad \dot{m}_1 s_1 + \dot{S}_{\text{gen}} + \int d\dot{Q}/T = \dot{m}_2 s_2 + \dot{m}_3 s_3$$

$$\text{Process:} \quad \dot{Q} = 0, \quad \text{irreversible (throttle)}$$

$$\text{B.1.1} \quad h_1 = 632.18 \text{ kJ/kg}, \quad s_1 = 1.8417 \text{ kJ/kg K}$$

$$\text{B.1.2} \quad h_3 = 2706.63 \text{ kJ/kg}, \quad s_3 = 7.1271 \text{ kJ/kg K},$$

$$h_2 = 504.68 \text{ kJ/kg}, \quad s_2 = 1.53 \text{ kJ/kg K}$$

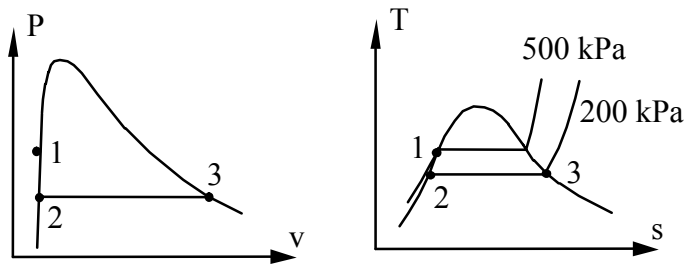
From the energy equation we solve for the flow rate

$$\dot{m}_3 = \dot{m}_1 (h_1 - h_2) / (h_3 - h_2) = 1.5 \times 0.0579 = 0.08685 \text{ kg/s}$$

$$\text{Continuity equation gives:} \quad \dot{m}_2 = \dot{m}_1 - \dot{m}_3 = 1.41315 \text{ kg/s}$$

Entropy equation now leads to

$$\begin{aligned} \dot{S}_{\text{gen}} &= \dot{m}_2 s_2 + \dot{m}_3 s_3 - \dot{m}_1 s_1 \\ &= 1.41315 \times 1.53 + 0.08685 \times 7.127 - 1.5 \times 1.8417 \\ &= \mathbf{0.01855 \text{ kW/K}} \end{aligned}$$



## 9.84

A steam turbine has an inlet of 2 kg/s water at 1000 kPa and 350°C with velocity of 15 m/s. The exit is at 100 kPa, 150°C and very low velocity. Find the power produced and the rate of entropy generation.

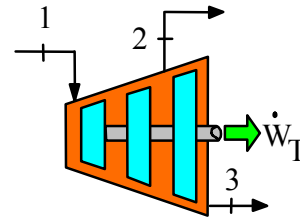
Solution:

C.V. Turbine. Steady flow and adiabatic.

$$\text{Continuity Eq.6.9: } \dot{m}_1 = \dot{m}_2 ;$$

$$\text{Energy Eq.6.10: } \dot{m}_1(h_1 + \frac{1}{2}\mathbf{V}^2) = \dot{m}_2 h_2 + \dot{W}$$

$$\text{Entropy Eq.9.7: } \dot{m}_1 s_1 + \dot{S}_{\text{gen}} = \dot{m}_2 s_2$$



States from Table B.1.3:  $h_1 = 3158 \text{ kJ/kg}$ ,  $s_1 = 7.301 \text{ kJ/kgK}$ ,

$h_2 = 2776 \text{ kJ/kg}$ ,  $s_2 = 7.613 \text{ kJ/kgK}$

$$\dot{W} = \dot{m}_1(h_1 + \frac{1}{2}\mathbf{V}^2 - h_2) = 2 (3158 + \frac{1}{2} \frac{15^2}{1000} - 2776) = \mathbf{764 \text{ kW}}$$

$$\dot{S}_{\text{gen}} = \dot{m}_1(s_2 - s_1) = 2 (7.613 - 7.301) = \mathbf{0.624 \text{ kW/K}}$$

## 9.85

A large supply line has a steady flow of R-410a at 1000 kPa, 60°C. It is used in three different adiabatic devices shown in Fig. P9.85, a throttle flow, an ideal nozzle and an ideal turbine. All the exit flows are at 300 kPa. Find the exit temperature and specific entropy generation for each device and the exit velocity of the nozzle.

Inlet state: B.4.2:  $h_i = 335.75 \text{ kJ/kg}$ ,  $s_i = 1.2019 \text{ kJ/kg-K}$

**C.V. Throttle**, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.6.13:  $h_i = h_e$  ( $Z_i = Z_e$  and  $V$ 's are small)

Entropy Eq.9.8:  $s_e = s_i + \int dq/T + s_{\text{gen}} = s_i + 0 + s_{\text{gen}}$

Exit state:  $h_e = h_i = 335.75 \text{ kJ/kg}$ ,  $T_e = 47.9^\circ\text{C}$ ,  $s_e = 1.332 \text{ kJ/kg-K}$   
 $s_{\text{gen}} = s_e - s_i = 1.332 - 1.2019 = 0.2 \text{ kJ/kg K}$

**C.V. Nozzle**, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.6.13:  $h_i = h_e + V_e^2/2$  ( $Z_i = Z_e$ )

Entropy Eq.9.8:  $s_e = s_i + \int dq/T + s_{\text{gen}} = s_i + 0 + 0$

The isentropic process ( $s_e = s_i$ ) gives from B.4.2

$$T_e = 4.6^\circ\text{C}, \quad s_{\text{gen}} = 0, \quad h_e = 296.775 \text{ kJ/kg}$$

The energy equation becomes

$$V_e^2/2 = h_i - h_e = 335.75 - 296.775 = 38.975 \text{ kJ/kg}$$

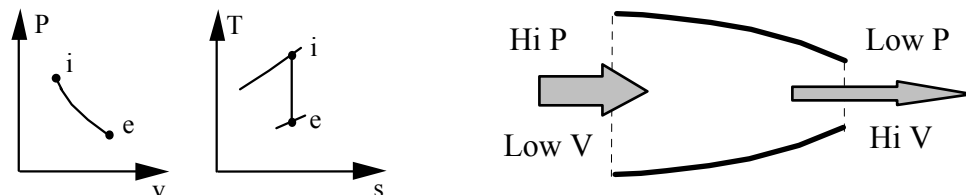
$$V_e = \sqrt{2 \times 38.975 \times 1000} = 279.2 \text{ m/s}$$

**Turbine:**

Process: Reversible and adiabatic  $\Rightarrow$  same as for nozzle except  $w$ ,  $V_e = 0$

Energy Eq.6.13:  $h_i = h_e + w$  ( $Z_i = Z_e$ )

$$T_e = 4.6^\circ\text{C}, \quad s_{\text{gen}} = 0, \quad h_e = 296.775 \text{ kJ/kg}$$



## 9.86

Two flowstreams of water, one at 0.6 MPa, saturated vapor, and the other at 0.6 MPa, 600°C, mix adiabatically in a steady flow process to produce a single flow out at 0.6 MPa, 400°C. Find the total entropy generation for this process.

Solution:

$$1: \text{B.1.2} \quad h_1 = 2756.8 \text{ kJ/kg}, \quad s_1 = 6.760 \text{ kJ/kg K}$$

$$2: \text{B.1.3} \quad h_2 = 3700.9 \text{ kJ/kg}, \quad s_2 = 8.2674 \text{ kJ/kg K}$$

$$3: \text{B.1.3} \quad h_3 = 3270.3 \text{ kJ/kg}, \quad s_3 = 7.7078 \text{ kJ/kg K}$$

$$\text{Continuity Eq.6.9:} \quad \dot{m}_3 = \dot{m}_1 + \dot{m}_2,$$

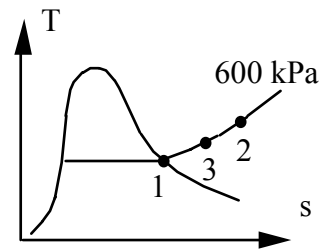
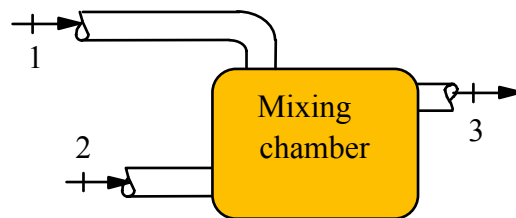
$$\text{Energy Eq.6.10:} \quad \dot{m}_3 h_3 = \dot{m}_1 h_1 + \dot{m}_2 h_2$$

$$\Rightarrow \dot{m}_1 / \dot{m}_3 = (h_3 - h_2) / (h_1 - h_2) = 0.456$$

$$\text{Entropy Eq.9.7:} \quad \dot{m}_3 s_3 = \dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{S}_{\text{gen}} \Rightarrow$$

$$\dot{S}_{\text{gen}} / \dot{m}_3 = s_3 - (\dot{m}_1 / \dot{m}_3) s_1 - (\dot{m}_2 / \dot{m}_3) s_2$$

$$= 7.7078 - 0.456 \times 6.760 - 0.544 \times 8.2674 = \mathbf{0.128 \text{ kJ/kg K}}$$



The mixing process generates entropy. The two inlet flows could have exchanged energy (they have different  $T$ ) through some heat engines and produced work, the process failed to do that, thus irreversible.

## 9.87

A mixing chamber receives 5 kg/min ammonia as saturated liquid at  $-20^{\circ}\text{C}$  from one line and ammonia at  $40^{\circ}\text{C}$ , 250 kPa from another line through a valve. The chamber also receives 325 kJ/min energy as heat transferred from a  $40^{\circ}\text{C}$  reservoir. This should produce saturated ammonia vapor at  $-20^{\circ}\text{C}$  in the exit line. What is the mass flow rate in the second line and what is the total entropy generation in the process?

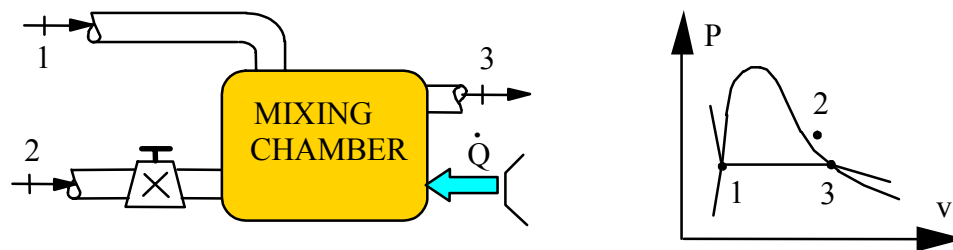
Solution:

CV: Mixing chamber out to reservoir

$$\text{Continuity Eq.6.9:} \quad \dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

$$\text{Energy Eq.6.10:} \quad \dot{m}_1 h_1 + \dot{m}_2 h_2 + \dot{Q} = \dot{m}_3 h_3$$

$$\text{Entropy Eq.9.7:} \quad \dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{Q}/T_{\text{res}} + \dot{S}_{\text{gen}} = \dot{m}_3 s_3$$



From the energy equation:

$$\begin{aligned} \dot{m}_2 &= [(\dot{m}_1(h_1 - h_3) + \dot{Q}) / (h_3 - h_2)] \\ &= [5 \times (89.05 - 1418.05) + 325] / (1418.05 - 1551.7) \\ &= \mathbf{47.288 \text{ kg/min}} \Rightarrow \dot{m}_3 = 52.288 \text{ kg/min} \end{aligned}$$

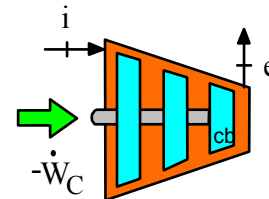
$$\begin{aligned} \dot{S}_{\text{gen}} &= \dot{m}_3 s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2 - \dot{Q}/T_{\text{res}} \\ &= 52.288 \times 5.6158 - 5 \times 0.3657 - 47.288 \times 5.9599 - 325/313.15 \\ &= \mathbf{8.94 \text{ kJ/K min}} \end{aligned}$$

## 9.88

A compressor in a commercial refrigerator receives R-410a at  $-25^{\circ}\text{C}$  and  $x = 1$ . The exit is at 1800 kPa and  $60^{\circ}\text{C}$ . Neglect kinetic energies and find the specific entropy generation.

Solution:

C.V. Compressor, steady state, single inlet and exit flow. For this device we also assume no heat transfer  $q = 0$  and  $Z_1 = Z_2$



$$\text{Entropy Eq.9.8:} \quad s_i + \int dq/T + s_{\text{gen}} = s_e = s_i + 0 + s_{\text{gen}}$$

$$\text{From Table B.4.1 :} \quad s_i = 1.0893 \text{ kJ/kgK}$$

$$\text{From Table B.4.2 :} \quad s_e = 1.1076 \text{ kJ/kgK}$$

Entropy generation becomes

$$s_{\text{gen}} = s_e - s_i = 1.1076 - 1.0893 = \mathbf{0.0183 \text{ kJ/kgK}}$$

## 9.89

A condenser in a power plant receives 5 kg/s steam at 15 kPa, quality 90% and rejects the heat to cooling water with an average temperature of 17°C. Find the power given to the cooling water in this constant pressure process and the total rate of entropy generation when condenser exit is saturated liquid.

Solution:

C.V. Condenser. Steady state with no shaft work term.

$$\text{Energy Eq.6.12:} \quad \dot{m} h_i + \dot{Q} = \dot{m} h_e$$

$$\text{Entropy Eq.9.8:} \quad \dot{m} s_i + \dot{Q}/T + \dot{S}_{\text{gen}} = \dot{m} s_e$$

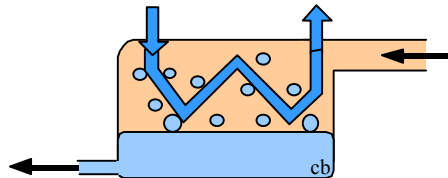
Properties are from Table B.1.2

$$h_i = 225.91 + 0.9 \times 2373.14 = 2361.74 \text{ kJ/kg}, \quad h_e = 225.91 \text{ kJ/kg}$$

$$s_i = 0.7548 + 0.9 \times 7.2536 = 7.283 \text{ kJ/kg K}, \quad s_e = 0.7548 \text{ kJ/kg K}$$

$$\dot{Q}_{\text{out}} = -\dot{Q} = \dot{m} (h_i - h_e) = 5(2361.74 - 225.91) = \mathbf{10679 \text{ kW}}$$

$$\begin{aligned} \dot{S}_{\text{gen}} &= \dot{m} (s_e - s_i) + \dot{Q}_{\text{out}}/T \\ &= 5(0.7548 - 7.283) + 10679/(273 + 17) \\ &= -32.641 + 36.824 = \mathbf{4.183 \text{ kW/K}} \end{aligned}$$



Often the cooling media flows inside a long pipe carrying the energy away.



## 9.90

Carbon dioxide at 300 K, 200 kPa is brought through a steady device where it is heated to 500 K by a 600 K reservoir in a constant pressure process. Find the specific work, specific heat transfer and specific entropy generation.

Solution:

C.V. Heater and walls out to the source. Steady single inlet and exit flows.

Since the pressure is constant and there are no changes in kinetic or potential energy between the inlet and exit flows the work is zero.  $w = 0$

$$\text{Continuity Eq.6.11: } \dot{m}_i = \dot{m}_e = \dot{m}$$

$$\text{Energy Eq.6.13: } h_i + q = h_e$$

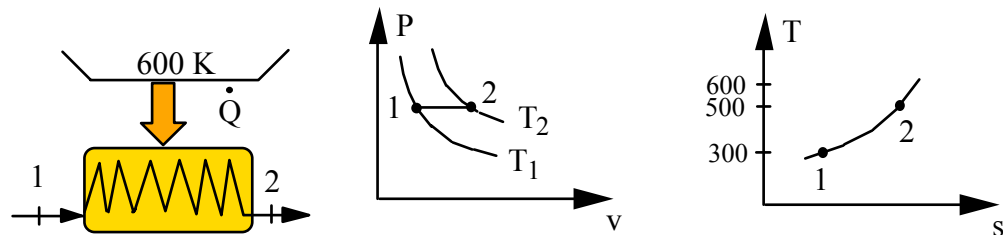
$$\text{Entropy Eq.9.8: } s_i + \int dq/T + s_{\text{gen}} = s_e = s_i + q/T_{\text{source}} + s_{\text{gen}}$$

Properties are from Table A.8 so the energy equation gives

$$q = h_e - h_i = 401.52 - 214.38 = \mathbf{187.1 \text{ kJ/kg}}$$

From the entropy equation

$$\begin{aligned} s_{\text{gen}} &= s_e - s_i - q/T_{\text{source}} = (5.3375 - 4.8631) - 187.1/600 \\ &= 0.4744 - 0.3118 = \mathbf{0.1626 \text{ kJ/kg K}} \end{aligned}$$



## 9.91

A heat exchanger that follows a compressor receives 0.1 kg/s air at 1000 kPa, 500 K and cools it in a constant pressure process to 320 K. The heat is absorbed by ambient air at 300 K. Find the total rate of entropy generation.

Solution:

C.V. Heat exchanger to ambient, steady constant pressure so no work.

$$\text{Energy Eq.6.12:} \quad \dot{m}h_i = \dot{m}h_e + \dot{Q}_{\text{out}}$$

$$\text{Entropy Eq.9.8:} \quad \dot{m}s_i + \dot{S}_{\text{gen}} = \dot{m}s_e + \dot{Q}_{\text{out}}/T$$

Using Table A.5 and Eq.8.25 for change in s

$$\dot{Q}_{\text{out}} = \dot{m}(h_i - h_e) = \dot{m}C_{p0}(T_i - T_e) = 0.1 \times 1.004(500 - 320) = 18.07 \text{ kW}$$

$$\begin{aligned} \dot{S}_{\text{gen}} &= \dot{m}(s_e - s_i) + \dot{Q}_{\text{out}}/T = \dot{m}C_{p0} \ln(T_e/T_i) + \dot{Q}_{\text{out}}/T \\ &= 0.1 \times 1.004 \ln(320/500) + 18.07/300 \\ &= \mathbf{0.0154 \text{ kW/K}} \end{aligned}$$

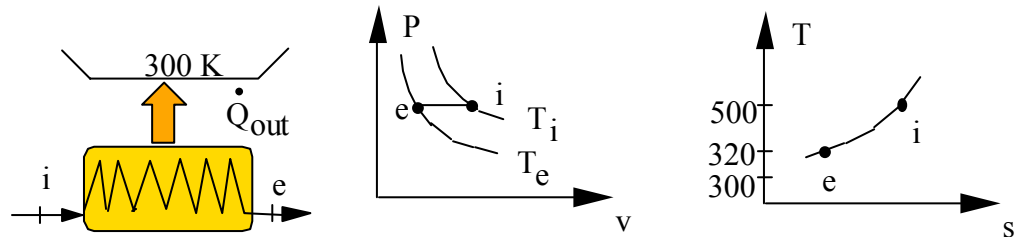
Using Table A.7.1 and Eq. 8.19 for change in entropy

$$h_{500} = 503.36 \text{ kJ/kg}, \quad h_{320} = 320.58 \text{ kJ/kg};$$

$$s_{T_{500}} = 7.38692 \text{ kJ/kg K}, \quad s_{T_{320}} = 6.93413 \text{ kJ/kg K}$$

$$\dot{Q}_{\text{out}} = \dot{m}(h_i - h_e) = 0.1 (503.36 - 320.58) = 18.19 \text{ kW}$$

$$\begin{aligned} \dot{S}_{\text{gen}} &= \dot{m}(s_e - s_i) + \dot{Q}_{\text{out}}/T \\ &= 0.1(6.93413 - 7.38692) + 18.19/300 \\ &= \mathbf{0.0156 \text{ kW/K}} \end{aligned}$$



## 9.92

Air at 1000 kPa, 300 K is throttled to 500 kPa. What is the specific entropy generation?

Solution:

C.V. Throttle, single flow, steady state. We neglect kinetic and potential energies and there are no heat transfer and shaft work terms.

Energy Eq. 6.13:  $h_i = h_e \Rightarrow T_i = T_e$  (ideal gas)

Entropy Eq. 9.9:  $s_e = s_i + \int_i^e \frac{dq}{T} + s_{\text{gen}} = s_i + s_{\text{gen}}$

Change in s Eq. 8.24:  $s_e - s_i = \int_i^e C_p \frac{dT}{T} - R \ln \frac{P_e}{P_i} = -R \ln \frac{P_e}{P_i}$

$$s_{\text{gen}} = s_e - s_i = -0.287 \ln \left( \frac{500}{1000} \right) = \mathbf{0.2 \frac{kJ}{kg \cdot K}}$$

## 9.93

Two flows of air both at 200 kPa; one has 1 kg/s at 400 K and the other has 2 kg/s at 290 K. The two flows are mixed together in an insulated box to produce a single exit flow at 200 kPa. Find the exit temperature and the total rate of entropy generation.

Solution:

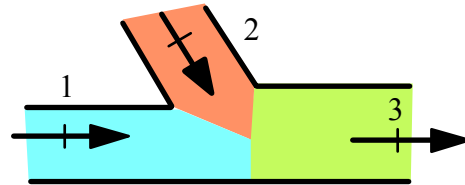
Continuity Eq.6.9:

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3 = 1 + 2 = 3 \text{ kg/s}$$

Energy Eq.6.10:

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

Entropy Eq.9.7:  $\dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{S}_{\text{gen}} = \dot{m}_3 s_3$



Using constant specific heats from A.5 and Eq.8.16 for  $s$  change.

Divide the energy equation with  $\dot{m}_3 C_{P0}$

$$T_3 = (\dot{m}_1/\dot{m}_3)T_1 + (\dot{m}_2/\dot{m}_3)T_2 = \frac{1}{3} \times 400 + \frac{2}{3} \times 290 = \mathbf{326.67 \text{ K}}$$

$$\begin{aligned} \dot{S}_{\text{gen}} &= \dot{m}_1(s_3 - s_1) + \dot{m}_2(s_3 - s_2) \\ &= 1 \times 1.004 \ln(326.67/400) + 2 \times 1.004 \ln(326.67/290) \\ &= \mathbf{0.0358 \text{ kW/K}} \end{aligned}$$

Using A.7.1 and Eq.8.19 for change in  $s$ .

$$h_3 = (\dot{m}_1/\dot{m}_3)h_1 + (\dot{m}_2/\dot{m}_3)h_2 = \frac{1}{3} \times 401.3 + \frac{2}{3} \times 290.43 = 327.39 \text{ kJ/kg}$$

From A.7.1:  $T_3 = \mathbf{326.77 \text{ K}}$   $s_{T_3} = 6.9548 \text{ kJ/kg K}$

$$\begin{aligned} \dot{S}_{\text{gen}} &= 1(6.9548 - 7.15926) + 2(6.9548 - 6.83521) \\ &= \mathbf{0.0347 \text{ kW/K}} \end{aligned}$$

The pressure correction part of the entropy terms cancel out as all three states have the same pressure.

**9.94**

Methane at 1 MPa, 300 K is throttled through a valve to 100 kPa. Assume no change in the kinetic energy and ideal gas behavior. What is the specific entropy generation?

$$\text{Continuity Eq.6.11: } \dot{m}_i = \dot{m}_e = \dot{m}$$

$$\text{Energy Eq.6.13: } h_i + 0 = h_e$$

$$\text{Entropy Eq.9.8, 9.9: } s_i + \int dq/T + s_{\text{gen}} = s_e = s_i + 0 + s_{\text{gen}}$$

Properties are from Table B.7.2 so the energy equation gives

$$h_e = h_i = 618.76 \text{ kJ/kg} \Rightarrow T_e = 296 \text{ K}, s_e = 11.5979 \text{ kJ/kg-K}$$

$$s_{\text{gen}} = s_e - s_i = 11.5979 - 10.4138 = \mathbf{1.184 \text{ kJ/kgK}}$$

## 9.95

Air at 327°C, 400 kPa with a volume flow 1 m<sup>3</sup>/s runs through an adiabatic turbine with exhaust pressure of 100 kPa. Neglect kinetic energies and use constant specific heats. Find the lowest and highest possible exit temperature. For each case find also the rate of work and the rate of entropy generation.

Solution:

C.V Turbine. Steady single inlet and exit flows,  $q = 0$ .

Inlet state: (T, P)  $v_i = RT_i / P_i = 0.287 \times 600 / 400 = 0.4305 \text{ m}^3/\text{kg}$

$$\dot{m} = \dot{V} / v_i = 1 / 0.4305 = 2.323 \text{ kg/s}$$

**The lowest exit T** is for maximum work out i.e. reversible case

Process: Reversible and adiabatic  $\Rightarrow$  constant  $s$  from Eq.9.8

$$\text{Eq.8.32: } T_e = T_i (P_e / P_i)^{\frac{k-1}{k}} = 600 \times (100/400)^{0.2857} = \mathbf{403.8 \text{ K}}$$

$$\Rightarrow w = h_i - h_e = C_{P0}(T_i - T_e) = 1.004 \times (600 - 403.8) = 197 \text{ kJ/kg}$$

$$\dot{W}_T = \dot{m}w = 2.323 \times 197 = \mathbf{457.6 \text{ kW}} \quad \text{and} \quad \dot{S}_{\text{gen}} = \mathbf{0}$$

**Highest exit T** occurs when there is no work out, throttling

$$q = \emptyset; \quad w = \emptyset \quad \Rightarrow \quad h_i - h_e = 0 \quad \Rightarrow \quad T_e = T_i = \mathbf{600 \text{ K}}$$

$$\dot{S}_{\text{gen}} = \dot{m}(s_e - s_i) = -\dot{m}R \ln \frac{P_e}{P_i} = -2.323 \times 0.287 \ln \frac{100}{400} = \mathbf{0.924 \text{ kW/K}}$$

## 9.96

In a heat-driven refrigerator with ammonia as the working fluid, a turbine with inlet conditions of 2.0 MPa, 70°C is used to drive a compressor with inlet saturated vapor at -20°C. The exhausts, both at 1.2 MPa, are then mixed together. The ratio of the mass flow rate to the turbine to the total exit flow was measured to be 0.62. Can this be true?

Solution:

Assume the compressor and the turbine are both adiabatic.

C.V. Total:

$$\text{Continuity Eq. 6.11: } \dot{m}_5 = \dot{m}_1 + \dot{m}_3$$

$$\text{Energy Eq. 6.10: } \dot{m}_5 h_5 = \dot{m}_1 h_1 + \dot{m}_3 h_3$$

$$\text{Entropy: } \dot{m}_5 s_5 = \dot{m}_1 s_1 + \dot{m}_3 s_3 + \dot{S}_{C.V.,gen}$$

$$s_5 = y s_1 + (1-y) s_3 + \dot{S}_{C.V.,gen}/\dot{m}_5$$

$$\text{Assume } y = \dot{m}_1/\dot{m}_5 = 0.62$$

$$\text{State 1: Table B.2.2 } h_1 = 1542.7 \text{ kJ/kg, } s_1 = 4.982 \text{ kJ/kg K,}$$

$$\text{State 3: Table B.2.1 } h_3 = 1418.1 \text{ kJ/kg, } s_3 = 5.616 \text{ kJ/kg K}$$

Solve for exit state 5 in the energy equation

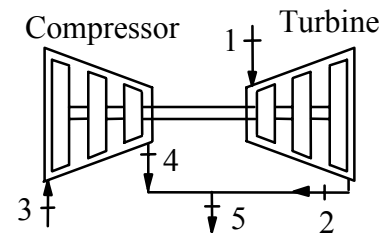
$$h_5 = y h_1 + (1-y) h_3 = 0.62 \times 1542.7 + (1 - 0.62) 1418.1 = 1495.4 \text{ kJ/kg}$$

$$\text{State 5: } h_5 = 1495.4 \text{ kJ/kg, } P_5 = 1200 \text{ kPa} \Rightarrow s_5 = 5.056 \text{ kJ/kg K}$$

Now check the 2nd law, entropy generation

$$\Rightarrow \dot{S}_{C.V.,gen}/\dot{m}_5 = s_5 - y s_1 - (1-y) s_3 = \mathbf{-0.1669 \text{ Impossible}}$$

The problem could also have been solved assuming a reversible process and then find the needed flow rate ratio  $y$ . Then  $y$  would have been found larger than 0.62 so the stated process can not be true.



## 9.97

A large supply line has a steady air flow at 500 K, 200 kPa. It is used in three different adiabatic devices shown in Fig. P9.85, a throttle flow, an ideal nozzle and an ideal turbine. All the exit flows are at 100 kPa. Find the exit temperature and specific entropy generation for each device and the exit velocity of the nozzle.

**C.V. Throttle**, Steady single inlet and exit flow, no work or heat transfer.

$$\text{Energy Eq.6.13: } h_i = h_e \quad (Z_i = Z_e \text{ and } V\text{'s are small})$$

$$\text{Entropy Eq.9.8: } s_e = s_i + \int dq/T + s_{\text{gen}} = s_i + 0 + s_{\text{gen}}$$

Since it is air we have  $h = h(T)$  so same  $h$  means same  $T_e = T_i = 500 \text{ K}$

$$s_{\text{gen}} = s_e - s_i = s_{T_e}^0 - s_{T_i}^0 - R \ln(P_e / P_i) = 0 - 0.287 \ln(1/2) = 0.2 \text{ kJ/kg K}$$

**C.V. Nozzle**, Steady single inlet and exit flow, no work or heat transfer.

$$\text{Energy Eq.6.13: } h_i = h_e + V_e^2/2 \quad (Z_i = Z_e)$$

$$\text{Entropy Eq.9.8: } s_e = s_i + \int dq/T + s_{\text{gen}} = s_i + 0 + 0$$

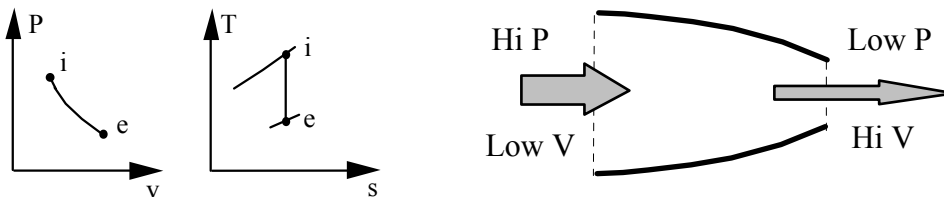
Use constant specific heat from Table A.5,  $C_{P_0} = 1.004 \frac{\text{kJ}}{\text{kg K}}$ ,  $k = 1.4$

The isentropic process ( $s_e = s_i$ ) gives Eq.8.32

$$\Rightarrow T_e = T_i (P_e/P_i)^{\frac{k-1}{k}} = 500 (100/200)^{0.2857} = 410 \text{ K}$$

The energy equation becomes:  $V_e^2/2 = h_i - h_e \cong C_p(T_i - T_e)$

$$V_e = \sqrt{2 C_p(T_i - T_e)} = \sqrt{2 \times 1.004(500 - 410) \times 1000} = 424 \text{ m/s}$$



**Turbine:**

Process: Reversible and adiabatic  $\Rightarrow$  constant  $s$  from Eq.9.8

$$\text{Eq.8.32: } T_e = T_i (P_e/P_i)^{\frac{k-1}{k}} = 500 \times (100/200)^{0.2857} = 410 \text{ K}$$

$$\Rightarrow w = h_i - h_e = C_{P_0}(T_i - T_e) = 1.004 \times (500 - 410) = 90 \text{ kJ/kg}$$



## 9.98

Repeat the previous problem for the throttle and the nozzle when the inlet air temperature is 2500 K and use the air tables.

**C.V. Throttle**, Steady single inlet and exit flow, no work or heat transfer.

$$\text{Energy Eq.6.13: } h_i = h_e \quad (Z_i = Z_e \text{ and } \mathbf{V}'\text{s are small})$$

$$\text{Entropy Eq.9.8: } s_e = s_i + \int dq/T + s_{\text{gen}} = s_i + 0 + s_{\text{gen}}$$

Since it is air we have  $h = h(T)$  so same  $h$  means same  $T_e = T_i = 2500 \text{ K}$

$$s_{\text{gen}} = s_e - s_i = s_{T_e}^o - s_{T_i}^o - R \ln(P_e / P_i) = 0 - 0.287 \ln(1/2) = 0.2 \text{ kJ/kg K}$$

**C.V. Nozzle**, Steady single inlet and exit flow, no work or heat transfer.

$$\text{Energy Eq.6.13: } h_i = h_e + \mathbf{V}_e^2/2 \quad (Z_i = Z_e)$$

$$\text{Entropy Eq.9.8: } s_e = s_i + \int dq/T + s_{\text{gen}} = s_i + 0 + 0$$

The isentropic process ( $s_e = s_i$ ) gives Eq.8.32

$$0 = s_e - s_i = s_{T_e}^o - s_{T_i}^o - R \ln(P_e / P_i)$$

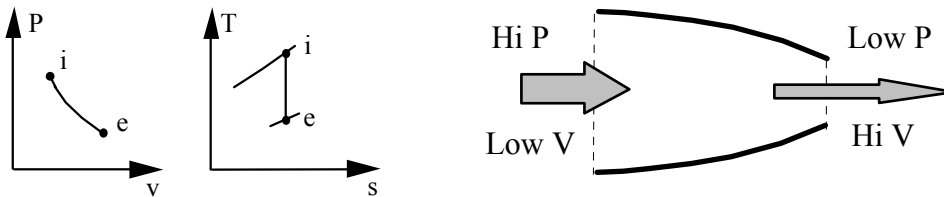
$$\Rightarrow s_{T_e}^o = s_{T_i}^o + R \ln(P_e / P_i) = 9.24781 + 0.287 \ln(100/200) = 9.04888$$

$$T = 2136.6 \text{ K}, \quad h_e = 2422.86 \text{ kJ/kg}$$

The energy equation becomes

$$\mathbf{V}_e^2/2 = h_i - h_e = 2883.06 - 2422.86 = 460.2 \text{ kJ/kg}$$

$$\mathbf{V}_e = \sqrt{2 \times 1000 \times 460.2} = 959 \text{ m/s}$$

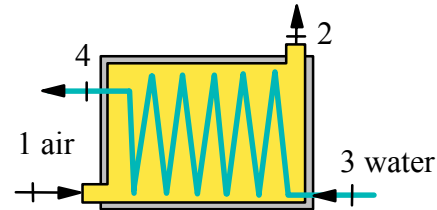


## 9.99

A counter flowing heat exchanger has one line with 2 kg/s at 125 kPa, 1000 K entering and the air is leaving at 100 kPa, 400 K. The other line has 0.5 kg/s water coming in at 200 kPa, 20°C and leaving at 200 kPa. What is the exit temperature of the water and the total rate of entropy generation?

Solution:

C.V. Heat exchanger, steady flow 1 inlet and 1 exit for air and water each. The two flows exchange energy with no heat transfer to/from the outside.



Energy Eq.6.10:  $\dot{m}_{\text{AIR}}\Delta h_{\text{AIR}} = \dot{m}_{\text{H}_2\text{O}}\Delta h_{\text{H}_2\text{O}}$

From A.7:  $h_1 - h_2 = 1046.22 - 401.3 = 644.92 \text{ kJ/kg}$

From B.1.2  $h_3 = 83.94 \text{ kJ/kg}; \quad s_3 = 0.2966 \text{ kJ/kg K}$

$$h_4 - h_3 = (\dot{m}_{\text{AIR}}/\dot{m}_{\text{H}_2\text{O}})(h_1 - h_2) = (2/0.5)644.92 = 2579.68 \text{ kJ/kg}$$

$$h_4 = h_3 + 2579.68 = 2663.62 \text{ kJ/kg} < h_g \quad \text{at } 200 \text{ kPa}$$

$$T_4 = T_{\text{sat}} = 120.23^\circ\text{C},$$

$$x_4 = (2663.62 - 504.68)/2201.96 = 0.9805,$$

$$s_4 = 1.53 + x_4 5.597 = 7.01786 \text{ kJ/kg K}$$

From entropy Eq.9.7

$$\begin{aligned} \dot{S}_{\text{gen}} &= \dot{m}_{\text{H}_2\text{O}}(s_4 - s_3) + \dot{m}_{\text{AIR}}(s_2 - s_1) \\ &= 0.5(7.01786 - 0.2966) + 2(7.1593 - 8.1349 - 0.287 \ln(100/125)) \\ &= 3.3606 - 1.823 = \mathbf{1.54 \text{ kW/K}} \end{aligned}$$

## 9.100

Saturated liquid nitrogen at 600 kPa enters a boiler at a rate of 0.005 kg/s and exits as saturated vapor. It then flows into a super heater also at 600 kPa where it exits at 600 kPa, 280 K. Assume the heat transfer comes from a 300 K source and find the rates of entropy generation in the boiler and the super heater.

Solution:

C.V.: boiler steady single inlet and exit flow, neglect KE, PE energies in flow

$$\text{Continuity Eq.: } \dot{m}_1 = \dot{m}_2 = \dot{m}_3$$

$$\text{Table B.6.1: } h_1 = -81.53 \text{ kJ/kg, } s_1 = 3.294 \text{ kJ/kgK,}$$

$$h_2 = 86.85 \text{ kJ/kg, } s_2 = 5.041 \text{ kJ/kgK}$$

$$\text{Table B.6.2: } h_3 = 289.05 \text{ kJ/kg, } s_3 = 6.238 \text{ kJ/kgK}$$

$$\text{Energy Eq.6.13: } q_{\text{boiler}} = h_2 - h_1 = 86.85 - (-81.53) = 168.38 \text{ kJ/kg}$$

$$\dot{Q}_{\text{boiler}} = \dot{m}_1 q_{\text{boiler}} = 0.005 \times 168.38 = 0.842 \text{ kW}$$

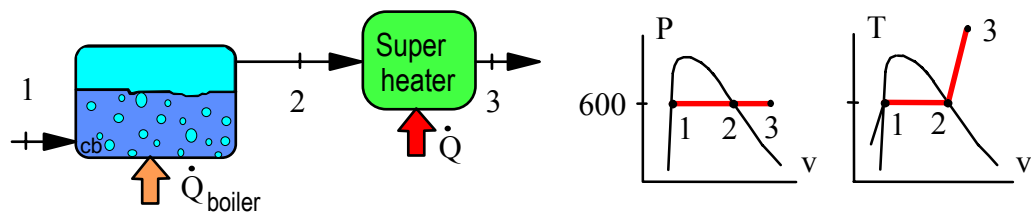
$$\begin{aligned} \text{Entropy Eq.: } \dot{S}_{\text{gen}} &= \dot{m}_1 (s_2 - s_1) - \dot{Q}_{\text{boiler}}/T_{\text{source}} \\ &= 0.005(5.041 - 3.294) - 0.842/300 = \mathbf{0.0059 \text{ kW/K}} \end{aligned}$$

C.V. Superheater (same approximations as for boiler)

$$\text{Energy Eq.6.13: } q_{\text{sup heater}} = h_3 - h_2 = 289.05 - 86.85 = 202.2 \text{ kJ/kg}$$

$$\dot{Q}_{\text{sup heater}} = \dot{m}_2 q_{\text{sup heater}} = 0.005 \times 202.2 = 1.01 \text{ kW}$$

$$\begin{aligned} \text{Entropy Eq.: } \dot{S}_{\text{gen}} &= \dot{m}_1 (s_3 - s_2) - \dot{Q}_{\text{sup heater}}/T_{\text{source}} \\ &= 0.005(6.238 - 5.041) - 1.01/300 = \mathbf{0.00262 \text{ kW/K}} \end{aligned}$$



## 9.101

One type of feedwater heater for preheating the water before entering a boiler operates on the principle of mixing the water with steam that has been bled from the turbine. For the states as shown in Fig. P9.59, calculate the rate of net entropy increase for the process, assuming the process to be steady flow and adiabatic.

Solution:

CV: Feedwater heater, Steady flow, no external heat transfer.

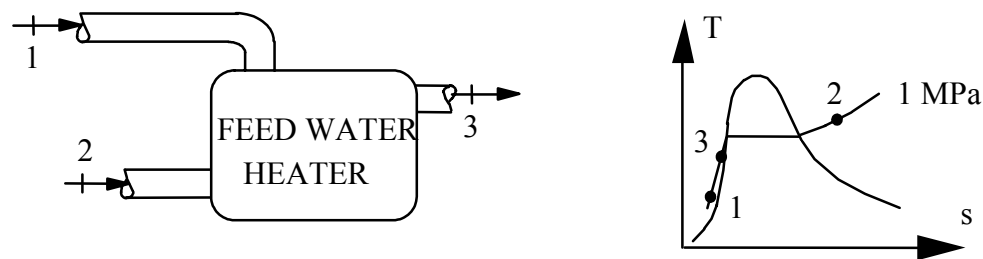
$$\text{Continuity Eq.6.9:} \quad \dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

$$\text{Energy Eq.6.10:} \quad \dot{m}_1 h_1 + (\dot{m}_3 - \dot{m}_1) h_2 = \dot{m}_3 h_3$$

Properties: All states are given by (P,T) table B.1.1 and B.1.3

$$h_1 = 168.42, \quad h_2 = 2828, \quad h_3 = 675.8 \quad \text{all kJ/kg}$$

$$s_1 = 0.572, \quad s_2 = 6.694, \quad s_3 = 1.9422 \quad \text{all kJ/kg K}$$



Solve for the flow rate from the energy equation

$$\dot{m}_1 = \frac{\dot{m}_3(h_3 - h_2)}{(h_1 - h_2)} = \frac{4(675.8 - 2828)}{(168.42 - 2828)} = 3.237 \text{ kg/s}$$

$$\Rightarrow \dot{m}_2 = 4 - 3.237 = 0.763 \text{ kg/s}$$

The second law for steady flow,  $\dot{S}_{CV} = 0$ , and no heat transfer, Eq.9.7:

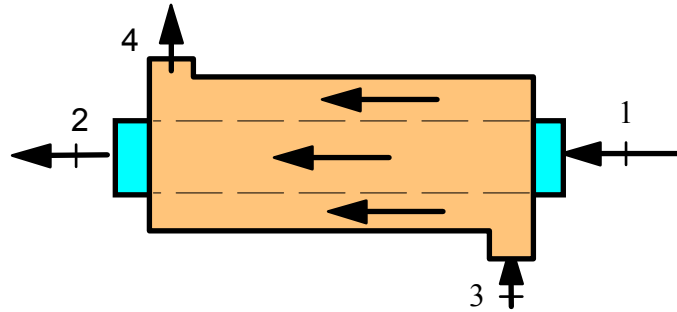
$$\begin{aligned} \dot{S}_{C.V.,gen} &= \dot{S}_{SURR} = \dot{m}_3 s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2 \\ &= 4(1.9422) - 3.237(0.572) - 0.763(6.694) = \mathbf{0.8097 \text{ kJ/K s}} \end{aligned}$$

## 9.102

A coflowing (same direction) heat exchanger has one line with 0.25 kg/s oxygen at 17°C, 200 kPa entering and the other line has 0.6 kg/s nitrogen at 150 kPa, 500 K entering. The heat exchanger is very long so the two flows exit at the same temperature. Use constant heat capacities and find the exit temperature and the total rate of entropy generation.

Solution:

C.V. Heat exchanger,  
steady 2 flows in and  
two flows out.



$$\text{Energy Eq.6.10: } \dot{m}_{\text{O}_2}h_1 + \dot{m}_{\text{N}_2}h_3 = \dot{m}_{\text{O}_2}h_2 + \dot{m}_{\text{N}_2}h_4$$

Same exit temperature so  $T_4 = T_2$  with values from Table A.5

$$\begin{aligned} \dot{m}_{\text{O}_2}C_{\text{P O}_2}T_1 + \dot{m}_{\text{N}_2}C_{\text{P N}_2}T_3 &= (\dot{m}_{\text{O}_2}C_{\text{P O}_2} + \dot{m}_{\text{N}_2}C_{\text{P N}_2})T_2 \\ T_2 &= \frac{0.25 \times 0.922 \times 290 + 0.6 \times 1.042 \times 500}{0.25 \times 0.922 + 0.6 \times 1.042} = \frac{379.45}{0.8557} \\ &= \mathbf{443.4 \text{ K}} \end{aligned}$$

Entropy Eq.9.7 gives for the generation

$$\begin{aligned} \dot{S}_{\text{gen}} &= \dot{m}_{\text{O}_2}(s_2 - s_1) + \dot{m}_{\text{N}_2}(s_4 - s_3) \\ &= \dot{m}_{\text{O}_2}C_{\text{P}} \ln(T_2/T_1) + \dot{m}_{\text{N}_2}C_{\text{P}} \ln(T_4/T_3) \\ &= 0.25 \times 0.922 \ln(443.4/290) + 0.6 \times 1.042 \ln(443.4/500) \\ &= 0.0979 - 0.0751 = \mathbf{0.0228 \text{ kW/K}} \end{aligned}$$

## 9.103

A steam turbine in a power plant receives steam at 3000 kPa, 500°C. The turbine has two exit flows, one is 20% of the flow at 1000 kPa, 350°C to a feedwater heater and the remainder flows out at 200 kPa, 200°C. Find the specific turbine work and the specific entropy generation both per kg flow in.

C.V. Steam turbine (  $x = 0.2 =$  extraction fraction)

$$\text{Energy Eq.6.13:} \quad w = h_1 - xh_2 - (1 - x)h_3$$

$$\text{Entropy Eq.9.8:} \quad s_2 = s_1 + s_{\text{gen HP}} \quad (\text{full flow rate})$$

$$\text{Entropy Eq.9.8:} \quad s_3 = s_2 + s_{\text{gen LP}} \quad [\text{flow rate is fraction } (1-x)]$$

$$\text{Overall entropy gen:} \quad s_{\text{gen HP}} = s_{\text{gen HP}} + (1 - x) s_{\text{gen LP}}$$

$$\text{Inlet state: Table B.1.3} \quad h_1 = 3456.48 \text{ kJ/kg}; \quad s_1 = 7.2337 \text{ kJ/kg K}$$

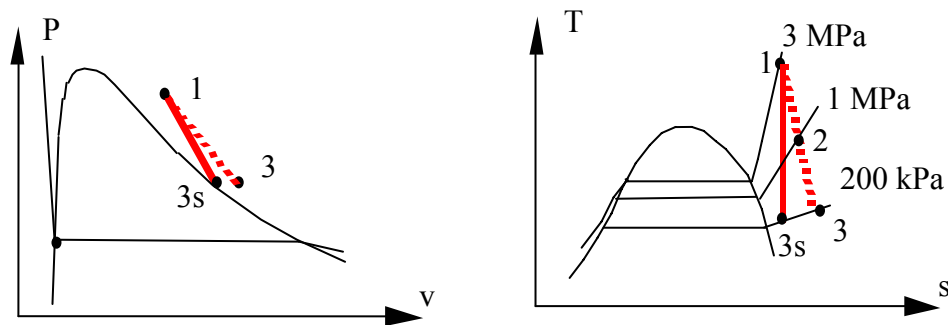
$$\text{Extraction state:} \quad h_2 = 3157.65 \text{ kJ/kg}, \quad s_2 = 7.3010 \text{ kJ/kg K}$$

$$\text{Exit (actual) state: Table B.1.3} \quad h_3 = 2870.46; \quad s_3 = 7.5066 \text{ kJ/kg K}$$

Actual turbine energy equation

$$w = 3456.48 - 0.2 \times 3157.65 - 0.8 \times 2870.46 = \mathbf{528.58 \text{ kJ/kg}}$$

$$s_{\text{gen tot}} = 7.301 - 7.2337 + 0.8 (7.5066 - 7.301) \\ = \mathbf{0.232 \text{ kJ/kgK}}$$



**9.104**

Carbon dioxide used as a natural refrigerant flows through a cooler at 10 MPa, which is supercritical so no condensation occurs. The inlet is at 200°C and the exit is at 40°C. Assume the heat transfer is to the ambient at 20°C and find the specific entropy generation.

C.V. Heat exchanger to ambient, steady constant pressure so no work.

Energy Eq.6.12:  $h_i = h_e + q_{\text{out}}$

Entropy Eq.9.8:  $s_i + s_{\text{gen}} = s_e + q_{\text{out}}/T$

Using Table B.3

$$q_{\text{out}} = (h_i - h_e) = 519.49 - 200.14 = 319.35 \text{ kJ/kg}$$

$$s_{\text{gen}} = s_e - s_i + q_{\text{out}}/T = 0.6906 - 1.5705 + \frac{319.35}{293.15 \text{ K}} = \mathbf{0.2095 \text{ kW/K}}$$

## 9.105

A supply of 5 kg/s ammonia at 500 kPa, 20°C is needed. Two sources are available one is saturated liquid at 20°C and the other is at 500 kPa and 140°C. Flows from the two sources are fed through valves to an insulated mixing chamber, which then produces the desired output state. Find the two source mass flow rates and the total rate of entropy generation by this setup.

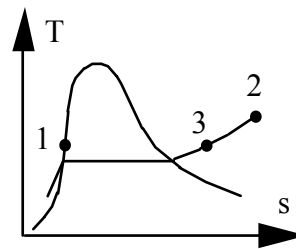
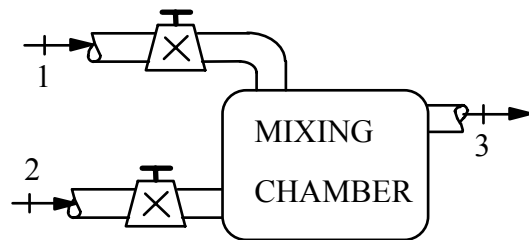
Solution:

C.V. mixing chamber + valve. Steady, no heat transfer, no work.

$$\text{Continuity Eq.6.9:} \quad \dot{m}_1 + \dot{m}_2 = \dot{m}_3;$$

$$\text{Energy Eq.6.10:} \quad \dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

$$\text{Entropy Eq.9.7:} \quad \dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{S}_{\text{gen}} = \dot{m}_3 s_3$$



$$\text{State 1: Table B.2.1} \quad h_1 = 273.4 \text{ kJ/kg}, \quad s_1 = 1.0408 \text{ kJ/kg K}$$

$$\text{State 2: Table B.2.2} \quad h_2 = 1773.8 \text{ kJ/kg}, \quad s_2 = 6.2422 \text{ kJ/kg K}$$

$$\text{State 3: Table B.2.2} \quad h_3 = 1488.3 \text{ kJ/kg}, \quad s_3 = 5.4244 \text{ kJ/kg K}$$

As all states are known the energy equation establishes the ratio of mass flow rates and the entropy equation provides the entropy generation.

$$\dot{m}_1 h_1 + (\dot{m}_3 - \dot{m}_2)h_2 = \dot{m}_3 h_3 \quad \Rightarrow \quad \dot{m}_1 = \dot{m}_3 \frac{h_3 - h_2}{h_1 - h_2} = 0.952 \text{ kg/s}$$

$$\dot{m}_2 = \dot{m}_3 - \dot{m}_1 = 4.05 \text{ kg/s}$$

$$\dot{S}_{\text{gen}} = 5 \times 5.4244 - 0.95 \times 1.0408 - 4.05 \times 6.2422 = \mathbf{0.852 \text{ kW/K}}$$



## **Transient processes**

## 9.106

Calculate the specific entropy generated in the filling process given in Example 6.11.

Solution:

C.V. Cannister filling process where:  ${}_1Q_2 = 0$ ;  ${}_1W_2 = 0$ ;  $m_1 = 0$

Continuity Eq.6.15:  $m_2 - 0 = m_{in}$ ;

Energy Eq.6.16:  $m_2u_2 - 0 = m_{in}h_{line} + 0 + 0 \Rightarrow u_2 = h_{line}$

Entropy Eq.9.11:  $m_2s_2 - 0 = m_{in}s_{line} + 0 + {}_1S_2_{gen}$

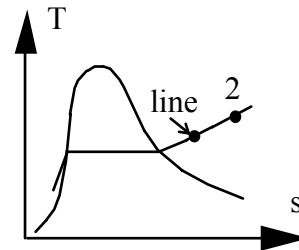
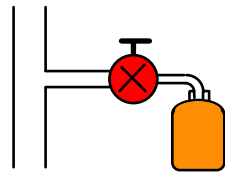
Inlet state : 1.4 MPa, 300°C,  $h_i = 3040.4$  kJ/kg,  $s_i = 6.9533$  kJ/kg K

final state: 1.4 MPa,  $u_2 = h_i = 3040.4$  kJ/kg

$$\Rightarrow T_2 = 452^\circ\text{C}, s_2 = 7.45896 \text{ kJ/kg K}$$

$${}_1S_2_{gen} = m_2(s_2 - s_i)$$

$${}_1S_2_{gen} = s_2 - s_i = 7.45896 - 6.9533 = \mathbf{0.506 \text{ kJ/kg K}}$$



## 9.107

An initially empty  $0.1 \text{ m}^3$  cannister is filled with R-410a from a line flowing saturated liquid at  $-5^\circ\text{C}$ . This is done quickly such that the process is adiabatic. Find the final mass, liquid and vapor volumes, if any, in the cannister. Is the process reversible?

Solution:

C.V. Cannister filling process where:  ${}_1Q_2 = 0$ ;  ${}_1W_2 = 0$ ;  $m_1 = 0$

Continuity Eq.6.15:  $m_2 - 0 = m_{\text{in}}$ ;

Energy Eq.6.16:  $m_2 u_2 - 0 = m_{\text{in}} h_{\text{line}} + 0 + 0 \Rightarrow u_2 = h_{\text{line}}$

1: Table B.4.1  $u_f = 49.65$ ,  $u_{fg} = 137.16$ ,  $h_f = 50.22$  all kJ/kg

2:  $P_2 = P_{\text{line}}$ ;  $u_2 = h_{\text{line}} \Rightarrow$  2 phase  $u_2 > u_f$ ;  $u_2 = u_f + x_2 u_{fg}$

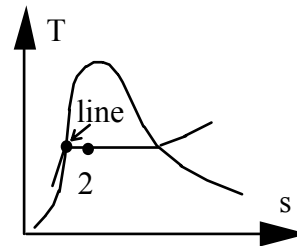
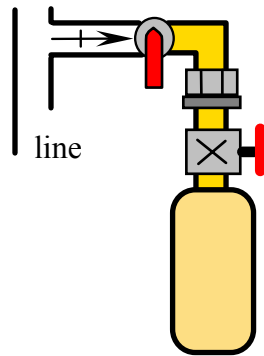
$$x_2 = (50.22 - 49.65)/137.16 = 0.004156$$

$$\Rightarrow v_2 = v_f + x_2 v_{fg} = 0.000841 + 0.004156 \times 0.03764 = 0.0009974 \text{ m}^3/\text{kg}$$

$$\Rightarrow m_2 = V/v_2 = \mathbf{100.26 \text{ kg}}; \quad m_f = 99.843 \text{ kg}; \quad m_g = 0.417 \text{ kg}$$

$$V_f = m_f v_f = \mathbf{0.084 \text{ m}^3}; \quad V_g = m_g v_g = \mathbf{0.016 \text{ m}^3}$$

**Process is irreversible** (throttling)  $s_2 > s_f$



**9.108**

Calculate the total entropy generated in the filling process given in Example 6.12.

Solution:

Since the solution to the problem is done in the example we will just add the second law analysis to that.

Initial state: Table B.1.2:  $s_1 = 6.9404 \text{ kJ/kg K}$

Final state: Table B.1.3:  $s_2 = 6.9533 + \frac{42}{50} \times (7.1359 - 6.9533) = 7.1067 \frac{\text{kJ}}{\text{kg K}}$

Inlet state: Table B.1.3:  $s_i = 6.9533 \text{ kJ/kg K}$

Entropy Eq.9.11:  $m_2 s_2 - m_1 s_1 = m_i s_i + {}_1S_2 \text{ gen}$

Now solve for the generation

$$\begin{aligned} {}_1S_2 \text{ gen} &= m_2 s_2 - m_1 s_1 - m_i s_i \\ &= 2.026 \times 7.1067 - 0.763 \times 6.9404 - 1.263 \times 6.9533 \\ &= \mathbf{0.32 \text{ kJ/K} > 0} \end{aligned}$$

## 9.109

A 1-m<sup>3</sup> rigid tank contains 100 kg R-410a at ambient temperature, 15°C. A valve on top of the tank is opened, and saturated vapor is throttled to ambient pressure, 100 kPa, and flows to a collector system. During the process the temperature inside the tank remains at 15°C. The valve is closed when no more liquid remains inside. Calculate the heat transfer to the tank and total entropy generation in the process.

Solution:

C.V. Tank out to surroundings. Rigid tank so no work term.

Continuity Eq.6.15:  $m_2 - m_1 = -m_e$  ;

Energy Eq.6.16:  $m_2u_2 - m_1u_1 = Q_{CV} - m_e h_e$

Entropy Eq.9.11:  $m_2s_2 - m_1s_1 = Q_{CV}/T_{SUR} - m_e s_e + S_{gen}$

State 1: Table B.4.1,  $v_1 = V_1/m_1 = 1/100 = 0.000904 + x_1 0.01955$

$$x_1 = 0.46527, \quad u_1 = 80.02 + 0.46527 \times 177.1 = 162.42 \text{ kJ/kg}$$

$$s_1 = 0.3083 + 0.46527 \times 0.6998 = 0.6339; \quad h_e = h_g = 282.79 \text{ kJ/kg}$$

State 2:  $v_2 = v_g = 0.02045$ ,  $u_2 = u_g = 257.12$ ,  $s_2 = 1.0081 \text{ kJ/kg K}$

Exit state:  $h_e = 282.79$ ,  $P_e = 100 \text{ kPa} \rightarrow T_e = -18.65^\circ\text{C}$ ,  $s_e = 1.2917 \text{ kJ/kgK}$

$$m_2 = 1/0.02045 = 48.9 \text{ kg}; \quad m_e = 100 - 48.9 = 51.1 \text{ kg}$$

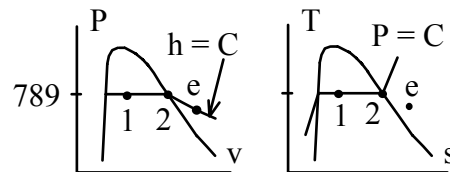
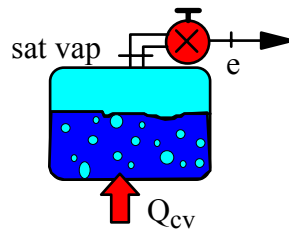
$$Q_{CV} = m_2u_2 - m_1u_1 + m_e h_e$$

$$= 48.9 \times 257.12 - 100 \times 162.42 + 51.1 \times 282.79 = \mathbf{10\ 782 \text{ kJ}}$$

$$S_{gen} = m_2s_2 - m_1s_1 + m_e s_e - Q_{CV}/T_{SUR}$$

$$= 48.9 \times 1.0081 - 100 \times 0.6339 + 51.1 \times 1.2917 - 10\ 782 / 288.15$$

$$= \mathbf{14.5 \text{ kJ/K}}$$



## 9.110

A 1 L can of R-134a is at room temperature 20°C with a quality of 50%. A leak in the top valve allows vapor to escape and heat transfer from the room takes place so we reach final state of 5°C with a quality of 100%. Find the mass that escaped, the heat transfer and the entropy generation not including that made in the valve.

CV The can of R-134a not including the nozzle/valve out to ambient 20°C

$$\text{Continuity Eq.: } m_2 - m_1 = -m_e$$

$$\text{Energy Eq.: } m_2 u_2 - m_1 u_1 = -m_e h_e + {}_1Q_2 - {}_1W_2$$

$$\text{Entropy Eq.: } m_2 s_2 - m_1 s_1 = -m_e s_e + \int dQ/T + {}_1S_2 \text{ gen}$$

$$\text{Process Eq.: } V = \text{constant} \Rightarrow {}_1W_2 = \int PdV = 0$$

$$\text{State 1: (T,x)} \quad v_1 = v_f + x_1 v_{fg} = 0.000817 + 0.5 \cdot 0.03524 = 0.018437 \text{ m}^3/\text{kg}$$

$$u_1 = u_f + x_1 u_{fg} = 227.03 + 0.5 \cdot 162.16 = 308.11 \text{ kJ/kg}$$

$$s_1 = s_f + x_1 s_{fg} = 1.0963 + 0.5 \cdot 0.622 = 1.4073 \text{ kJ/kg K}$$

$$m_1 = V / v_1 = 0.001 / 0.018437 = 0.05424 \text{ kg}$$

State 2: (T,x)

$$v_2 = v_g = 0.05833 \text{ m}^3/\text{kg}, \quad u_2 = u_g = 380.85 \text{ kJ/kg}, \quad s_2 = s_g = 1.7239 \text{ kJ/kg K}$$

$$m_2 = V/v_2 = 0.001 / 0.05833 = 0.017144 \text{ kg}$$

Exit state e: Saturated vapor starting at 20°C ending at 5°C so we take an average

$$h_e = 0.5(h_{e1} + h_{e2}) = 0.5 (409.84 + 401.32) = 405.58 \text{ kJ/kg}$$

$$s_e = 0.5(s_{e1} + s_{e2}) = 0.5 (1.7183 + 1.7239) = 1.7211 \text{ kJ/kg K}$$

$$m_e = m_1 - m_2 = \mathbf{0.0371 \text{ kg}}$$

The heat transfer from the energy equation becomes

$${}_1Q_2 = m_2 u_2 - m_1 u_1 + m_e h_e = 6.5293 - 16.7119 + 15.047 = \mathbf{4.864 \text{ kJ}}$$

$${}_1S_2 \text{ gen} = m_2 s_2 - m_1 s_1 + m_e s_e - {}_1Q_2/T_{\text{amb}}$$

$$= 0.029555 - 0.076332 + 0.063853 - 0.016592 = \mathbf{0.000484 \text{ kJ/K}}$$

## 9.111

An empty cannister of  $0.002 \text{ m}^3$  is filled with R-134a from a line flowing saturated liquid R-134a at  $0^\circ\text{C}$ . The filling is done quickly so it is adiabatic. Find the final mass in the cannister and the total entropy generation.

Solution:

C.V. Cannister filling process where:  ${}_1Q_2 = 0$ ;  ${}_1W_2 = 0$ ;  $m_1 = 0$

Continuity Eq.6.15:  $m_2 - 0 = m_{\text{in}}$  ;

Energy Eq.6.16:  $m_2 u_2 - 0 = m_{\text{in}} h_{\text{line}} + 0 + 0 \Rightarrow u_2 = h_{\text{line}}$

Entropy Eq.9.11:  $m_2 s_2 - 0 = m_{\text{in}} s_{\text{line}} + 0 + {}_1S_2_{\text{gen}}$

Inlet state: Table B.5.1  $h_{\text{line}} = 200 \text{ kJ/kg}$ ,  $s_{\text{line}} = 1.0 \text{ kJ/kg K}$

State 2:  $P_2 = P_{\text{line}}$  and  $u_2 = h_{\text{line}} = 200 \text{ kJ/kg} > u_f$

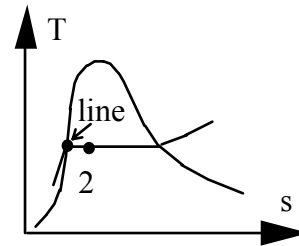
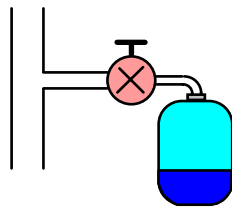
$$x_2 = (200 - 199.77) / 178.24 = 0.00129$$

$$v_2 = 0.000773 + x_2 0.06842 = 0.000861 \text{ m}^3/\text{kg}$$

$$s_2 = 1.0 + x_2 0.7262 = 1.000937 \text{ kJ/kg K}$$

$$m_2 = V / v_2 = 0.002 / 0.000861 = \mathbf{2.323 \text{ kg}}$$

$${}_1S_2_{\text{gen}} = m_2(s_2 - s_{\text{line}}) = 2.323 (1.00094 - 1) = \mathbf{0.00218 \text{ kJ/K}}$$



## 9.112

An  $0.2 \text{ m}^3$  initially empty container is filled with water from a line at  $500 \text{ kPa}$ ,  $200^\circ\text{C}$  until there is no more flow. Assume the process is adiabatic and find the final mass, final temperature and the total entropy generation.

Solution:

C.V. The container volume and any valve out to line.

$$\text{Continuity Eq.6.15: } m_2 - m_1 = m_2 = m_i$$

$$\text{Energy Eq.6.16: } m_2 u_2 - m_1 u_1 = m_2 u_2 = {}_1Q_2 - {}_1W_2 + m_i h_i = m_i h_i$$

$$\text{Entropy Eq.9.11: } m_2 s_2 - m_1 s_1 = m_2 s_2 = \int dQ/T + {}_1S_{2 \text{ gen}} + m_i s_i$$

Process: Adiabatic  ${}_1Q_2 = 0$ , Rigid  ${}_1W_2 = 0$  Flow stops  $P_2 = P_{\text{line}}$

$$\text{State i: } h_i = 2855.37 \text{ kJ/kg}; \quad s_i = 7.0592 \text{ kJ/kg K}$$

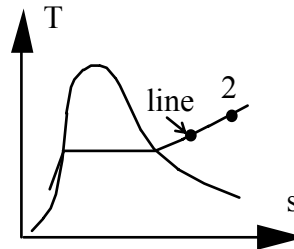
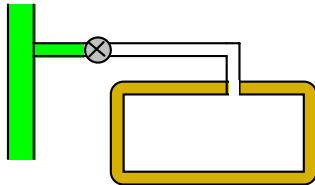
$$\text{State 2: } 500 \text{ kPa, } u_2 = h_i = 2855.37 \text{ kJ/kg} \Rightarrow \text{Table B.1.3}$$

$$T_2 \cong 332.9^\circ\text{C}, \quad s_2 = 7.5737 \text{ kJ/kg}, \quad v_2 = 0.55387 \text{ m}^3/\text{kg}$$

$$m_2 = V/v_2 = 0.2/0.55387 = \mathbf{0.361 \text{ kg}}$$

From the entropy equation

$$\begin{aligned} {}_1S_{2 \text{ gen}} &= m_2 s_2 - m_2 s_i \\ &= 0.361(7.5737 - 7.0592) = \mathbf{0.186 \text{ kJ/K}} \end{aligned}$$





## 9.113

A 10 m tall 0.1 m diameter pipe is filled with liquid water at 20°C. It is open at the top to the atmosphere, 100 kPa, and a small nozzle is mounted in the bottom. The water is now let out through the nozzle splashing out to the ground until the pipe is empty. Find the water initial exit velocity, the average kinetic energy in the exit flow and the total entropy generation for the process.

$$\text{Total mass: } m = \rho AH = \rho \frac{\pi}{4} D^2 H = 998 \frac{\pi}{4} 0.1^2 \times 10 = 78.383 \text{ kg}$$

$$\text{Bernoulli: } \frac{1}{2} \mathbf{V}^2 = gH \quad \Rightarrow \quad \mathbf{V}_1 = \sqrt{2gH_1} = \sqrt{2 \times 9.807 \times 10} = \mathbf{14 \text{ m/s}}$$

$$\frac{1}{2} \mathbf{V}_{\text{avg}}^2 = gH_{\text{avg}} = g \frac{1}{2} H_1 = 9.807 \times 5 = \mathbf{49 \text{ m}^2/\text{s}^2} \quad (\text{J/kg})$$

All the energy (average kinetic energy) is dispersed in the ambient at 20°C so

$$S_{\text{gen}} = \frac{Q}{T} = \frac{m}{2T} \mathbf{V}_{\text{avg}}^2 = \frac{78.383 \text{ kg} \times 49 \text{ J/kg}}{293.15 \text{ K}} = \mathbf{13.1 \text{ J/K}}$$

## 9.114

Air from a line at 12 MPa, 15°C, flows into a 500-L rigid tank that initially contained air at ambient conditions, 100 kPa, 15°C. The process occurs rapidly and is essentially adiabatic. The valve is closed when the pressure inside reaches some value,  $P_2$ . The tank eventually cools to room temperature, at which time the pressure inside is 5 MPa. What is the pressure  $P_2$ ? What is the net entropy change for the overall process?

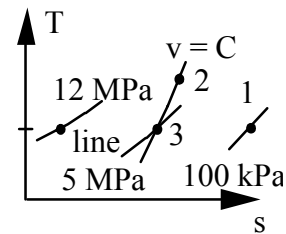
Solution:

CV: Tank. Mass flows in, so this is transient. Find the mass first

$$m_1 = P_1 V / RT_1 = \frac{100 \times 0.5}{0.287 \times 288.2} = 0.604 \text{ kg}$$

Fill to  $P_2$ , then cool to  $T_3 = 15^\circ\text{C}$ ,  $P_3 = 5 \text{ MPa}$

$$\begin{aligned} m_3 = m_2 = P_3 V / RT_3 \\ = \frac{5000 \times 0.5}{0.287 \times 288.2} = 30.225 \text{ kg} \end{aligned}$$



$$\text{Mass: } m_i = m_2 - m_1 = 30.225 - 0.604 = 29.621 \text{ kg}$$

In the process 1-2 heat transfer = 0

$$\text{1st law Eq.6.16: } m_i h_i = m_2 u_2 - m_1 u_1 ; \quad m_i C_{p0} T_i = m_2 C_{v0} T_2 - m_1 C_{v0} T_1$$

$$T_2 = \frac{(29.621 \times 1.004 + 0.604 \times 0.717) \times 288.2}{30.225 \times 0.717} = 401.2 \text{ K}$$

$$P_2 = m_2 R T_2 / V = (30.225 \times 0.287 \times 401.2) / 0.5 = \mathbf{6.960 \text{ MPa}}$$

Consider now the total process from the start to the finish at state 3.

$$\text{Energy Eq.6.16: } Q_{CV} + m_i h_i = m_2 u_3 - m_1 u_1 = m_2 h_3 - m_1 h_1 - (P_3 - P_1) V$$

$$\text{But, since } T_i = T_3 = T_1, \quad m_i h_i = m_2 h_3 - m_1 h_1$$

$$\Rightarrow Q_{CV} = -(P_3 - P_1) V = -(5000 - 100) 0.5 = -2450 \text{ kJ}$$

From Eqs.9.24-9.26

$$\Delta S_{NET} = m_3 s_3 - m_1 s_1 - m_i s_i - Q_{CV} / T_0 = m_3 (s_3 - s_i) - m_1 (s_1 - s_i) - Q_{CV} / T_0$$

$$= 30.225 \left[ 0 - 0.287 \ln \frac{5}{12} \right] - 0.604 \left[ 0 - 0.287 \ln \frac{0.1}{12} \right] + (2450 / 288.2)$$

$$= \mathbf{15.265 \text{ kJ/K}}$$

## 9.115

An initially empty canister of volume  $0.2 \text{ m}^3$  is filled with carbon dioxide from a line at  $1000 \text{ kPa}$ ,  $500 \text{ K}$ . Assume the process is adiabatic and the flow continues until it stops by itself. Use constant heat capacity to solve for the final mass and temperature of the carbon dioxide in the canister and the total entropy generated by the process.

Solution:

C.V. Cannister + valve out to line. No boundary/shaft work,  $m_1 = 0$ ;  $Q = 0$ .

$$\text{Continuity Eq.6.15: } m_2 - 0 = m_i$$

$$\text{Energy Eq.6.16: } m_2 u_2 - 0 = m_i h_i$$

$$\text{Entropy Eq.9.11: } m_2 s_2 - 0 = m_i s_i + {}_1S_2 \text{ gen}$$

$$\text{State 2: } P_2 = P_i \text{ and } u_2 = h_i = h_{\text{line}} = h_2 - RT_2 \quad (\text{ideal gas})$$

$$\text{To reduce or eliminate guess use: } h_2 - h_{\text{line}} = C_{P0}(T_2 - T_{\text{line}})$$

$$\text{Energy Eq. becomes: } C_{P0}(T_2 - T_{\text{line}}) - RT_2 = 0$$

$$T_2 = T_{\text{line}} C_{P0}/(C_{P0} - R) = T_{\text{line}} C_{P0}/C_{V0} = k T_{\text{line}}$$

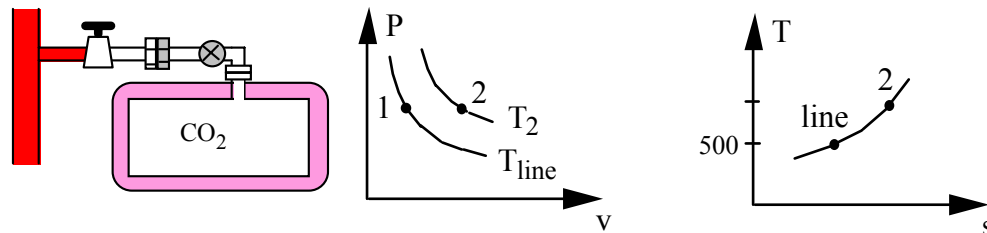
$$\text{Use A.5: } C_p = 0.842 \frac{\text{kJ}}{\text{kg K}}, k = 1.289 \Rightarrow T_2 = 1.289 \times 500 = \mathbf{644.5 \text{ K}}$$

$$m_2 = P_2 V / RT_2 = 1000 \times 0.2 / (0.1889 \times 644.5) = \mathbf{1.643 \text{ kg}}$$

$$\begin{aligned} {}_1S_2 \text{ gen} &= m_2 (s_2 - s_i) = m_2 [C_p \ln(T_2 / T_{\text{line}}) - R \ln(P_2 / P_{\text{line}})] \\ &= 1.644 [0.842 \times \ln(1.289) - 0] = \mathbf{0.351 \text{ kJ/K}} \end{aligned}$$

$$\text{If we use A.8 at } 550 \text{ K: } C_p = 1.045 \frac{\text{kJ}}{\text{kg K}}, k = 1.22$$

$$\Rightarrow T_2 = 610 \text{ K}, m_2 = 1.735 \text{ kg}$$



## 9.116

A can of volume  $0.2 \text{ m}^3$  is empty and filled with carbon dioxide from a line at  $3000 \text{ kPa}$ ,  $60^\circ\text{C}$ . The process is adiabatic and stops when the can is full. Use Table B.3 to find the final temperature and the entropy generation.

Solution:

C.V. Cannister filling process where:  ${}_1Q_2 = 0$ ;  ${}_1W_2 = 0$ ;  $m_1 = 0$

Continuity Eq.6.15:  $m_2 - 0 = m_{\text{in}}$ ;

Energy Eq.6.16:  $m_2 u_2 - 0 = m_{\text{in}} h_{\text{line}} + 0 + 0 \Rightarrow u_2 = h_{\text{line}}$

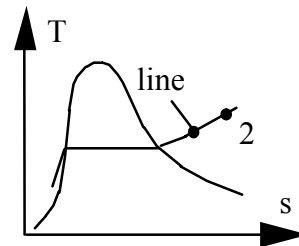
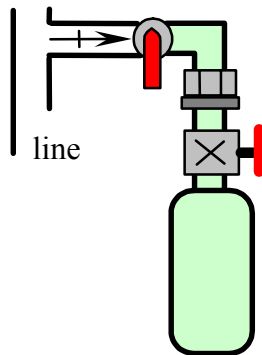
Entropy Eq.9.11:  $m_2 s_2 - 0 = m_{\text{in}} s_{\text{line}} + {}_1S_2_{\text{gen}}$

1: Table B.3.2  $h_{\text{line}} = 400.19 \text{ kJ/kg}$ ,  $s_{\text{line}} = 1.4773 \text{ kJ/kg-K}$

2:  $P_2 = P_{\text{line}}$ ;  $u_2 = h_{\text{line}} \Rightarrow T_2 = \mathbf{129.6^\circ\text{C}}$ ,  $s_2 = 1.6735 \text{ kJ/kg-K}$

$v_2 = 0.024012 \text{ m}^3/\text{kg} \Rightarrow m_2 = V/v_2 = 8.329 \text{ kg}$

${}_1S_2_{\text{gen}} = m_2 s_2 - m_{\text{in}} s_{\text{line}} = m_2 (s_2 - s_{\text{line}})$   
 $= 8.329 (1.6735 - 1.4773) = \mathbf{1.634 \text{ kJ/K}}$



## 9.117

A cook filled a pressure cooker with 3 kg water at 20°C and a small amount of air and forgot about it. The pressure cooker has a vent valve so if  $P > 200$  kPa steam escapes to maintain a pressure of 200 kPa. How much entropy was generated in the throttling of the steam through the vent to 100 kPa when half the original mass has escaped?

Solution:

The pressure cooker goes through a transient process as it heats water up to the boiling temperature at 200 kPa then heats more as saturated vapor at 200 kPa escapes. The throttling process is steady state as it flows from saturated vapor at 200 kPa to 100 kPa which we assume is a constant  $h$  process.

C.V. Pressure cooker, no work.

$$\text{Continuity Eq.6.15:} \quad m_2 - m_1 = -m_e$$

$$\text{Energy Eq.6.16:} \quad m_2 u_2 - m_1 u_1 = -m_e h_e + {}_1Q_2$$

$$\text{Entropy Eq.9.11:} \quad m_2 s_2 - m_1 s_1 = -m_e s_e + \int dQ/T + {}_1S_2 \text{ gen}$$

$$\text{State 1: } v_1 = v_f = 0.001002 \text{ m}^3/\text{kg} \quad V = m_1 v_1 = 0.003006 \text{ m}^3$$

$$\text{State 2: } m_2 = m_1/2 = 1.5 \text{ kg, } v_2 = V/m_2 = 2v_1, P_2 = 200 \text{ kPa}$$

$$\text{Exit: } h_e = h_g = 2706.63 \text{ kJ/kg, } s_e = s_g = 7.1271 \text{ kJ/kg K}$$

So we can find the needed heat transfer and entropy generation if we know the C.V. surface temperature  $T$ . If we assume  $T$  for water then  ${}_1S_2 \text{ gen} = 0$ , which is an internally reversible externally irreversible process, there is a  $\Delta T$  between the water and the source.

C.V. Valve, steady flow from state e (200 kPa) to state 3 (at 100 kPa).

$$\text{Energy Eq.:} \quad h_3 = h_e$$

$$\text{Entropy Eq.:} \quad s_3 = s_e + eS_3 \text{ gen} \quad \text{generation in valve (throttle)}$$

$$\text{State 3: } 100 \text{ kPa, } h_3 = 2706.63 \text{ kJ/kg} \quad \text{Table B.1.3} \Rightarrow$$

$$T_3 = 99.62 + (150 - 99.62) \frac{2706.63 - 2675.46}{2776.38 - 2675.46} = 115.2^\circ\text{C}$$

$$s_3 = 7.3593 + (7.6133 - 7.3593) 0.30886 = 7.4378 \text{ kJ/kg K}$$

$$eS_3 \text{ gen} = m_e(s_3 - s_e) = 1.5 (7.4378 - 7.1271) = \mathbf{0.466 \text{ kJ/K}}$$

## 9.118

A balloon is filled with air from a line at 200 kPa, 300 K to a final state of 110 kPa, 300 K with a mass of 0.1 kg air. Assume the pressure is proportional to the balloon volume as:  $P = 100 \text{ kPa} + CV$ . Find the heat transfer to/from the ambient at 300 K and the total entropy generation.

C.V. Balloon out to the ambient. Assume  $m_1 = 0$

Continuity Eq.6.15:  $m_2 - 0 = m_{in}$  ;

Energy Eq.6.16:  $m_2 u_2 - 0 = m_{in} h_{in} + {}_1Q_2 - {}_1W_2$

Entropy Eq.9.11:  $m_2 s_2 - 0 = m_{in} s_{in} + \int \frac{dQ}{T} + {}_1S_2 \text{ gen} = m_{in} s_{in} + \frac{{}_1Q_2}{T} + {}_1S_2 \text{ gen}$

Process Eq.:  $P = A + C V$ ,  $A = 100 \text{ kPa}$

State 2 (P, T):

$$V_2 = m_2 R T_2 / P_2 = \frac{0.1 \times 0.287 \times 300}{110} = 0.078273 \text{ m}^3$$

$$P_2 = A + C V_2 \quad \Rightarrow \quad C = (P_2 - 100) / V_2 = 127.758 \text{ kPa/m}^3$$

Inlet state:  $h_{in} = h_2 = u_2 + P_2 v_2$ ,  $s_{in} = s_2 - R \ln\left(\frac{P_{in}}{P_2}\right)$

$$\begin{aligned} {}_1W_2 &= \int P dV = \int A + C V dV = A (V_2 - 0) + \frac{1}{2} C (V_2^2 - 0) \\ &= 100 \times 0.078273 + \frac{1}{2} 127.758 \times 0.078273^2 \\ &= 8.219 \text{ kJ} \quad [ = \frac{1}{2} (P_o + P_2) V_2 = \text{area in P-V diagram} ] \end{aligned}$$

$$\begin{aligned} {}_1Q_2 &= m_2 (u_2 - h_{line}) + {}_1W_2 = -P_2 V_2 + {}_1W_2 \\ &= -110 \times 0.078273 + 8.219 = \mathbf{-0.391 \text{ kJ}} \end{aligned}$$

$$\begin{aligned} {}_1S_2 \text{ gen} &= m_2 (s_2 - s_{in}) - \frac{{}_1Q_2}{T} = m_2 R \ln\left(\frac{P_{in}}{P_2}\right) - \frac{{}_1Q_2}{T} \\ &= 0.1 \times 0.287 \ln\left(\frac{200}{110}\right) + \frac{0.391}{300} = \mathbf{0.0185 \text{ kJ/K}} \end{aligned}$$

## Device efficiency

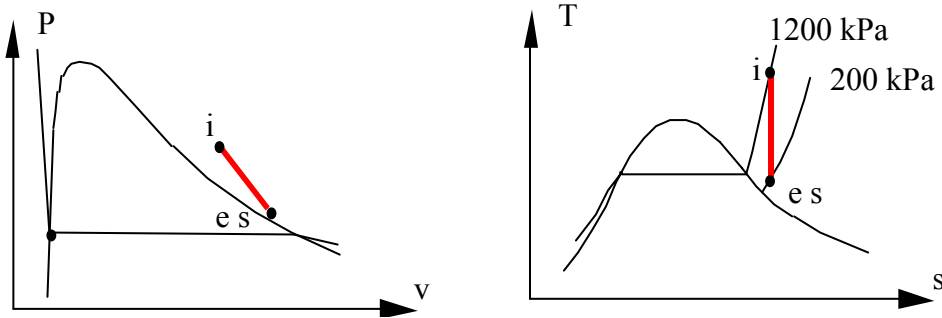
## 9.119

A steam turbine inlet is at 1200 kPa, 500°C. The exit is at 200 kPa. What is the lowest possible exit temperature? Which efficiency does that correspond to?

We would expect the lowest possible exit temperature when the maximum amount of work is taken out. This happens in a reversible process so if we assume it is adiabatic this becomes an isentropic process.

$$\text{Exit: } 200 \text{ kPa, } s = s_{\text{in}} = 7.6758 \text{ kJ/kg K} \Rightarrow \mathbf{T = 241.9^\circ\text{C}}$$

The efficiency from Eq.9.26 measures the turbine relative to an isentropic turbine, so the **efficiency** will be **100%**.





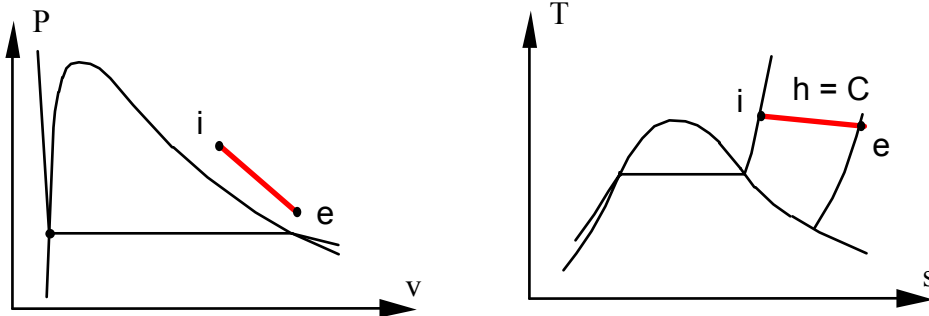
## 9.120

A steam turbine inlet is at 1200 kPa, 500°C. The exit is at 200 kPa. What is the highest possible exit temperature? Which efficiency does that correspond to?

The highest possible exit temperature would be if we did not get any work out, i.e. the turbine broke down. Now we have a throttle process with constant  $h$  assuming we do not have a significant exit velocity.

$$\text{Exit: } 200 \text{ kPa, } h = h_{\text{in}} = 3476.28 \text{ kJ/kg} \Rightarrow \mathbf{T = 495^\circ\text{C}}$$

$$\text{Efficiency: } \eta = \frac{w}{w_s} = \mathbf{0}$$



Remark: Since process is irreversible there is no area under curve in T-s diagram that correspond to a  $q$ , nor is there any area in the P-v diagram corresponding to a shaft work.

## 9.121

A steam turbine inlet is at 1200 kPa, 500°C. The exit is at 200 kPa, 275°C. What is the isentropic efficiency?

$$\text{Inlet: } h_{\text{in}} = 3476.28 \text{ kJ/kg}, \quad s_{\text{in}} = 7.6758 \text{ kJ/kg K}$$

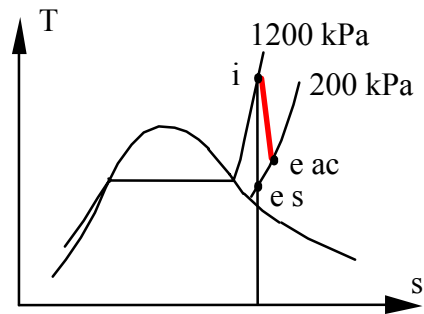
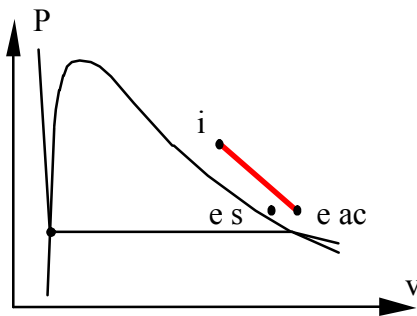
$$\text{Exit: } h_{\text{ex}} = 3021.4 \text{ kJ/kg}, \quad s_{\text{ex}} = 7.8006 \text{ kJ/kg K}$$

$$\text{Ideal Exit: } 200 \text{ kPa}, \quad s = s_{\text{in}} = 7.6758 \text{ kJ/kg K} \Rightarrow h_s = 2954.7 \text{ kJ/kg}$$

$$w_{\text{ac}} = h_{\text{in}} - h_{\text{ex}} = 3476.28 - 3021.4 = 454.9 \text{ kJ/kg}$$

$$w_s = h_{\text{in}} - h_s = 3476.28 - 2954.7 = 521.6 \text{ kJ/kg}$$

$$\eta = \frac{w_{\text{ac}}}{w_s} = \frac{454.9}{521.6} = \mathbf{0.872}$$

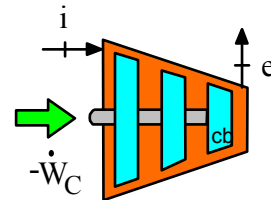


## 9.122

A compressor in a commercial refrigerator receives R-22 at  $-25^{\circ}\text{C}$  and  $x = 1$ . The exit is at 1000 kPa and  $60^{\circ}\text{C}$ . Neglect kinetic energies and find the isentropic compressor efficiency.

Solution:

C.V. Compressor, steady state, single inlet and exit flow. For this device we also assume no heat transfer  $q = 0$  and  $Z_1 = Z_2$



$$\text{Energy Eq.6.13:} \quad h_i + 0 = w_C + h_e;$$

$$\text{Entropy Eq.9.8:} \quad s_i + \int dq/T + s_{\text{gen}} = s_e = s_i + 0 + s_{\text{gen}}$$

$$\text{From Table B.4.1 :} \quad h_i = 239.92 \text{ kJ/kg}, \quad s_i = 0.9685 \text{ kJ/kgK}$$

$$\text{From Table B.4.2 :} \quad h_e = 286.97 \text{ kJ/kg}, \quad s_e = 0.9893 \text{ kJ/kgK}$$

Energy equation gives

$$w_{C \text{ ac}} = h_i - h_e = 239.92 - 286.97 = -47.05 \text{ kJ/kgK}$$

The ideal compressor has an exit state e,s: 1000 kPa, 0.9685 kJ/kgK

$$\text{Table B.4.2} \Rightarrow T_{e,s} \cong 51.4^{\circ}\text{C}, \quad h_{e,s} = 280.1 \text{ kJ/kg}$$

$$w_{C s} = 239.9 - 280.1 = -40.2 \text{ kJ/kg}$$

The isentropic efficiency measures the actual compressor to the ideal one

$$\eta = w_{C s} / w_{C \text{ ac}} = -40.2 / -47.05 = \mathbf{0.854}$$

**9.123**

The exit velocity of a nozzle is 500 m/s. If  $\eta_{\text{nozzle}} = 0.88$  what is the ideal exit velocity?

The nozzle efficiency is given by Eq. 9.29 and since we have the actual exit velocity we get

$$\begin{aligned} \mathbf{V}_{e s}^2 &= \mathbf{V}_{a c}^2 / \eta_{\text{nozzle}} \Rightarrow \\ \mathbf{V}_{e s} &= \mathbf{V}_{a c} / \sqrt{\eta_{\text{nozzle}}} = 500 / \sqrt{0.88} = \mathbf{533 \text{ m/s}} \end{aligned}$$

## 9.124

Find the isentropic efficiency of the R-134a compressor in Example 6.10

Solution:

State 1: Table B.5.2  $h_1 = 387.2 \text{ kJ/kg}$ ;  $s_1 = 1.7665 \text{ kJ/kg K}$

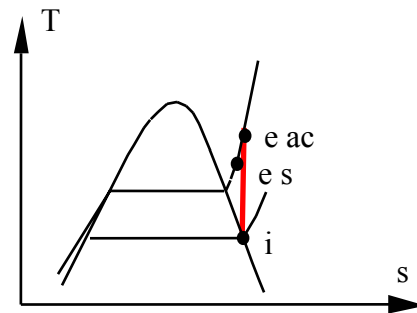
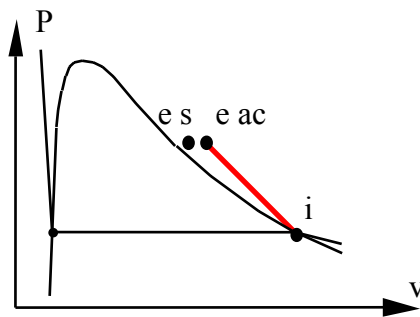
State 2ac:  $h_2 = 435.1 \text{ kJ/kg}$

State 2s:  $s = 1.7665 \text{ kJ/kg K}$ ,  $800 \text{ kPa} \Rightarrow h = 431.8 \text{ kJ/kg}$ ;  $T = 46.8^\circ\text{C}$

$$-w_{cs} = h_{2s} - h_1 = 431.8 - 387.2 = 44.6 \text{ kJ/kg}$$

$$-w_{ac} = 5/0.1 = 50 \text{ kJ/kg}$$

$$\eta = w_{cs} / w_{ac} = 44.6/50 = \mathbf{0.89}$$



## 9.125

Steam enters a turbine at 300°C, 600 kPa and exhausts as saturated vapor at 20 kPa. What is the isentropic efficiency?

Solution:

C.V. Turbine. Steady single inlet and exit flow.

To get the efficiency we must compare the actual turbine to the ideal one (the reference).

Energy Eq.6.13:  $w_T = h_1 - h_2$  ;

Entropy Eq.9.8:  $s_{2s} = s_1 + s_{gen} = s_1$

Process: Ideal  $s_{gen} = 0$

State 1: Table B.1.3  $h_1 = 3061.63$  kJ/kg,  $s_1 = 7.3723$  kJ/kg K

State 2s: 20 kPa,  $s_{2s} = s_1 = 7.3723$  kJ/kg K  $< s_g$  so two-phase

$$x_{2s} = \frac{s - s_f}{s_{fg}} = \frac{7.3723 - 0.8319}{7.0766} = 0.92423$$

$$h_{2s} = h_f + x_{2s} h_{fg} = 251.38 + x_{2s} \times 2358.33 = 2431.0 \text{ kJ/kg}$$

$$w_{Ts} = h_1 - h_{2s} = 3061.63 - 2431.0 = 630.61 \text{ kJ/kg}$$

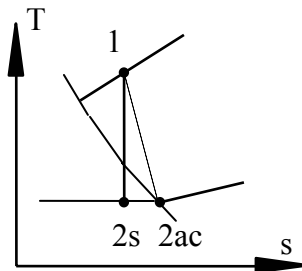
State 2ac: Table B.1.2  $h_{2ac} = 2609.7$  kJ/kg,  $s_{2ac} = 7.9085$  kJ/kg K

Now we can consider the actual turbine from energy Eq.6.13:

$$w_{ac}^T = h_1 - h_{2ac} = 3061.63 - 2609.7 = 451.93$$

Then the efficiency from Eq. 9.26

$$\eta_T = w_{ac}^T / w_{Ts} = 451.93/630.61 = \mathbf{0.717}$$



## 9.126

An emergency drain pump should be able to pump  $0.1 \text{ m}^3/\text{s}$  liquid water at  $15^\circ\text{C}$ ,  $10 \text{ m}$  vertically up delivering it with a velocity of  $20 \text{ m/s}$ . It is estimated that the pump, pipe and nozzle have a combined isentropic efficiency expressed for the pump as  $60\%$ . How much power is needed to drive the pump?

Solution:

C.V. Pump, pipe and nozzle together. Steady flow, no heat transfer.

Consider the ideal case first (it is the reference for the efficiency).

$$\text{Energy Eq.6.12: } \dot{m}_i(h_i + \mathbf{V}_i^2/2 + gZ_i) + \dot{W}_{\text{in}} = \dot{m}_e(h_e + \mathbf{V}_e^2/2 + gZ_e)$$

Solve for work and use reversible process Eq.9.15

$$\begin{aligned}\dot{W}_{\text{ins}} &= \dot{m} [h_e - h_i + (\mathbf{V}_e^2 - \mathbf{V}_i^2)/2 + g(Z_e - Z_i)] \\ &= \dot{m}[(P_e - P_i)v + \mathbf{V}_e^2/2 + g\Delta Z]\end{aligned}$$

$$\dot{m} = \dot{V}/v = 0.1/0.001001 = 99.9 \text{ kg/s}$$

$$\begin{aligned}\dot{W}_{\text{ins}} &= 99.9[0 + (20^2/2) \times (1/1000) + 9.807 \times (10/1000)] \\ &= 99.9(0.2 + 0.09807) = 29.8 \text{ kW}\end{aligned}$$

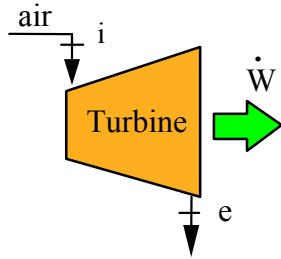
With the estimated efficiency the actual work, Eq.9.27 is

$$\dot{W}_{\text{in actual}} = \dot{W}_{\text{ins}}/\eta = 29.8/0.6 = 49.7 \text{ kW} = \mathbf{50 \text{ kW}}$$

## 9.127

A gas turbine with air flowing in at 1200 kPa, 1200 K has an exit pressure of 200 kPa and an isentropic efficiency of 87%. Find the exit temperature.

Solution:



C.V. Ideal air turbine.

Adiabatic:  $q = 0$ , reversible:  $s_{gen} = 0$

$$\text{Energy Eq.6.13: } w_T = h_i - h_e,$$

$$\text{Entropy Eq.9.8: } s_e = s_i$$

$$\text{Table A.7: } h_i = 1277.8 \text{ kJ/kg, } s_{Ti}^o = 8.34596 \text{ kJ/kg K}$$

The constant  $s$  process is written from Eq.8.28 as

$$\Rightarrow s_{Te}^o = s_{Ti}^o + R \ln\left(\frac{P_e}{P_i}\right) = 8.34596 + 0.287 \ln\left(\frac{200}{1200}\right) = 7.83173 \text{ kJ/kg K}$$

$$\text{Interpolate in A.7.1 } \Rightarrow T_{e_s} = 761.9 \text{ K, } h_{e_s} = 780.52 \text{ kJ/kg}$$

$$w_{T_s} = h_i - h_{e_s} = 1277.81 - 780.52 = 497.3 \text{ kJ/kg}$$

The actual turbine then has

$$w_{T_{ac}} = \eta_T w_{T_s} = 0.87 \times 497.3 = 432.65 \text{ kJ/kg} = h_i - h_{e_{ac}}$$

$$h_{e_{ac}} = h_i - w_{T_{ac}} = 1277.81 - 432.65 = 845.16 \text{ kJ/kg}$$

$$\text{Interpolate in A.7.1 } \Rightarrow T_{e_{ac}} = \mathbf{820.8 \text{ K}}$$

If constant specific heats are used we get

Table A.5:  $C_{P0} = 1.004 \text{ kJ/kg K}$ ,  $R = 0.287 \text{ kJ/kg K}$ ,  $k = 1.4$ , then from Eq.8.32

$$T_{e_s} = T_i \left(\frac{P_e}{P_i}\right)^{\frac{k-1}{k}} = 1200 \left(\frac{200}{1200}\right)^{0.286} = 719.2 \text{ K}$$

$$w_{T_s} = C_{P0}(T_i - T_{e_s}) = 1.004(1200 - 719.2) = 482.72 \text{ kJ/kg}$$

The actual turbine then has

$$w_{T_{ac}} = \eta_T w_{T_s} = 0.87 \times 482.72 = 419.97 \text{ kJ/kg} = C_{P0}(T_i - T_{e_{ac}})$$

$$T_{e_{ac}} = T_i - w_{T_{ac}}/C_{P0} = 1200 - 419.97/1.004 = \mathbf{781.7 \text{ K}}$$



## 9.128

A gas turbine with air flowing in at 1200 kPa, 1200 K has an exit pressure of 200 kPa. Find the lowest possible exit temperature. Which efficiency does that correspond to?

Solution:

Look at the T-s diagram for the possible processes. We notice that the lowest exit T is for the isentropic process (the ideal turbine)

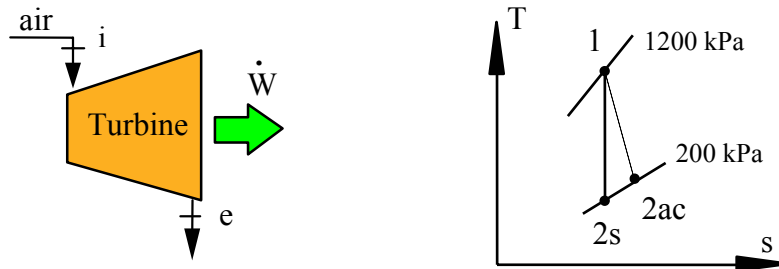


Table A.7:  $h_i = 1277.8 \text{ kJ/kg}$ ,  $s_{Ti}^o = 8.34596 \text{ kJ/kg K}$

The constant s process is written from Eq.8.28 as

$$\Rightarrow s_{Te}^o = s_{Ti}^o + R \ln\left(\frac{P_e}{P_i}\right) = 8.34596 + 0.287 \ln\left(\frac{200}{1200}\right) = 7.83173 \text{ kJ/kg K}$$

Interpolate in A.7.1  $\Rightarrow T_{e_s} = \mathbf{761.9 \text{ K}}$

This is an efficiency of **100%**

## 9.129

Liquid water enters a pump at 15°C, 100 kPa, and exits at a pressure of 5 MPa. If the isentropic efficiency of the pump is 75%, determine the enthalpy (steam table reference) of the water at the pump exit.

Solution:

$$\text{CV: pump } \dot{Q}_{\text{CV}} \approx 0, \quad \Delta \text{KE} \approx 0, \quad \Delta \text{PE} \approx 0$$

$$\text{2nd law, reversible (ideal) process: } s_{\text{es}} = s_i \Rightarrow$$

Eq.9.18 for work term.

$$\begin{aligned} w_s &= -\int_i^{\text{es}} v \, dP \approx -v_i(P_e - P_i) = -0.001001 \text{ m}^3/\text{kg} (5000 - 100) \text{ kPa} \\ &= -4.905 \text{ kJ/kg} \end{aligned}$$

$$\text{Real process Eq.9.28: } w = w_s/\eta_s = -4.905/0.75 = -6.54 \text{ kJ/kg}$$

$$\text{Energy Eq.6.13: } h_e = h_i - w = 62.99 + 6.54 = \mathbf{69.53 \text{ kJ/kg}}$$

## 9.130

Ammonia is brought from saturated vapor at 300 kPa to 1400 kPa, 140°C in a steady flow adiabatic compressor. Find the compressor specific work, entropy generation and its isentropic efficiency.

C.V. Actual Compressor, assume adiabatic and neglect kinetic energies.

$$\text{Energy Eq.6.13:} \quad w_C = h_i - h_e$$

$$\text{Entropy Eq.9.9:} \quad s_e = s_i + s_{\text{gen}}$$

$$\text{States: 1: B.2.2} \quad h_i = 1431.7 \text{ kJ/kg}, \quad s_i = 5.4565 \text{ kJ/kg-K}$$

$$2: \text{B.2.2} \quad h_e = 1752.8 \text{ kJ/kg}, \quad s_e = 5.7023 \text{ kJ/kg-K}$$

$$-w_C = h_e - h_i = 1752.8 - 1431.7 = 321.1 \text{ kJ/kg}$$

Ideal compressor. We find the exit state from (P,s).

$$\text{State 2s: } P_e, s_{e_s} = s_i = 5.4565 \text{ kJ/kg-K} \Rightarrow h_{e_s} = 1656.08 \text{ kJ/kg}$$

$$-w_{C_s} = h_{2s} - h_i = 1656.08 - 1431.7 = 224.38 \text{ kJ/kg}$$

$$\eta_C = -w_{C_s} / -w_C = \frac{224.38}{321.1} = \mathbf{0.699}$$

## 9.131

A centrifugal compressor takes in ambient air at 100 kPa, 15°C, and discharges it at 450 kPa. The compressor has an isentropic efficiency of 80%. What is your best estimate for the discharge temperature?

Solution:

C.V. Compressor. Assume adiabatic, no kinetic energy is important.

$$\text{Energy Eq.6.13:} \quad w = h_1 - h_2$$

$$\text{Entropy Eq.9.8:} \quad s_2 = s_1 + s_{\text{gen}}$$

We have two different cases, the ideal and the actual compressor.

We will solve using constant specific heat.

State 2 for the ideal,  $s_{\text{gen}} = 0$  so  $s_2 = s_1$  and it becomes:

$$\text{Eq.8.23:} \quad T_{2s} = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = 288.15 (450 / 100)^{0.2857} = 442.83 \text{ K}$$

$$w_s = h_1 - h_{2s} = C_p (T_1 - T_{2s}) = 1.004 (288.15 - 442.83) = -155.3 \text{ kJ/kg}$$

The actual work from definition Eq.9.27 and then energy equation:

$$w_{\text{ac}} = w_s / \eta = -155.3 / 0.8 = -194.12 \text{ kJ/kg} = h_1 - h_2 = C_p (T_1 - T_2)$$

$$\begin{aligned} \Rightarrow T_2 &= T_1 - w_{\text{ac}} / C_p \\ &= 288.15 + 194.12 / 1.004 = \mathbf{481.5 \text{ K}} \end{aligned}$$

Solving using Table A.7.1 instead will give

$$\text{State 1: Table A.7.1:} \quad s_{T1}^{\circ} = 6.82869 \text{ kJ/kg K}$$

Now constant s for the ideal is done with Eq.8.19

$$s_{T2s}^{\circ} = s_{T1}^{\circ} + R \ln \left( \frac{P_2}{P_1} \right) = 6.82869 + 0.287 \ln \left( \frac{450}{100} \right) = 7.26036 \text{ kJ/kg K}$$

$$\text{From A.7.1:} \quad T_{2s} = 442.1 \text{ K} \quad \text{and} \quad h_{2s} = 443.86 \text{ kJ/kg}$$

$$w_s = h_1 - h_{2s} = 288.57 - 443.86 = -155.29 \text{ kJ/kg}$$

The actual work from definition Eq.9.27 and then energy equation:

$$w_{\text{ac}} = w_s / \eta = -155.29 / 0.8 = -194.11 \text{ kJ/kg}$$

$$\Rightarrow h_2 = 194.11 + 288.57 = 482.68, \quad \text{Table A.7.1:} \quad T_2 = \mathbf{480 \text{ K}}$$

The answer is very close to the previous one due to the modest T's.

## 9.132

A compressor is used to bring saturated water vapor at 1 MPa up to 17.5 MPa, where the actual exit temperature is 650°C. Find the isentropic compressor efficiency and the entropy generation.

Solution:

C.V. Compressor. Assume adiabatic and neglect kinetic energies.

Energy Eq.6.13:  $w = h_1 - h_2$

Entropy Eq.9.9:  $s_2 = s_1 + s_{\text{gen}}$

We have two different cases, the ideal and the actual compressor.

States: 1: B.1.2  $h_1 = 2778.1 \text{ kJ/kg}$ ,  $s_1 = 6.5865 \text{ kJ/kg K}$

2ac: B.1.3  $h_{2,\text{AC}} = 3693.9 \text{ kJ/kg}$ ,  $s_{2,\text{AC}} = 6.7357 \text{ kJ/kg K}$

2s: B.1.3 (P, s = s<sub>1</sub>)  $h_{2,s} = 3560.1 \text{ kJ/kg}$

IDEAL:

$$-w_{c,s} = h_{2,s} - h_1 = 782 \text{ kJ/kg}$$

ACTUAL:

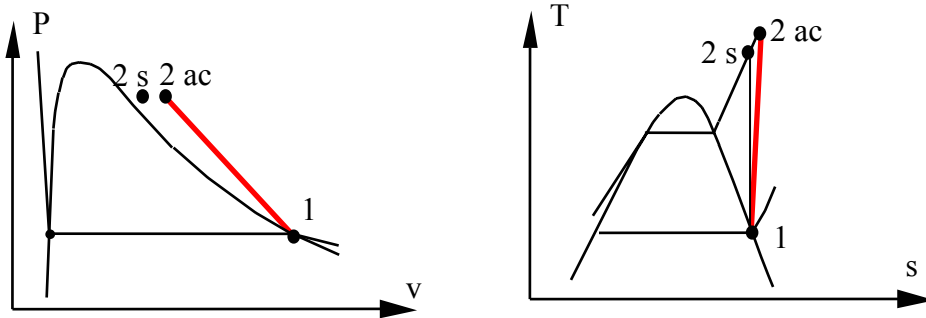
$$-w_{c,\text{AC}} = h_{2,\text{AC}} - h_1 = 915.8 \text{ kJ/kg}$$

Definition Eq.9.28:

$$\eta_c = w_{c,s}/w_{c,\text{AC}} = \mathbf{0.8539 \sim 85\%}$$

Entropy Eq.9.8:

$$s_{\text{gen}} = s_{2,\text{ac}} - s_1 = 6.7357 - 6.5865 = \mathbf{0.1492 \text{ kJ/kg K}}$$



## 9.133

A refrigerator uses carbon dioxide that is brought from 1 MPa,  $-20^{\circ}\text{C}$  to 6 MPa using 2 kW power input to the compressor with a flow rate of 0.02 kg/s. Find the compressor exit temperature and its isentropic efficiency.

C.V. Actual Compressor, assume adiabatic and neglect kinetic energies.

$$\text{Energy Eq.6.13:} \quad -w_C = h_2 - h_1 = \frac{\dot{W}}{\dot{m}} = \frac{2 \text{ kW}}{0.02 \text{ kg/s}} = 100 \text{ kJ/kg}$$

$$\text{Entropy Eq.9.9:} \quad s_2 = s_1 + s_{\text{gen}}$$

$$\text{States: 1: B.3.2} \quad h_1 = 342.31 \text{ kJ/kg}, \quad s_1 = 1.4655 \text{ kJ/kg-K}$$

$$2: \text{ B.3.2} \quad h_2 = h_1 - w_C = 442.31 \text{ kJ/kg} \Rightarrow T_2 = 117.7^{\circ}\text{C}$$

Ideal compressor. We find the exit state from (P,s).

$$\text{State 2s: } P_2, s_{2s} = s_1 = 1.4655 \text{ kJ/kg-K} \Rightarrow h_{2s} = 437.55 \text{ kJ/kg}$$

$$-w_{C_s} = h_{2s} - h_1 = 437.55 - 342.31 = 95.24 \text{ kJ/kg}$$

$$\eta_C = -w_{C_s} / -w_C = \frac{95.24}{100} = \mathbf{0.952}$$

## 9.134

Find the isentropic efficiency for the compressor in Problem 6.57.

A compressor in an air-conditioner receives saturated vapor R-410a at 400 kPa and brings it to 1.8 MPa, 60°C in an adiabatic compression. Find the flow rate for a compressor work of 2 kW?

C.V. Actual Compressor, assume adiabatic and neglect kinetic energies.

$$\text{Energy Eq.6.13:} \quad w = h_1 - h_2$$

$$\text{Entropy Eq.9.9:} \quad s_2 = s_1 + s_{\text{gen}}$$

$$\text{States: 1: B.4.2} \quad h_1 = 271.9 \text{ kJ/kg}, \quad s_1 = 1.0779 \text{ kJ/kg-K}$$

$$2: \text{B.4.2} \quad h_2 = 323.92 \text{ kJ/kg}$$

$$-w_C = h_2 - h_1 = 323.92 - 271.9 = 52.02 \text{ kJ/kg}$$

Ideal compressor. We find the exit state from (P,s).

$$\text{State 2s: } P_2, s_{2s} = s_1 = 1.0779 \text{ kJ/kg-K} \Rightarrow h_{2s} = 314.33 \text{ kJ/kg}$$

$$-w_{C_s} = h_{2s} - h_1 = 314.33 - 271.9 = 42.43 \text{ kJ/kg}$$

$$\eta_C = -w_{C_s} / -w_C = \frac{42.43}{52.02} = \mathbf{0.816}$$

## 9.135

A pump receives water at 100 kPa, 15°C and a power input of 1.5 kW. The pump has an isentropic efficiency of 75% and it should flow 1.2 kg/s delivered at 30 m/s exit velocity. How high an exit pressure can the pump produce?

Solution:

CV Pump. We will assume the ideal and actual pumps have same exit pressure, then we can analyse the ideal pump.

Specific work:  $w_{ac} = 1.5/1.2 = 1.25 \text{ kJ/kg}$

Ideal work Eq.9.27:  $w_s = \eta w_{ac} = 0.75 \times 1.25 = 0.9375 \text{ kJ/kg}$

As the water is incompressible (liquid) we get

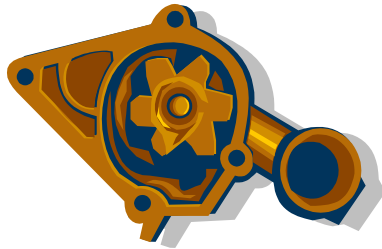
Energy Eq.9.14:

$$\begin{aligned} w_s &= (P_e - P_i)v + \mathbf{V}_e^2/2 = (P_e - P_i)0.001001 + (30^2/2)/1000 \\ &= (P_e - P_i)0.001001 + 0.45 \end{aligned}$$

Solve for the pressure difference

$$P_e - P_i = (w_s - 0.45)/0.001001 = 487 \text{ kPa}$$

$$\mathbf{P_e = 587 \text{ kPa}}$$



Water pump from a car



## 9.136

A turbine receives air at 1500 K, 1000 kPa and expands it to 100 kPa. The turbine has an isentropic efficiency of 85%. Find the actual turbine exit air temperature and the specific entropy increase in the actual turbine.

Solution:

C.V. Turbine. steady single inlet and exit flow.

To analyze the actual turbine we must first do the ideal one (the reference).

Energy Eq.6.13:  $w_T = h_1 - h_2$  ;

Entropy Eq.9.8:  $s_2 = s_1 + s_{\text{gen}} = s_1$

Entropy change in Eq.8.19 and Table A.7.1:

$$s_{T2}^o = s_{T1}^o + R \ln(P_2/P_1) = 8.61208 + 0.287 \ln(100/1000) = 7.95124$$

Interpolate in A.7  $\Rightarrow T_{2s} = 849.2$ ,  $h_{2s} = 876.56 \Rightarrow$

$$w_T = 1635.8 - 876.56 = 759.24 \text{ kJ/kg}$$

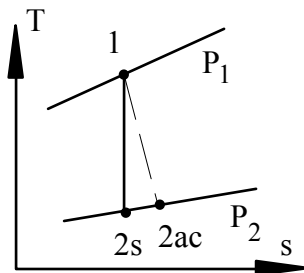
Now we can consider the actual turbine from Eq.9.26 and Eq.6.13:

$$w_{ac}^T = \eta_T w_T = 0.85 \times 759.24 = 645.35 = h_1 - h_{2ac}$$

$$\Rightarrow h_{2ac} = h_1 - w_{ac}^T = 990.45 \Rightarrow \mathbf{T_{2ac} = 951 \text{ K}}$$

The entropy balance equation is solved for the generation term

$$s_{\text{gen}} = s_{2ac} - s_1 = 8.078 - 8.6121 - 0.287 \ln(100/1000) = \mathbf{0.1268 \text{ kJ/kg K}}$$



## 9.137

Carbon dioxide,  $\text{CO}_2$ , enters an adiabatic compressor at 100 kPa, 300 K, and exits at 1000 kPa, 520 K. Find the compressor efficiency and the entropy generation for the process.

Solution:

C.V. Ideal compressor. We will assume constant heat capacity.

Energy Eq.6.13:  $w_c = h_1 - h_2$ ,

Entropy Eq.9.8:  $s_2 = s_1 : T_{2s} = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = 300 \left( \frac{1000}{100} \right)^{0.2242} = 502.7 \text{ K}$

$$w_{cs} = C_p(T_1 - T_{2s}) = 0.842(300 - 502.7) = -170.67 \text{ kJ/kg}$$

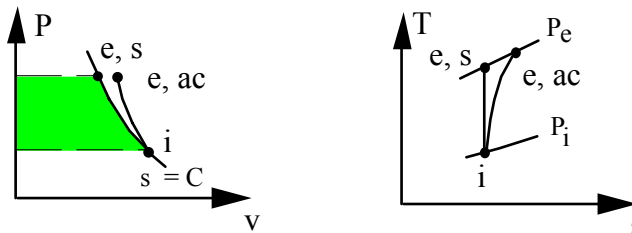
C.V. Actual compressor

$$w_{cac} = C_p(T_1 - T_{2ac}) = 0.842(300 - 520) = -185.2 \text{ kJ/kg}$$

$$\eta_c = w_{cs}/w_{cac} = -170.67/(-185.2) = \mathbf{0.92}$$

Use Eq.8.16 for the change in entropy

$$\begin{aligned} s_{gen} &= s_{2ac} - s_1 = C_p \ln(T_{2ac}/T_1) - R \ln(P_2/P_1) \\ &= 0.842 \ln(520/300) - 0.1889 \ln(1000/100) = \mathbf{0.028 \text{ kJ/kg K}} \end{aligned}$$



Constant heat capacity is not the best approximation. It would be more accurate to use Table A.8. Entropy change in Eq.8.19 and Table A.8:

$$s_{T2}^0 = s_{T1}^0 + R \ln(P_2/P_1) = 4.8631 + 0.1889 \ln(1000/100) = 5.29806$$

Interpolate in A.8  $\Rightarrow T_{2s} = 481 \text{ K}, h_{2s} = 382.807 \text{ kJ/kg} \Rightarrow$

$$-w_{cs} = 382.807 - 214.38 = 168.43 \text{ kJ/kg}; -w_{cac} = 422.12 - 214.38 = 207.74 \text{ kJ/kg}$$

$$\eta_c = w_{cs}/w_{cac} = -168.43/(-207.74) = \mathbf{0.81}$$

$$s_{gen} = s_{2ac} - s_1 = 5.3767 - 4.8631 - 0.1889 \ln(10) = 0.0786 \text{ kJ/kgK}$$

## 9.138

A small air turbine with an isentropic efficiency of 80% should produce 270 kJ/kg of work. The inlet temperature is 1000 K and it exhausts to the atmosphere. Find the required inlet pressure and the exhaust temperature.

Solution:

C.V. Turbine actual energy Eq.6.13:

$$w = h_i - h_{e,ac} = 270 \text{ kJ/kg}$$

Table A.7:  $h_i = 1046.22 \Rightarrow h_{e,ac} = 776.22 \text{ kJ/kg}$ ,  $T_e = 757.9 \text{ K}$

C.V. Ideal turbine, Eq.9.27 and energy Eq.6.13:

$$w_s = w/\eta_s = 270/0.8 = 337.5 = h_i - h_{e,s} \Rightarrow h_{e,s} = 708.72 \text{ kJ/kg}$$

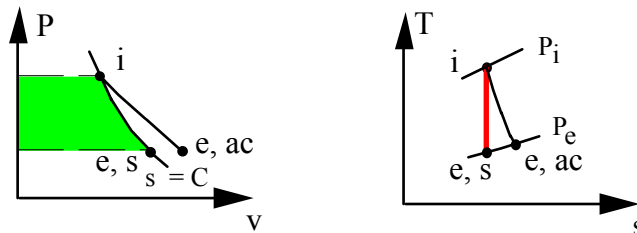
From Table A.7:  $T_{e,s} = 695.5 \text{ K}$

Entropy Eq.9.8:  $s_i = s_{e,s}$  adiabatic and reversible

To relate the entropy to the pressure use Eq.8.19 inverted and standard entropy from Table A.7.1:

$$P_e/P_i = \exp[(s_{Te}^0 - s_{Ti}^0)/R] = \exp[(7.733 - 8.13493)/0.287] = 0.2465$$

$$P_i = P_e / 0.2465 = 101.3/0.2465 = 411 \text{ kPa}$$



If constant heat capacity were used

$$T_e = T_i - w/C_p = 1000 - 270/1.004 = 731 \text{ K}$$

C.V. Ideal turbine, Eq.9.26 and energy Eq.6.13:

$$w_s = w/\eta_s = 270/0.8 = 337.5 \text{ kJ/kg} = h_i - h_{e,s} = C_p(T_i - T_{e,s})$$

$$T_{e,s} = T_i - w_s/C_p = 1000 - 337.5/1.004 = 663.8 \text{ K}$$

Eq.9.8 (adiabatic and reversible) gives constant s and relation is Eq.8.23

$$P_e/P_i = (T_e/T_i)^{k/(k-1)} \Rightarrow P_i = 101.3 (1000/663.8)^{3.5} = 425 \text{ kPa}$$

## 9.139

The small turbine in Problem 9.31 was ideal. Assume instead the isentropic turbine efficiency is 88%. Find the actual specific turbine work and the entropy generated in the turbine.

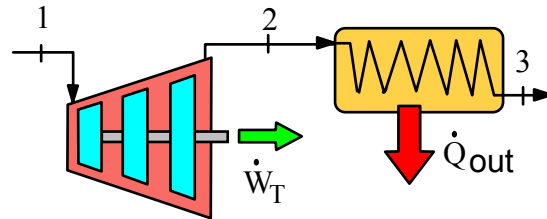
Solution:

Continuity Eq.6.11: (Steady)

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m}$$

Turbine: Energy Eq.6.13:

$$w_T = h_1 - h_2$$



Entropy Eq.9.8:  $s_2 = s_1 + s_{T,gen}$

Inlet state: Table B.1.3  $h_1 = 3917.45$  kJ/kg,  $s_1 = 7.9487$  kJ/kg K

Ideal turbine  $s_{T,gen} = 0$ ,  $s_2 = s_1 = 7.9487 = s_{f2} + x s_{fg2}$

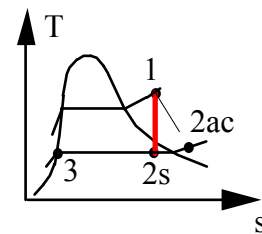
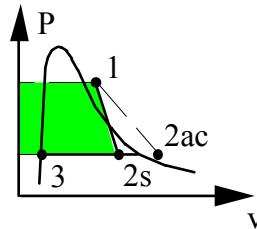
State 2:  $P = 10$  kPa,  $s_2 < s_g \Rightarrow$  saturated 2-phase in Table B.1.2

$$\Rightarrow x_{2,s} = (s_1 - s_{f2})/s_{fg2} = (7.9487 - 0.6492)/7.501 = 0.9731$$

$$\Rightarrow h_{2,s} = h_{f2} + x \times h_{fg2} = 191.8 + 0.9731 \times 2392.8 = 2520.35$$
 kJ/kg

$$w_{T,s} = h_1 - h_{2,s} = 1397.05$$
 kJ/kg

Explanation for the reversible work term is in sect. 9.3 Eq.9.14



$$w_{T,AC} = \eta \times w_{T,s} = \mathbf{1229.9}$$
 kJ/kg

$$= h_1 - h_{2,AC} \Rightarrow h_{2,AC} = h_1 - w_{T,AC} = 2687.5$$
 kJ/kg

$$\Rightarrow T_{2,AC} = 100^\circ\text{C}, \quad s_{2,AC} = 8.4479$$
 kJ/kg-K

$$s_{T,gen} = s_{2,AC} - s_1 = \mathbf{0.4992}$$
 kJ/kg K

## 9.140

A compressor in an industrial air-conditioner compresses ammonia from a state of saturated vapor at 150 kPa to a pressure 800 kPa. At the exit, the temperature is measured to be 100°C and the mass flow rate is 0.5 kg/s. What is the required motor size for this compressor and what is its isentropic efficiency?

C.V. Compressor. Assume adiabatic and neglect kinetic energies.

Energy Eq.6.13:  $w = h_1 - h_2$

Entropy Eq.9.8:  $s_2 = s_1 + s_{\text{gen}}$

We have two different cases, the ideal and the actual compressor.

States: 1: B.2.2:  $h_1 = 1410.9 \text{ kJ/kg}$ ,  $v_1 = 0.7787 \text{ m}^3/\text{kg}$ ,  $s_1 = 5.6983 \text{ kJ/kg K}$

2ac: B.2.3  $h_{2,\text{AC}} = 1670.6 \text{ kJ/kg}$ ,  $v_{2,\text{AC}} = 0.21949 \text{ m}^3/\text{kg}$

2s: B.2.3 (P, s = s<sub>1</sub>)  $h_{2,s} = 1649.8 \text{ kJ/kg}$ ,  $T_{2,s} = 91.4^\circ\text{C}$

ACTUAL:

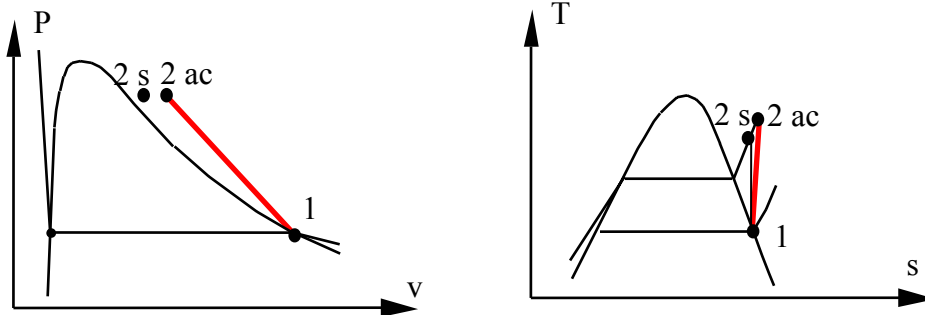
$$-w_{\text{C,AC}} = h_{2,\text{AC}} - h_1 = 1670.6 - 1410.9 = 259.7 \text{ kJ/kg}$$

$$\dot{W}_{\text{in}} = \dot{m} (-w_{\text{C,AC}}) = 0.5 \text{ kg/s} \times 259.7 \text{ kJ/kg} = \mathbf{130 \text{ kW}}$$

IDEAL:

$$-w_{\text{c,s}} = h_{2,s} - h_1 = 1649.8 - 1410.9 = 238.9 \text{ kJ/kg}$$

Definition Eq.9.27:  $\eta_{\text{c}} = w_{\text{c,s}}/w_{\text{c,AC}} = \mathbf{0.92}$



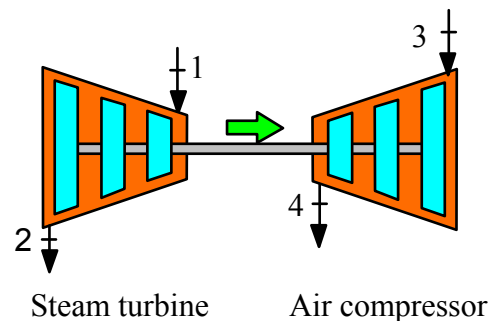
## 9.141

Repeat Problem 9.45 assuming the steam turbine and the air compressor each have an isentropic efficiency of 80%.

A certain industrial process requires a steady supply of saturated vapor steam at 200 kPa, at a rate of 0.5 kg/s. Also required is a steady supply of compressed air at 500 kPa, at a rate of 0.1 kg/s. Both are to be supplied by the process shown in Fig. P9.41. Steam is expanded in a turbine to supply the power needed to drive the air compressor, and the exhaust steam exits the turbine at the desired state. Air into the compressor is at the ambient conditions, 100 kPa, 20°C. Give the required steam inlet pressure and temperature, assuming that both the turbine and the compressor are reversible and adiabatic.

Solution:

C.V. Each device. Steady flow.  
Both adiabatic ( $q = 0$ ) and actual devices ( $s_{\text{gen}} > 0$ ) given by  $\eta_{sT}$  and  $\eta_{sc}$ .



$$\text{Air Eq. 8.32, } T_{4s} = T_3(P_4/P_3)^{\frac{k-1}{k}} = 293.2 \left( \frac{500}{100} \right)^{0.286} = 464.6 \text{ K}$$

$$\dot{W}_{Cs} = \dot{m}_3(h_3 - h_{4s}) = 0.1 \times 1.004(293.2 - 464.6) = -17.21 \text{ kW}$$

$$\dot{W}_{Cs} = \dot{m}_3(h_3 - h_4) = \dot{W}_{Cs} / \eta_{sc} = -17.2 / 0.80 = -21.5 \text{ kW}$$

Now the actual turbine must supply the actual compressor work. The actual state 2 is given so we must work backwards to state 1.

$$\begin{aligned} \dot{W}_T = +21.5 \text{ kW} &= \dot{m}_1(h_1 - h_2) = 0.5(h_1 - 2706.6) \\ \Rightarrow h_1 &= 2749.6 \text{ kJ/kg} \end{aligned}$$

$$\text{Also, } \eta_{sT} = 0.80 = (h_1 - h_2)/(h_1 - h_{2s}) = 43/(2749.6 - h_{2s})$$

$$\Rightarrow h_{2s} = 2695.8 \text{ kJ/kg}$$

$$2695.8 = 504.7 + x_{2s}(2706.6 - 504.7) \Rightarrow x_{2s} = 0.9951$$

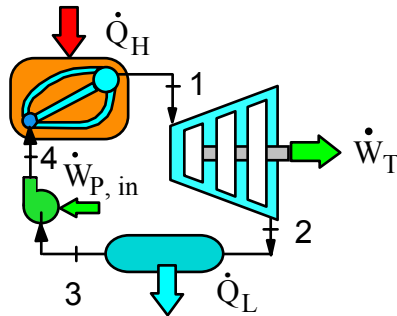
$$s_{2s} = 1.5301 + 0.9951(7.1271 - 1.5301) = 7.0996 \text{ kJ/kg K}$$

$$(s_1 = s_{2s}, h_1) \rightarrow P_1 = \mathbf{269 \text{ kPa}}, \quad T_1 = \mathbf{143.5^\circ\text{C}}$$

## 9.142

Repeat Problem 9.47 assuming the turbine and the pump each have an isentropic efficiency of 85%.

Solution:



$$P_1 = P_4 = 20 \text{ MPa}$$

$$T_1 = 700 \text{ }^\circ\text{C}$$

$$P_2 = P_3 = 20 \text{ kPa}$$

$$T_3 = 40 \text{ }^\circ\text{C}$$

$$\eta_P = \eta_T = 85\%$$

- a) State 1: (P, T) Table B.1.3  $h_1 = 3809.1 \text{ kJ/kg}$ ,  $s_1 = 6.7993 \text{ kJ/kg K}$

C.V. Turbine. First we do the ideal, then the actual.

$$\text{Entropy Eq.9.8: } s_2 = s_1 = 6.7993 \text{ kJ/kg K}$$

$$\text{Table B.1.2 } s_2 = 0.8319 + x_2 \times 7.0766 \Rightarrow x_2 = 0.8433$$

$$h_{2s} = 251.4 + 0.8433 \times 2358.33 = 2240.1 \text{ kJ/kg}$$

$$\text{Energy Eq.6.13: } w_{T_s} = h_1 - h_{2s} = 1569 \text{ kJ/kg}$$

$$w_{T,AC} = \eta_T w_{T_s} = \mathbf{1333.65} = h_1 - h_{2,AC}$$

$$h_{2,AC} = h_1 - w_{T,AC} = 2475.45 \text{ kJ/kg;}$$

$$x_{2,AC} = (2475.45 - 251.4)/2358.3 = \mathbf{0.943}, \quad T_{2,AC} = \mathbf{60.06}^\circ\text{C}$$

- b)

State 3: (P, T) Compressed liquid, take sat. liq. Table B.1.1

$$h_3 = 167.54 \text{ kJ/kg}, \quad v_3 = 0.001008 \text{ m}^3/\text{kg}$$

$$w_{P_s} = -v_3(P_4 - P_3) = -0.001008(20000 - 20) = -20.1 \text{ kJ/kg}$$

$$-w_{P,AC} = -w_{P,s}/\eta_P = 20.1/0.85 = \mathbf{23.7} = h_{4,AC} - h_3$$

$$h_{4,AC} = \mathbf{191.2} \quad T_{4,AC} \cong 45.7^\circ\text{C}$$

- c) The heat transfer in the boiler is from energy Eq.6.13

$$q_{\text{boiler}} = h_1 - h_4 = 3809.1 - 191.2 = 3617.9 \text{ kJ/kg}$$

$$w_{\text{net}} = 1333.65 - 23.7 = \mathbf{1310 \text{ kJ/kg}}$$

$$\eta_{\text{TH}} = w_{\text{net}}/q_{\text{boiler}} = \frac{1310}{3617.9} = \mathbf{0.362}$$

## 9.143

Assume an actual compressor has the same exit pressure and specific heat transfer as the ideal isothermal compressor in Problem 9.23 with an isothermal efficiency of 80%. Find the specific work and exit temperature for the actual compressor.

Solution:

C.V. Compressor. Steady, single inlet and single exit flows.

$$\text{Energy Eq.6.13: } h_i + q = w + h_e;$$

$$\text{Entropy Eq.9.8: } s_i + q/T = s_e$$

$$\text{Inlet state: Table B.5.2, } h_i = 403.4 \text{ kJ/kg, } s_i = 1.8281 \text{ kJ/kg K}$$

$$\text{Exit state: Table B.5.1, } h_e = 398.36 \text{ kJ/kg, } s_e = 1.7262 \text{ kJ/kg K}$$

$$q = T(s_e - s_i) = 273.15(1.7262 - 1.8281) = -27.83 \text{ kJ/kg}$$

$$w = 403.4 + (-27.83) - 398.36 = -22.8 \text{ kJ/kg}$$

From Eq.9.28 for a cooled compressor

$$w_{ac} = w_T / \eta = -22.8 / 0.8 = \mathbf{28.5 \text{ kJ/kg}}$$

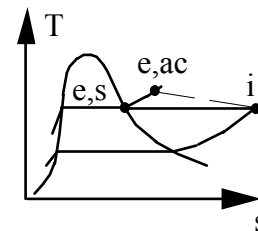
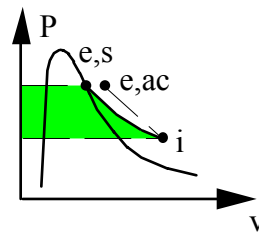
Now the energy equation gives

$$h_e = h_i + q - w_{ac} = 403.4 + (-27.83) + 28.5 = 404.07$$

$$T_{e,ac} \approx \mathbf{6^\circ\text{C}} \quad P_e = 294 \text{ kPa}$$

Explanation for the reversible work term is in Sect. 9.3

Eqs. 9.13 and 9.14





## 9.144

Air enters an insulated turbine at 50°C, and exits the turbine at -30°C, 100 kPa. The isentropic turbine efficiency is 70% and the inlet volumetric flow rate is 20 L/s. What is the turbine inlet pressure and the turbine power output?

Solution:

C.V.: Turbine,  $\eta_s = 0.7$ , Insulated

Air table A.5:  $C_p = 1.004$  kJ/kg K,  $R = 0.287$  kJ/kg K,  $k = 1.4$

Inlet:  $T_i = 50^\circ\text{C}$ ,  $\dot{V}_i = 20$  L/s =  $0.02$  m<sup>3</sup>/s ;

$$\dot{m} = P\dot{V}/RT = 100 \times 0.02 / (0.287 \times 323.15) = 0.099 \text{ kg/s}$$

Exit (actual):  $T_e = -30^\circ\text{C}$ ,  $P_e = 100$  kPa

1<sup>st</sup> Law Steady state Eq.6.13:  $q_T + h_i = h_e + w_T$ ;  $q_T = 0$

Assume Constant Specific Heat

$$w_T = h_i - h_e = C_p(T_i - T_e) = 80.3 \text{ kJ/kg}$$

$$w_{Ts} = w/\eta = 114.7 \text{ kJ/kg}, \quad w_{Ts} = C_p(T_i - T_{es})$$

Solve for  $T_{es} = 208.9$  K

Isentropic Process Eq.8.32:  $P_e = P_i (T_e / T_i)^{\frac{k}{k-1}} \Rightarrow P_i = 461 \text{ kPa}$

$$\dot{W}_T = \dot{m}w_T = 0.099 \times 80.3 = 7.98 \text{ kW}$$

## 9.145

Find the isentropic efficiency of the nozzle in example 6.4.

Solution:

C.V. adiabatic nozzle with known inlet state and velocity.

Inlet state: B.1.3  $h_i = 2850.1$  kJ/kg;  $s_i = 6.9665$  kJ/kg K

Process ideal: adiabatic and reversible Eq.9.8 gives constant  $s$   
ideal exit, (150 kPa,  $s$ );  $x_{es} = (6.9665 - 1.4335)/5.7897 = 0.9557$

$$h_{es} = h_f + x_{es} h_{fg} = 2594.9 \text{ kJ/kg}$$

$$V_{es}^2/2 = h_i - h_{es} + V_i^2/2 = 2850.1 - 2594.9 + (50^2)/2000 = 256.45 \text{ kJ/kg}$$

$$V_{es} = 716.2 \text{ m/s}$$

From Eq.9.29,

$$\eta_{noz.} = (V_e^2/2)/(V_{es}^2/2) = 180/256.45 = \mathbf{0.70}$$

## 9.146

Air enters an insulated compressor at ambient conditions, 100 kPa, 20°C, at the rate of 0.1 kg/s and exits at 200°C. The isentropic efficiency of the compressor is 70%. What is the exit pressure? How much power is required to drive the compressor? Assume the ideal and actual compressor has the same exit pressure.

Solution:

C.V. Compressor:  $P_1, T_1, T_e(\text{real}), \eta_{s \text{ COMP}}$  known, assume constant  $C_{P0}$

Energy Eq.6.13 for real:  $-w = C_{P0}(T_e - T_i) = 1.004(200 - 20) = 180.72$

Ideal  $-w_s = -w \times \eta_s = 180.72 \times 0.70 = 126.5$

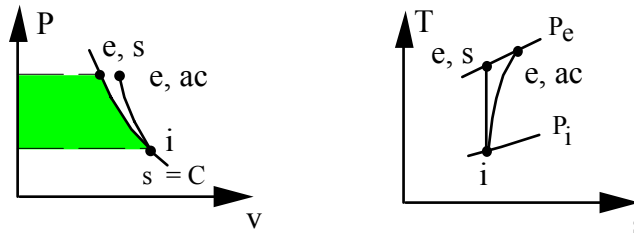
Energy Eq.6.13 for ideal:

$126.5 = C_{P0}(T_{es} - T_i) = 1.004(T_{es} - 293.2), T_{es} = 419.2 \text{ K}$

Constant entropy for ideal as in Eq.8.23:

$$P_e = P_i (T_{es}/T_i)^{\frac{k}{k-1}} = 100(419.2/293.20)^{3.5} = \mathbf{349 \text{ kPa}}$$

$$-\dot{W}_{\text{REAL}} = \dot{m}(-w) = 0.1 \times 180.72 = \mathbf{18.07 \text{ kW}}$$



## 9.147

A nozzle in a high pressure liquid water sprayer has an area of  $0.5 \text{ cm}^2$ . It receives water at 250 kPa,  $20^\circ\text{C}$  and the exit pressure is 100 kPa. Neglect the inlet kinetic energy and assume a nozzle isentropic efficiency of 85%. Find the ideal nozzle exit velocity and the actual nozzle mass flow rate.

Solution:

C.V. Nozzle. Liquid water is incompressible  $v \approx \text{constant}$ , no work, no heat transfer  $\Rightarrow$  Bernoulli Eq.9.16

$$\frac{1}{2}V_{\text{ex}}^2 - 0 = v(P_i - P_e) = 0.001002 (250 - 100) = 0.1503 \text{ kJ/kg}$$

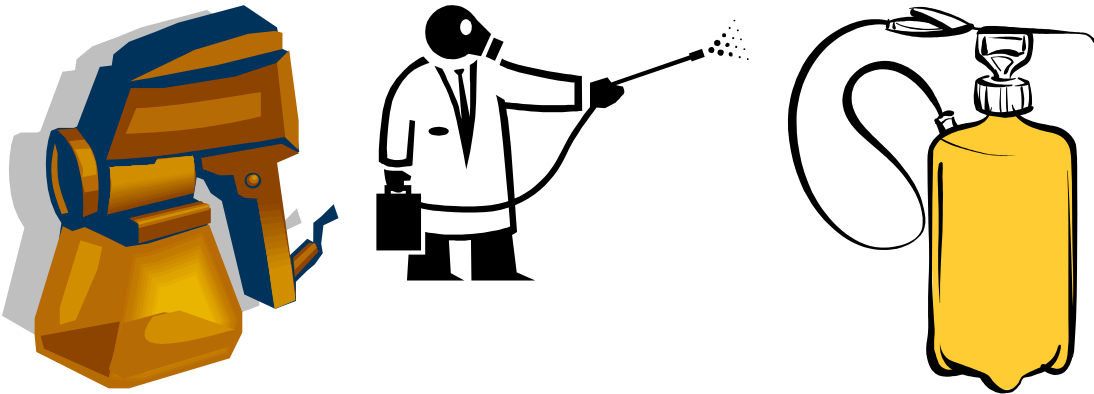
$$V_{\text{ex}} = \sqrt{2 \times 0.1503 \times 1000 \text{ J/kg}} = 17.34 \text{ m s}^{-1}$$

This was the ideal nozzle now we can do the actual nozzle, Eq. 9.29

$$\frac{1}{2}V_{\text{ex ac}}^2 = \eta \frac{1}{2}V_{\text{ex}}^2 = 0.85 \times 0.1503 = 0.12776 \text{ kJ/kg}$$

$$V_{\text{ex ac}} = \sqrt{2 \times 0.12776 \times 1000 \text{ J/kg}} = 15.99 \text{ m s}^{-1}$$

$$\dot{m} = \rho A V_{\text{ex ac}} = A V_{\text{ex ac}} / v = 0.5 \times 10^{-4} \times 15.99 / 0.001002 = 0.798 \text{ kg/s}$$



These are examples of relatively low pressure spray systems.

## 9.148

Redo Problem 9.64 if the water pump has an isentropic efficiency of 85% (hose, nozzle included).

Solution:

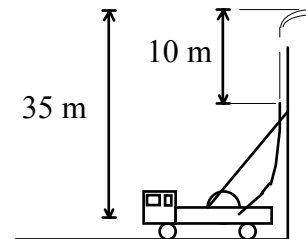
C.V.: pump + hose + water column, height difference 35 m.  $\mathbf{V}$  is velocity.

$$\text{Continuity Eq.6.11: } \dot{m}_{\text{in}} = \dot{m}_{\text{ex}} = (\rho A \mathbf{V})_{\text{nozzle}}$$

$$\text{Energy Eq.6.12: } \dot{m}(-w_p) + \dot{m}(h + \mathbf{V}^2/2 + gz)_{\text{in}} = \dot{m}(h + \mathbf{V}^2/2 + gz)_{\text{ex}}$$

$$\text{Process: } \quad h_{\text{in}} \cong h_{\text{ex}}, \quad \mathbf{V}_{\text{in}} \cong \mathbf{V}_{\text{ex}} = 0, \\ z_{\text{ex}} - z_{\text{in}} = 35 \text{ m}, \quad \rho = 1/v \cong 1/v_f$$

$$-w_p = g(z_{\text{ex}} - z_{\text{in}}) = 9.80665(35 - 0) = 343.2 \text{ J/kg}$$



The velocity in nozzle is such that it can rise 10 m, so make that column C.V.

$$gz_{\text{noz}} + \frac{1}{2}\mathbf{V}_{\text{noz}}^2 = gz_{\text{ex}} + 0$$

$$\Rightarrow \mathbf{V}_{\text{noz}} = \sqrt{2g(z_{\text{ex}} - z_{\text{noz}})} = \sqrt{2 \times 9.81 \times 10} = 14 \text{ m/s}$$

$$\dot{m} = (\pi/v_f) (D^2/4) \mathbf{V}_{\text{noz}} = (\pi/4) 0.025^2 \times 14 / 0.001 = 6.873 \text{ kg/s}$$

$$-\dot{W}_p = \dot{m}(-w_p)/\eta = 6.872 \times 0.343/0.85 = \mathbf{2.77 \text{ kW}}$$

## 9.149

Air flows into an insulated nozzle at 1 MPa, 1200 K with 15 m/s and mass flow rate of 2 kg/s. It expands to 650 kPa and exit temperature is 1100 K. Find the exit velocity, and the nozzle efficiency.

Solution:

C.V. Nozzle. Steady 1 inlet and 1 exit flows, no heat transfer, no work.

$$\text{Energy Eq.6.13: } h_i + (1/2)V_i^2 = h_e + (1/2)V_e^2$$

$$\text{Entropy Eq.9.8: } s_i + s_{\text{gen}} = s_e$$

Ideal nozzle  $s_{\text{gen}} = 0$  and assume same exit pressure as actual nozzle. Instead of using the standard entropy from Table A.7 and Eq.8.19 let us use a constant heat capacity at the average T and Eq.8.23. First from A.7.1

$$C_{p\ 1150} = \frac{1277.81 - 1161.18}{1200 - 1100} = 1.166 \text{ kJ/kg K;}$$

$$C_v = C_{p\ 1150} - R = 1.166 - 0.287 = 0.8793, \quad k = C_{p\ 1150}/C_v = 1.326$$

Notice how they differ from Table A.5 values.

$$T_{e\ s} = T_i (P_e/P_i)^{\frac{k-1}{k}} = 1200 \left( \frac{650}{1000} \right)^{0.24585} = 1079.4 \text{ K}$$

$$\begin{aligned} \frac{1}{2}V_{e\ s}^2 &= \frac{1}{2}V_i^2 + C(T_i - T_{e\ s}) = \frac{1}{2} \times 15^2 + 1.166(1200 - 1079.4) \times 1000 \\ &= 112.5 + 140619.6 = 140732 \text{ J/kg} \quad \Rightarrow \quad V_{e\ s} = 530.5 \text{ m/s} \end{aligned}$$

Actual nozzle with given exit temperature

$$\begin{aligned} \frac{1}{2}V_{e\ ac}^2 &= \frac{1}{2}V_i^2 + h_i - h_{e\ ac} = 112.5 + 1.166(1200 - 1100) \times 1000 \\ &= 116712.5 \text{ J/kg} \\ \Rightarrow V_{e\ ac} &= \mathbf{483 \text{ m/s}} \end{aligned}$$

$$\begin{aligned} \eta_{\text{noz}} &= \left( \frac{1}{2}V_{e\ ac}^2 - \frac{1}{2}V_i^2 \right) / \left( \frac{1}{2}V_{e\ s}^2 - \frac{1}{2}V_i^2 \right) = \\ &= (h_i - h_{e, AC}) / (h_i - h_{e, s}) = \frac{116600}{140619.6} = \mathbf{0.829} \end{aligned}$$

## 9.150

A nozzle is required to produce a flow of air at 200 m/s at 20°C, 100 kPa. It is estimated that the nozzle has an isentropic efficiency of 92%. What nozzle inlet pressure and temperature is required assuming the inlet kinetic energy is negligible?

Solution:

C.V. Air nozzle:  $P_e$ ,  $T_e(\text{real})$ ,  $V_e(\text{real})$ ,  $\eta_s(\text{real})$

For the real process:  $h_i = h_e + V_e^2/2$  or

$$T_i = T_e + V_e^2/2C_{p0} = 293.2 + 200^2/2 \times 1000 \times 1.004 = \mathbf{313.1 \text{ K}}$$

For the ideal process, from Eq.9.29:

$$V_{es}^2/2 = V_e^2/2\eta_s = 200^2/2 \times 1000 \times 0.92 = 21.74 \text{ kJ/kg}$$

and  $h_i = h_{es} + (V_{es}^2/2)$

$$T_{es} = T_i - V_{es}^2/(2C_{p0}) = 313.1 - 21.74/1.004 = 291.4 \text{ K}$$

The constant s relation in Eq.8.23 gives

$$\Rightarrow P_i = P_e (T_i/T_{es})^{\frac{k}{k-1}} = 100 \left( \frac{313.1}{291.4} \right)^{3.50} = \mathbf{128.6 \text{ kPa}}$$

## 9.151

A water-cooled air compressor takes air in at 20°C, 90 kPa and compresses it to 500 kPa. The isothermal efficiency is 80% and the actual compressor has the same heat transfer as the ideal one. Find the specific compressor work and the exit temperature.

Solution:

Ideal isothermal compressor exit 500 kPa, 20°C

Reversible process:  $dq = T ds \Rightarrow q = T(s_e - s_i)$

$$q = T(s_e - s_i) = T[s_{Te}^0 - s_{Ti}^0 - R \ln(P_e / P_i)]$$

$$= -RT \ln(P_e / P_i) = -0.287 \times 293.15 \ln(500/90) = -144.3 \text{ kJ/kg}$$

As same temperature for the ideal compressor  $h_e = h_i \Rightarrow$

$$w = q = -144.3 \text{ kJ/kg} \Rightarrow w_{ac} = w/\eta = -180.3 \text{ kJ/kg}, \quad q_{ac} = q$$

Now for the actual compressor energy equation becomes

$$q_{ac} + h_i = h_{e ac} + w_{ac} \Rightarrow$$

$$h_{e ac} - h_i = q_{ac} - w_{ac} = -144.3 - (-180.3) = 36 \text{ kJ/kg} \approx C_p (T_{e ac} - T_i)$$

$$T_{e ac} = T_i + 36/1.004 = 55.9^\circ\text{C}$$



## Review Problems

**9.152**

A flow of saturated liquid R-410a at 200 kPa in an evaporator is brought to a state of superheated vapor at 200 kPa, 40°C. Assume the process is reversible find the specific heat transfer and specific work.

C.V. Evaporator. From the device we know that potential and kinetic energies are not important (see chapter 6).

Since the pressure is constant and the process is reversible from Eq.9.14

$$w = -\int v dP + 0 + 0 - 0 = 0$$

From energy equation

$$h_i + q = w + h_e = h_e; \quad q = h_e - h_i$$

State i:  $h_i = 4.18 \text{ kJ/kg}$ , State e:  $h_e = 328.68 \text{ kJ/kg}$

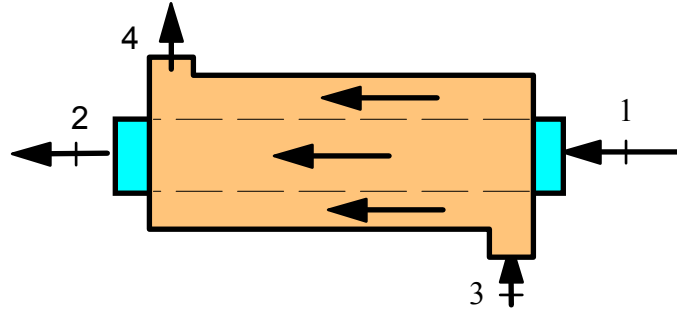
$$q = h_e - h_i = 328.68 - 4.18 = \mathbf{324.5 \text{ kJ/kg}}$$

## 9.153

A coflowing heat exchanger has one line with 2 kg/s saturated water vapor at 100 kPa entering. The other line is 1 kg/s air at 200 kPa, 1200 K. The heat exchanger is very long so the two flows exit at the same temperature. Find the exit temperature by trial and error. Calculate the rate of entropy generation.

Solution:

C.V. Heat exchanger,  
steady 2 flows in and  
two flows out.  
No W, no external Q



Flows:  $\dot{m}_1 = \dot{m}_2 = \dot{m}_{\text{H}_2\text{O}}$ ;  $\dot{m}_3 = \dot{m}_4 = \dot{m}_{\text{air}}$

Energy:  $\dot{m}_{\text{H}_2\text{O}} (h_2 - h_1) = \dot{m}_{\text{air}} (h_3 - h_4)$

State 1: Table B.1.2  $h_1 = 2675.5$  kJ/kg State 2: 100 kPa,  $T_2$

State 3: Table A.7  $h_3 = 1277.8$  kJ/kg, State 4: 200 kPa,  $T_2$

Only one unknown  $T_2$  and one equation the energy equation:

$$2(h_2 - 2675.5) = 1(1277.8 - h_4) \Rightarrow 2h_2 + h_4 = 6628.8 \text{ kW}$$

At 500 K:  $h_2 = 2902.0$ ,  $h_4 = 503.36 \Rightarrow \text{LHS} = 6307$  too small

At 700 K:  $h_2 = 3334.8$ ,  $h_4 = 713.56 \Rightarrow \text{LHS} = 7383$  too large

Linear interpolation  $T_2 = 560$  K,  $h_2 = 3048.3$ ,  $h_4 = 565.47 \Rightarrow \text{LHS} = 6662$

Final states are with  $T_2 = 554.4$  K = 281 °C

H<sub>2</sub>O: Table B.1.3,  $h_2 = 3036.8$  kJ/kg,  $s_2 = 8.1473$ ,  $s_1 = 7.3593$  kJ/kg K

AIR: Table A.7,  $h_4 = 559.65$  kJ/kg,  $s_{T4} = 7.4936$ ,  $s_{T3} = 8.3460$  kJ/kg K

The entropy balance equation, Eq.9.7, is solved for the generation term:

$$\begin{aligned} \dot{S}_{\text{gen}} &= \dot{m}_{\text{H}_2\text{O}} (s_2 - s_1) + \dot{m}_{\text{air}} (s_4 - s_3) \\ &= 2(8.1473 - 7.3593) + 1(7.4936 - 8.3460) = \mathbf{0.724 \text{ kW/K}} \end{aligned}$$

No pressure correction is needed as the air pressure for 4 and 3 is the same.

## 9.154

A flow of R-410a at 2000 kPa, 40°C in an isothermal expander is brought to a state of 1000 kPa in a reversible process. Find the specific heat transfer and work.

C.V. Expander. Steady reversible, single inlet and exit flow. Some  $q$  and  $w$

Energy Eq.6.13: 
$$h_i + q = w + h_e;$$

Entropy Eq.9.8: 
$$s_i + \int dq/T + s_{\text{gen}} = s_e$$

Process:  $T = \text{constant}$  so  $\int dq/T = q/T$  and reversible  $s_{\text{gen}} = 0$

State i:  $h_i = 295.49 \text{ kJ/kg}$ ,  $s_i = 1.0099 \text{ kJ/kg-K}$

State e:  $h_e = 316.05 \text{ kJ/kg}$ ,  $s_e = 1.1409 \text{ kJ/kg-K}$

From entropy equation

$$q = T (s_e - s_i) = 313.15 \text{ K} (1.1409 - 1.0099) \text{ kJ/kg-K} = \mathbf{41.023 \text{ kJ/kg}}$$

From the energy equation

$$w = h_i - h_e + q = 295.49 - 316.05 + 41.023 = \mathbf{330.46 \text{ kJ/kg}}$$

## 9.155

A vortex tube has an air inlet flow at 20°C, 200 kPa and two exit flows of 100 kPa, one at 0°C and the other at 40°C. The tube has no external heat transfer and no work and all the flows are steady and have negligible kinetic energy. Find the fraction of the inlet flow that comes out at 0°C. Is this setup possible?

Solution:

C.V. The vortex tube. Steady, single inlet and two exit flows. No q or w.

$$\text{Continuity Eq.: } \dot{m}_1 = \dot{m}_2 + \dot{m}_3 ; \quad \text{Energy: } \dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{m}_3 h_3$$

$$\text{Entropy: } \dot{m}_1 s_1 + \dot{S}_{\text{gen}} = \dot{m}_2 s_2 + \dot{m}_3 s_3$$

States all given by temperature and pressure. Use constant heat capacity to evaluate changes in h and s. Solve for  $x = \dot{m}_2/\dot{m}_1$  from the energy equation

$$\begin{aligned} \dot{m}_3/\dot{m}_1 &= 1 - x; & h_1 &= x h_2 + (1-x) h_3 \\ \Rightarrow x &= (h_1 - h_3)/(h_2 - h_3) = (T_1 - T_3)/(T_2 - T_3) = (20-40)/(0-40) = 0.5 \end{aligned}$$

Evaluate the entropy generation

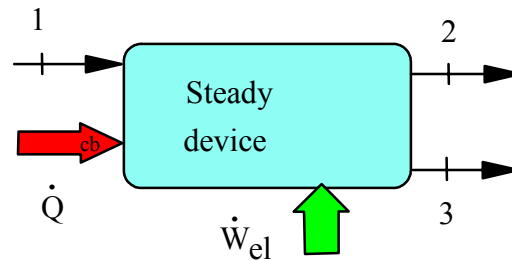
$$\begin{aligned} \dot{S}_{\text{gen}}/\dot{m}_1 &= x s_2 + (1-x)s_3 - s_1 = 0.5(s_2 - s_1) + 0.5(s_3 - s_1) \\ &= 0.5 [C_p \ln(T_2 / T_1) - R \ln(P_2 / P_1)] + 0.5[C_p \ln(T_3 / T_1) - R \ln(P_3/ P_1)] \\ &= 0.5 \left[ 1.004 \ln\left(\frac{273.15}{293.15}\right) - 0.287 \ln\left(\frac{100}{200}\right) \right] \\ &\quad + 0.5 \left[ 1.004 \ln\left(\frac{313.15}{293.15}\right) - 0.287 \ln\left(\frac{100}{200}\right) \right] \\ &= \mathbf{0.1966 \text{ kJ/kg K} > 0} \quad \mathbf{\text{So this is possible.}} \end{aligned}$$

## 9.156

A stream of ammonia enters a steady flow device at 100 kPa, 50°C, at the rate of 1 kg/s. Two streams exit the device at equal mass flow rates; one is at 200 kPa, 50°C, and the other as saturated liquid at 10°C. It is claimed that the device operates in a room at 25°C on an electrical power input of 250 kW. Is this possible?

Solution:

Control volume: Steady device out to ambient 25°C.



$$\text{Energy Eq.6.10:} \quad \dot{m}_1 h_1 + \dot{Q} + \dot{W}_{el} = \dot{m}_2 h_2 + \dot{m}_3 h_3$$

$$\text{Entropy Eq.9.7:} \quad \dot{m}_1 s_1 + \dot{Q}/T_{\text{room}} + \dot{S}_{\text{gen}} = \dot{m}_2 s_2 + \dot{m}_3 s_3$$

$$\text{State 1: Table B.2.2, } h_1 = 1581.2 \text{ kJ/kg, } s_1 = 6.4943 \text{ kJ/kg K}$$

$$\text{State 2: Table B.2.2 } h_2 = 1576.6 \text{ kJ/kg, } s_2 = 6.1453 \text{ kJ/kg K}$$

$$\text{State 3: Table B.2.1 } h_3 = 226.97 \text{ kJ/kg, } s_3 = 0.8779 \text{ kJ/kg K}$$

From the energy equation

$$\dot{Q} = 0.5 \times 1576.6 + 0.5 \times 226.97 - 1 \times 1581.2 - 250 = -929.4 \text{ kW}$$

From the entropy equation

$$\begin{aligned} \dot{S}_{\text{gen}} &= 0.5 \times 6.1453 + 0.5 \times 0.8779 - 1 \times 6.4943 - (-929.4)/298.15 \\ &= 0.1345 \text{ kW/K} > 0 \end{aligned}$$

**since  $\dot{S}_{\text{gen}} > 0$  this is possible**

## 9.157

In a heat-powered refrigerator, a turbine is used to drive the compressor using the same working fluid. Consider the combination shown in Fig. P9.157 where the turbine produces just enough power to drive the compressor and the two exit flows are mixed together. List any assumptions made and find the ratio of mass flow rates  $\dot{m}_3/\dot{m}_1$  and  $T_5$  ( $x_5$  if in two-phase region) if the turbine and the compressor are reversible and adiabatic

Solution:

CV: compressor

$$s_{2S} = s_1 = 1.0779 \text{ kJ/kg K} \rightarrow h_{2S} = 317.43 \text{ kJ/kg}$$

$$w_{SC} = h_1 - h_{2S} = 271.89 - 317.43 = -45.54 \text{ kJ/kg}$$

CV: turbine

$$s_{4S} = s_3 = 1.0850 \text{ kJ/kgK} \text{ and } P_{4S} \Rightarrow h_{4S} = 319.72 \text{ kJ/kg}$$

$$w_{ST} = h_3 - h_{4S} = 341.29 - 319.72 = 21.57 \text{ kJ/kg}$$

$$\text{As } \dot{w}_{\text{TURB}} = -\dot{w}_{\text{COMP}}, \quad \dot{m}_3/\dot{m}_1 = -\frac{w_{SC}}{w_{ST}} = \frac{45.54}{21.57} = \mathbf{2.111}$$

CV: mixing portion

$$\dot{m}_1 h_{2S} + \dot{m}_3 h_{4S} = (\dot{m}_1 + \dot{m}_3) h_5$$

$$1 \times 317.43 + 2.111 \times 319.72 = 3.111 h_5$$

$$\Rightarrow h_5 = 318.984 \text{ kJ/kg} \quad \Rightarrow \quad T_5 = \mathbf{58.7^\circ\text{C}}$$

## 9.158

Carbon dioxide flows through a device entering at 300 K, 200 kPa and leaving at 500 K. The process is steady state polytropic with  $n = 3.8$  and heat transfer comes from a 600 K source. Find the specific work, specific heat transfer and the specific entropy generation due to this process.

Solution:

C.V. Steady state device, single inlet and single exit flow.

Energy Eq.6.13:  $h_i + q = h_e + w$  Neglect kinetic, potential energies

Entropy Eq.9.8:  $s_i + \int dq/T + s_{gen} = s_e$

Process Eq.8.28:

$$P_e = P_i (T_e / T_i)^{\frac{n}{n-1}} = 200(500/300)^{\frac{3.8}{2.8}} = 400 \text{ kPa}$$

and the process leads to Eq.9.17 for the work term

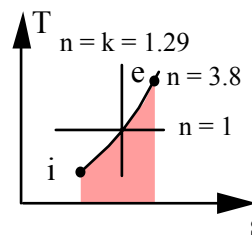
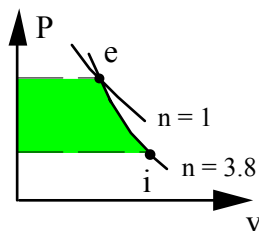
$$w = -\frac{n}{n-1} R (T_e - T_i) = -\frac{3.8}{2.8} \times 0.1889 \times (500 - 300) = \mathbf{-51.3 \text{ kJ/kg}}$$

Energy equation gives

$$q = h_e - h_i + w = 401.52 - 214.38 - 51.3 = \mathbf{135.8 \text{ kJ/kg}}$$

Entropy equation gives (CV out to source)

$$\begin{aligned} s_{gen} &= s_e - s_i - q/T_{source} = s_{T_e}^o - s_{T_i}^o - R \ln(P_e / P_i) - q/T_{source} \\ &= 5.3375 - 4.8631 - 0.1889 \ln(400/200) - (135.8/600) \\ &= \mathbf{0.117 \text{ kJ/kg K}} \end{aligned}$$



Notice:

$dP > 0$   
so  $dw < 0$

$ds > 0$   
so  $dq > 0$

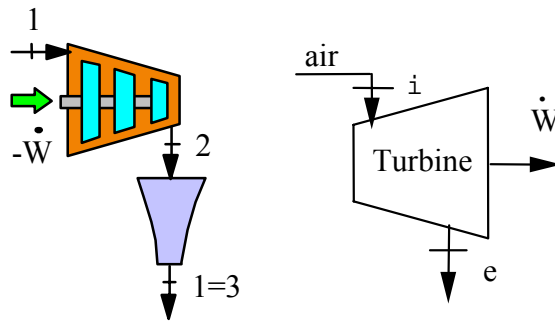
Notice process is externally irreversible,  $\Delta T$  between source and  $\text{CO}_2$



## 9.159

Air at 100 kPa, 17°C is compressed to 400 kPa after which it is expanded through a nozzle back to the atmosphere. The compressor and the nozzle are both reversible and adiabatic and kinetic energy in and out of the compressor can be neglected. Find the compressor work and its exit temperature and find the nozzle exit velocity.

Solution:



Separate control volumes around compressor and nozzle. For ideal compressor we have inlet : 1 and exit : 2

Adiabatic :  $q = 0$ .  
Reversible:  $s_{\text{gen}} = 0$

$$\text{Energy Eq.6.13: } h_1 + 0 = w_C + h_2;$$

$$\text{Entropy Eq.9.8: } s_1 + 0/T + 0 = s_2$$

$$-w_C = h_2 - h_1, \quad s_2 = s_1$$

Properties Table A.5 air:  $C_{P0} = 1.004 \text{ kJ/kg K}$ ,  $R = 0.287 \text{ kJ/kg K}$ ,  $k = 1.4$

Process gives constant  $s$  (isentropic) which with constant  $C_{P0}$  gives Eq.8.32

$$\Rightarrow T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = 290 \left( \frac{400}{100} \right)^{0.2857} = \mathbf{430.9 \text{ K}}$$

$$\Rightarrow -w_C = C_{P0}(T_2 - T_1) = 1.004 (430.9 - 290) = \mathbf{141.46 \text{ kJ/kg}}$$

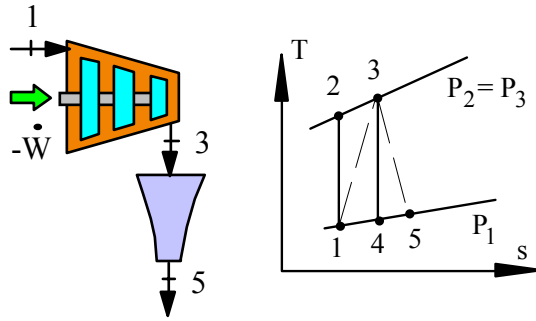
The ideal nozzle then expands back down to  $P_1$  (constant  $s$ ) so state 3 equals state 1. The energy equation has no work but kinetic energy and gives:

$$\frac{1}{2}V^2 = h_2 - h_1 = -w_C = 141\,460 \text{ J/kg} \quad (\text{remember conversion to J})$$

$$\Rightarrow V_3 = \sqrt{2 \times 141\,460} = \mathbf{531.9 \text{ m/s}}$$

## 9.160

Assume both the compressor and the nozzle in Problem 9.37 have an isentropic efficiency of 90% the rest being unchanged. Find the actual compressor work and its exit temperature and find the actual nozzle exit velocity.



C.V. Ideal compressor, inlet: 1 exit: 2

Adiabatic :  $q = 0$ .

Reversible:  $s_{\text{gen}} = 0$

$$\text{Energy Eq.6.13: } h_1 + 0 = w_C + h_2;$$

$$\text{Entropy Eq.9.8: } s_1 + 0/T + 0 = s_2$$

$$-w_{Cs} = h_2 - h_1, \quad s_2 = s_1$$

Properties use air Table A.5:  $C_{p0} = 1.004 \frac{\text{kJ}}{\text{kg K}}$ ,  $R = 0.287 \frac{\text{kJ}}{\text{kg K}}$ ,  $k = 1.4$ ,

Process gives constant  $s$  (isentropic) which with constant  $C_{p0}$  gives Eq.8.32

$$\Rightarrow T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = 290 \left( \frac{400}{100} \right)^{0.2857} = \mathbf{430.9 \text{ K}}$$

$$\Rightarrow -w_{Cs} = C_{p0}(T_2 - T_1) = 1.004 (430.9 - 290) = \mathbf{141.46 \text{ kJ/kg}}$$

The ideal nozzle then expands back down to state 1 (constant  $s$ ). The actual compressor discharges at state 3 however, so we have:

$$w_C = w_{Cs}/\eta_C = \mathbf{-157.18} \Rightarrow T_3 = T_1 - w_C/C_p = \mathbf{446.6 \text{ K}}$$

Nozzle receives air at 3 and exhausts at 5. We must do the ideal (exit at 4) first.

$$s_4 = s_3 \Rightarrow \text{Eq.8.32: } T_4 = T_3 \left( \frac{P_4}{P_3} \right)^{\frac{k-1}{k}} = 300.5 \text{ K}$$

$$\frac{1}{2} \mathbf{V}_s^2 = C_p(T_3 - T_4) = 146.68 \Rightarrow \frac{1}{2} \mathbf{V}_{ac}^2 = 132 \text{ kJ/kg} \Rightarrow \mathbf{V}_{ac} = 513.8 \text{ m/s}$$

If we need it, the actual nozzle exit (5) can be found:

$$T_5 = T_3 - \mathbf{V}_{ac}^2/2C_p = 315 \text{ K}$$

## 9.161

An insulated piston/cylinder contains R-410a at 20°C, 85% quality, at a cylinder volume of 50 L. A valve at the closed end of the cylinder is connected to a line flowing R-410a at 2 MPa, 60°C. The valve is now opened, allowing R-410a to flow in, and at the same time the external force on the piston is decreased, and the piston moves. When the valve is closed, the cylinder contents are at 800 kPa, 20°C, and a positive work of 50 kJ has been done against the external force. What is the final volume of the cylinder? Does this process violate the second law of thermodynamics?

Solution:

C.V. Cylinder volume. A transient problem.

$$\text{Continuity Eq.:} \quad m_2 - m_1 = m_i$$

$$\text{Energy Eq.:} \quad m_2 u_2 - m_1 u_1 = {}_1Q_2 + m_i h_i - {}_1W_2$$

$$\text{Entropy Eq.:} \quad m_2 s_2 - m_1 s_1 = {}_1Q_2/T + m_i s_i + {}_1S_2 \text{ gen}$$

$$\text{Process:} \quad {}_1Q_2 = 0, \quad {}_1W_2 = 50 \text{ kJ}$$

$$\text{State 1: } T_1 = 20^\circ\text{C}, \quad x_1 = 0.85, \quad V_1 = 50 \text{ L} = 0.05 \text{ m}^3$$

$$P_1 = P_g = 1444.2 \text{ kPa}, \quad u_1 = u_f + x_1 u_{fg} = 232.62 \text{ kJ/kg}$$

$$v_1 = v_f + x_1 v_{fg} = 0.000923 + 0.85 \times 0.01666 = 0.015084 \text{ m}^3/\text{kg},$$

$$s_1 = s_f + x_1 s_{fg} = 0.3357 + 0.85 \times 0.6627 = 0.8990 \text{ kJ/kg K}$$

$$m_1 = V_1/v_1 = 0.050 / 0.015084 = 3.3148 \text{ kg}$$

$$\text{State 2: } T_2 = 20^\circ\text{C}, \quad P_2 = 800 \text{ kPa, superheated}, \quad v_2 = 0.03693 \text{ m}^3/\text{kg},$$

$$u_2 = 270.47 \text{ kJ/kg}, \quad s_2 = 1.1105 \text{ kJ/kg K}$$

$$\text{Inlet: } T_i = 60^\circ\text{C}, \quad P_i = 2 \text{ MPa}, \quad h_i = 320.62 \text{ kJ/kg}, \quad s_i = 1.0878 \text{ kJ/kg K}$$

Solve for the mass  $m_2$  from the energy equation (the only unknown)

$$\begin{aligned} m_2 &= [m_1 u_1 - {}_1W_2 - m_1 h_i] / [u_2 - h_i] \\ &= \frac{3.3148 \times 232.62 - 50 - 3.3148 \times 320.62}{270.47 - 320.62} = 6.8136 \text{ kg} \end{aligned}$$

$$V_2 = m_2 v_2 = 0.2516 \text{ m}^3$$

Now check the second law

$$\begin{aligned} {}_1S_2 \text{ gen} &= m_2 s_2 - m_1 s_1 - {}_1Q_2/T - m_i s_i \\ &= 6.8136 \times 1.1105 - 3.3148 \times 0.8990 - 0 - (6.8136 - 3.3148)1.0878 \\ &= 0.7805 \text{ kJ/K} \geq 0, \quad \text{Satisfies 2}^{\text{nd}} \text{ Law} \end{aligned}$$

## 9.162

A certain industrial process requires a steady 0.5 kg/s supply of compressed air at 500 kPa, at a maximum temperature of 30°C. This air is to be supplied by installing a compressor and aftercooler, see Fig. P9.46. Local ambient conditions are 100 kPa, 20°C. Using an isentropic compressor efficiency of 80%, determine the power required to drive the compressor and the rate of heat rejection in the aftercooler.

Air table A.5:  $R = 0.287 \text{ kJ/kg-K}$ ,  $C_p = 1.004 \text{ kJ/kg-K}$ ,  $k = 1.4$

State 1:  $T_1 = T_0 = 20^\circ\text{C}$ ,  $P_1 = P_0 = 100 \text{ kPa}$ ,  $\dot{m} = 0.5 \text{ kg/s}$

State 2:  $P_2 = P_3 = 500 \text{ kPa}$

State 3:  $T_3 = 30^\circ\text{C}$ ,  $P_3 = 500 \text{ kPa}$

We have  $\eta_s = 80\% = w_{Cs}/w_{Cac}$

Compressor: First do the ideal (Isentropic)

$$T_{2s} = T_1 (P_2/P_1)^{\frac{k-1}{k}} = 293.15 (500/100)^{0.2857} = 464.6 \text{ K}$$

Energy Eq.:  $q_c + h_1 = h_2 + w_c$ ;  $q_c = 0$ , assume constant specific heat

$$w_{Cs} = C_p(T_1 - T_{2s}) = 1.004(293.15 - 464.6) = -172.0 \text{ kJ/kg}$$

$$\eta_s = w_{Cs}/w_{Cac}, \quad w_{Cac} = w_{Cs}/\eta_s = -215, \quad \dot{W}_C = \dot{m}w_C = \mathbf{-107.5 \text{ kW}}$$

$$w_{Cac} = C_p(T_1 - T_2), \text{ solve for } T_2 = 507.5 \text{ K}$$

Aftercooler:

Energy Eq.:  $q + h_2 = h_3 + w$ ;  $w = 0$ , assume constant specific heat

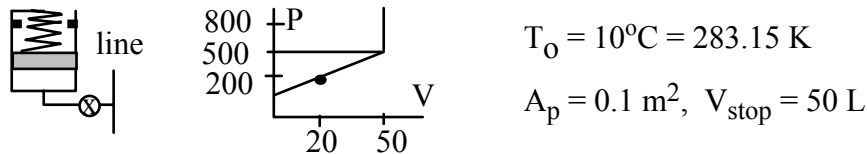
$$q = C_p(T_3 - T_2) = 1.004(303.15 - 507.5) = -205 \text{ kJ/kg},$$

$$\dot{Q} = \dot{m}q = \mathbf{-102.5 \text{ kW}}$$

## 9.163

A frictionless piston/cylinder is loaded with a linear spring, spring constant 100 kN/m and the piston cross-sectional area is 0.1 m<sup>2</sup>. The cylinder initial volume of 20 L contains air at 200 kPa and ambient temperature, 10°C. The cylinder has a set of stops that prevent its volume from exceeding 50 L. A valve connects to a line flowing air at 800 kPa, 50°C. The valve is now opened, allowing air to flow in until the cylinder pressure reaches 800 kPa, at which point the temperature inside the cylinder is 80°C. The valve is then closed and the process ends.

- Is the piston at the stops at the final state?
- Taking the inside of the cylinder as a control volume, calculate the heat transfer during the process.
- Calculate the net entropy change for this process.



Air from Table A.5:  $R = 0.287$ ,  $C_p = 1.004$ ,  $C_v = 0.717\text{ kJ/kg}\cdot\text{K}$

State 1:  $T_1 = 10^\circ\text{C}$ ,  $P_1 = 200\text{ kPa}$ ,  $V_1 = 20\text{ L} = 0.02\text{ m}^3$ ,

$$m_1 = P_1 V_1 / RT_1 = 200 \times 0.02 / (0.287 \times 283.15) = 0.0492\text{ kg}$$

State 2:  $T_2 = 80^\circ\text{C}$ ,  $P_2 = 800\text{ kPa}$ , Inlet:  $T_i = 50^\circ\text{C}$ ,  $P_i = 800\text{ kPa}$

$$\text{a) } P_{\text{stop}} = P_1 + \frac{k_s}{A_p} (V_{\text{stop}} - V_1) = 500\text{ kPa}, \mathbf{P_2 > P_{\text{stop}} \rightarrow \text{Piston hits stops}}$$

$$V_2 = V_{\text{stop}} = 50\text{ L}, m_2 = PV/RT = 0.3946\text{ kg}$$

b) 1<sup>st</sup> Law:  ${}_1Q_2 + m_i h_i = m_2 u_2 - m_1 u_1 + m_e h_e + {}_1W_2$ ;  $m_e = 0$ ,  $m_i = m_2 - m_1$

$${}_1W_2 = \int P dV = (P_1 + P_{\text{stop}})(V_{\text{stop}} - V_1)/2 = 10.5\text{ kJ}$$

Assume constant specific heat

$${}_1Q_2 = m_2 C_v T_2 - m_1 C_v T_1 - (m_2 - m_1) C_p T_i + {}_1W_2 = \mathbf{-11.6\text{ kJ}}$$

c) 2<sup>nd</sup> Law:

$$\Delta S_{\text{net}} = m_2 s_2 - m_1 s_1 - m_i s_i - \frac{Q_{\text{cv}}}{T_0} = m_2 (s_2 - s_i) - m_1 (s_1 - s_i) - \frac{Q_{\text{cv}}}{T_0}$$

$$s_2 - s_i = C_p \ln(T_2 / T_i) - R \ln(P_2 / P_i) = 0.08907\text{ kJ/kg}\cdot\text{K} \quad (P_2 = P_i)$$

$$s_1 - s_i = C_p \ln(T_1 / T_i) - R \ln(P_1 / P_i) = 0.26529\text{ kJ/kg}\cdot\text{K}$$

$$\Delta S_{\text{net}} = \mathbf{0.063\text{ kJ/K}}$$

## 9.164

Air enters an insulated turbine at 50°C, and exits the turbine at -30°C, 100 kPa. The isentropic turbine efficiency is 70% and the inlet volumetric flow rate is 20 L/s. What is the turbine inlet pressure and the turbine power output?

C.V.: Turbine,  $\eta_s = 0.7$ , Insulated

Air:  $C_p = 1.004$  kJ/kg-K,  $R = 0.287$  kJ/kg-K,  $k = 1.4$

Inlet:  $T_i = 50^\circ\text{C}$ ,  $\dot{V}_i = 20$  L/s =  $0.02$  m<sup>3</sup>/s

Exit:  $T_e = -30^\circ\text{C}$ ,  $P_e = 100$  kPa

a) 1<sup>st</sup> Law steady flow:  $q + h_i = h_e + w_T$ ;  $q = 0$

Assume Constant Specific Heat

$$w_T = h_i - h_e = C_p(T_i - T_e) = 80.3 \text{ kJ/kg}$$

$$w_{Ts} = w/\eta = 114.7 \text{ kJ/kg}, \quad w_{Ts} = C_p(T_i - T_{es})$$

Solve for  $T_{es} = 208.9$  K

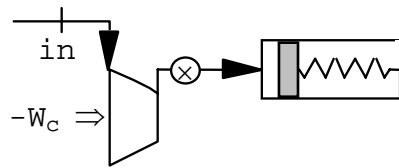
$$\text{Isentropic Process: } P_e = P_i (T_e / T_i)^{\frac{k}{k-1}} \Rightarrow \mathbf{P_i = 461 \text{ kPa}}$$

$$\text{b) } \dot{W}_T = \dot{m}w_T; \quad \dot{m} = P\dot{V}/RT = 0.099 \text{ kg/s} \Rightarrow \mathbf{\dot{W}_T = 7.98 \text{ kW}}$$

## 9.165

An initially empty spring-loaded piston/cylinder requires 100 kPa to float the piston. A compressor with a line and valve now charges the cylinder with water to a final pressure of 1.4 MPa at which point the volume is  $0.6 \text{ m}^3$ , state 2. The inlet condition to the reversible adiabatic compressor is saturated vapor at 100 kPa. After charging the valve is closed and the water eventually cools to room temperature,  $20^\circ\text{C}$ , state 3. Find the final mass of water, the piston work from 1 to 2, the required compressor work, and the final pressure,  $P_3$ .

Solution:



Process 1→2: transient, adiabatic.  
for C.V. compressor + cylinder  
Assume process is reversible

$$\text{Continuity: } m_2 - 0 = m_{\text{in}}, \quad \text{Energy: } m_2 u_2 - 0 = (m_{\text{in}} h_{\text{in}}) - W_c - {}_1W_2$$

$$\text{Entropy Eq.: } m_2 s_2 - 0 = m_{\text{in}} s_{\text{in}} + 0 \Rightarrow s_2 = s_{\text{in}}$$

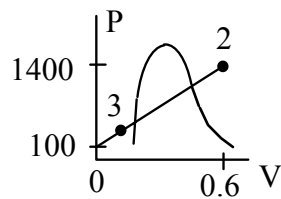
$$\text{Inlet state: Table B.1.2, } h_{\text{in}} = 2675.5 \text{ kJ/kg, } s_{\text{in}} = 7.3594 \text{ kJ/kg K}$$

$${}_1W_2 = \int P dV = \frac{1}{2} (P_{\text{float}} + P_2)(V_2 - 0) = \frac{1}{2} (100 + 1400)0.6 = \mathbf{450 \text{ kJ}}$$

$$\text{State 2: } P_2, s_2 = s_{\text{in}} \text{ Table B.1.3 } \Rightarrow v_2 = 0.2243, u_2 = 2984.4 \text{ kJ/kg}$$

$$m_2 = V_2/v_2 = 0.6/0.2243 = \mathbf{2.675 \text{ kg}}$$

$$W_c = m_{\text{in}} h_{\text{in}} - m_2 u_2 - {}_1W_2 = 2.675 \times (2675.5 - 2984.4) - 450 = \mathbf{-1276.3 \text{ kJ}}$$



State 3 must be on line &  $20^\circ\text{C}$

Assume 2-phase  $\Rightarrow P_3 = P_{\text{sat}}(20^\circ\text{C}) = 2.339 \text{ kPa}$

less than  $P_{\text{float}}$  so compressed liquid

$$\text{Table B.1.1: } v_3 \cong v_f(20^\circ\text{C}) = 0.001002 \Rightarrow V_3 = m_3 v_3 = 0.00268 \text{ m}^3$$

$$\text{On line: } P_3 = 100 + (1400 - 100) \times 0.00268/0.6 = \mathbf{105.8 \text{ kPa}}$$

## 9.166

Consider the scheme shown in Fig. P9.166 for producing fresh water from salt water. The conditions are as shown in the figure. Assume that the properties of salt water are the same as for pure water, and that the pump is reversible and adiabatic.

- Determine the ratio  $(\dot{m}_7/\dot{m}_1)$ , the fraction of salt water purified.
- Determine the input quantities,  $w_P$  and  $q_H$ .
- Make a second law analysis of the overall system.

C.V. Flash evaporator: Steady flow, no external  $q$ , no work.

$$\text{Energy Eq.: } \dot{m}_1 h_4 = (\dot{m}_1 - \dot{m}_7) h_5 + \dot{m}_7 h_6$$

$$\text{Table B.1.1 or } 632.4 = (1 - (\dot{m}_7/\dot{m}_1)) 417.46 + (\dot{m}_7/\dot{m}_1) 2675.5$$

$$\Rightarrow \dot{m}_7/\dot{m}_1 = \mathbf{0.0952}$$

C.V. Pump steady flow, incompressible liq.:

$$w_P = -\int v dP \approx -v_1(P_2 - P_1) = -0.001001(700 - 100) = \mathbf{-0.6 \text{ kJ/kg}}$$

$$h_2 = h_1 - w_P = 62.99 + 0.6 = 63.6 \text{ kJ/kg}$$

C.V. Heat exchanger:  $h_2 + (\dot{m}_7/\dot{m}_1)h_6 = h_3 + (\dot{m}_7/\dot{m}_1)h_7$

$$63.6 + 0.0952 \times 2675.5 = h_3 + 0.0952 \times 146.68 \Rightarrow h_3 = 304.3 \text{ kJ/kg}$$

C.V. Heater:  $q_H = h_4 - h_3 = 632.4 - 304.3 = \mathbf{328.1 \text{ kJ/kg}}$

CV: entire unit, entropy equation per unit mass flow rate at state 1

$$\begin{aligned} S_{\text{C.V.,gen}} &= -q_H/T_H + (1 - (\dot{m}_7/\dot{m}_1))s_5 + (\dot{m}_7/\dot{m}_1)s_7 - s_1 \\ &= (-328.1/473.15) + 0.9048 \times 1.3026 + 0.0952 \times 0.5053 - 0.2245 \\ &= \mathbf{0.3088 \text{ kJ/K kg } m_1} \end{aligned}$$



## 9.167

A rigid 1.0 m<sup>3</sup> tank contains water initially at 120°C, with 50 % liquid and 50% vapor, by volume. A pressure-relief valve on the top of the tank is set to 1.0 MPa (the tank pressure cannot exceed 1.0 MPa - water will be discharged instead). Heat is now transferred to the tank from a 200°C heat source until the tank contains saturated vapor at 1.0 MPa. Calculate the heat transfer to the tank and show that this process does not violate the second law.

Solution:

C.V. Tank and walls out to the source. Neglect storage in walls. There is flow out and no boundary or shaft work.

$$\text{Continuity Eq.6.15: } m_2 - m_1 = -m_e$$

$$\text{Energy Eq.6.16: } m_2 u_2 - m_1 u_1 = -m_e h_e + {}_1Q_2$$

$$\text{Entropy Eq.9.12: } m_2 s_2 - m_1 s_1 = -m_e s_e + \int dQ/T + {}_1S_2 \text{ gen}$$

State 1:  $T_1 = 120^\circ\text{C}$ , Table B.1.1

$$v_f = 0.00106 \text{ m}^3/\text{kg}, \quad m_{\text{liq}} = 0.5V_1/v_f = 471.7 \text{ kg}$$

$$v_g = 0.8919 \text{ m}^3/\text{kg}, \quad m_g = 0.5V_1/v_g = 0.56 \text{ kg},$$

$$m_1 = 472.26 \text{ kg}, \quad x_1 = m_g/m_1 = 0.001186$$

$$u_1 = u_f + x_1 u_{fg} = 503.5 + 0.001186 \times 2025.8 = 505.88 \text{ kJ/kg},$$

$$s_1 = s_f + x_1 s_{fg} = 1.5275 + 0.001186 \times 5.602 = 1.5341 \text{ kJ/kg-K}$$

State 2:  $P_2 = 1.0 \text{ MPa}$ , sat. vap.  $x_2 = 1.0$ ,  $V_2 = 1 \text{ m}^3$

$$v_2 = v_g = 0.19444 \text{ m}^3/\text{kg}, \quad m_2 = V_2/v_2 = 5.14 \text{ kg}$$

$$u_2 = u_g = 2583.6 \text{ kJ/kg}, \quad s_2 = s_g = 6.5864 \text{ kJ/kg-K}$$

Exit:  $P_e = 1.0 \text{ MPa}$ , sat. vap.  $x_e = 1.0$ ,  $h_e = h_g = 2778.1 \text{ kJ/kg}$ ,

$$s_e = s_g = 6.5864 \text{ kJ/kg}, \quad m_e = m_1 - m_2 = 467.12 \text{ kg}$$

From the energy equation we get

$${}_1Q_2 = m_2 u_2 - m_1 u_1 + m_e h_e = \mathbf{1\ 072\ 080\ kJ}$$

From the entropy Eq.9.24 (with 9.25 and 9.26) we get

$${}_1S_2 \text{ gen} = m_2 s_2 - m_1 s_1 + m_e s_e - \frac{{}_1Q_2}{T_H}; \quad T_H = 200^\circ\text{C} = 473 \text{ K}$$

$${}_1S_2 \text{ gen} = \Delta S_{\text{net}} = \mathbf{120.4\ kJ \geq 0} \quad \text{Process Satisfies 2<sup>nd</sup> Law}$$

## 9.168

A jet-ejector pump, shown schematically in Fig. P9.168, is a device in which a low-pressure (secondary) fluid is compressed by entrainment in a high-velocity (primary) fluid stream. The compression results from the deceleration in a diffuser. For purposes of analysis this can be considered as equivalent to the turbine-compressor unit shown in Fig. P9.157 with the states 1, 3, and 5 corresponding to those in Fig. P9.168. Consider a steam jet-pump with state 1 as saturated vapor at 35 kPa; state 3 is 300 kPa, 150°C; and the discharge pressure,  $P_5$ , is 100 kPa.

- a. Calculate the ideal mass flow ratio,  $\dot{m}_1/\dot{m}_3$ .
- b. The efficiency of a jet pump is defined as  $\eta = (\dot{m}_1/\dot{m}_3)_{\text{actual}} / (\dot{m}_1/\dot{m}_3)_{\text{ideal}}$  for the same inlet conditions and discharge pressure. Determine the discharge temperature of the jet pump if its efficiency is 10%.

a) ideal processes (isen. comp. & exp.)

expands 3-4s }  
comp 1-2s } then mix at const. P

$$s_{4s} = s_3 = 7.0778 = 1.3026 + x_{4s} \times 6.0568 \Rightarrow x_{4s} = 0.9535$$

$$h_{4s} = 417.46 + 0.9535 \times 2258.0 = 2570.5 \text{ kJ/kg}$$

$$s_{2s} = s_1 = 7.7193 \rightarrow T_{2s} = 174^\circ\text{C} \quad \& \quad h_{2s} = 2823.8 \text{ kJ/kg}$$

$$\dot{m}_1(h_{2s} - h_1) = \dot{m}_3(h_3 - h_{4s})$$

$$\Rightarrow (\dot{m}_1/\dot{m}_3)_{\text{IDEAL}} = \frac{2761.0 - 2570.5}{2823.8 - 2631.1} = \mathbf{0.9886}$$

b) real processes with jet pump eff. = 0.10

$$\Rightarrow (\dot{m}_1/\dot{m}_3)_{\text{ACTUAL}} = 0.10 \times 0.9886 = 0.09886$$

$$\text{1st law } \dot{m}_1 h_1 + \dot{m}_3 h_3 = (\dot{m}_1 + \dot{m}_3) h_5$$

$$0.09886 \times 2631.1 + 1 \times 2761.0 = 1.09896 h_5$$

$$\text{State 5: } h_5 = 2749.3 \text{ kJ/kg, } P_5 = 100 \text{ kPa} \Rightarrow T_5 = \mathbf{136.5^\circ\text{C}}$$

## 9.169

A horizontal, insulated cylinder has a frictionless piston held against stops by an external force of 500 kN. The piston cross-sectional area is  $0.5 \text{ m}^2$ , and the initial volume is  $0.25 \text{ m}^3$ . Argon gas in the cylinder is at 200 kPa,  $100^\circ\text{C}$ . A valve is now opened to a line flowing argon at 1.2 MPa,  $200^\circ\text{C}$ , and gas flows in until the cylinder pressure just balances the external force, at which point the valve is closed. Use constant heat capacity to verify that the final temperature is 645 K and find the total entropy generation.

Solution:

The process has inlet flow, no work (volume constant) and no heat transfer.

$$\text{Continuity Eq.6.15: } m_2 - m_1 = m_i$$

$$\text{Energy Eq.6.16: } m_2 u_2 - m_1 u_1 = m_i h_i$$

$$m_1 = P_1 V_1 / RT_1 = 200 \times 0.25 / (0.2081 \times 373.15) = 0.644 \text{ kg}$$

$$\text{Force balance: } P_2 A = F \quad \Rightarrow \quad P_2 = \frac{500}{0.5} = 1000 \text{ kPa}$$

For argon use constant heat capacities so the energy equation is:

$$m_2 C_{V0} T_2 - m_1 C_{V0} T_1 = (m_2 - m_1) C_{P0} T_{in}$$

We know  $P_2$  so only 1 unknown for state 2.

$$\text{Use ideal gas law to write } m_2 T_2 = P_2 V_1 / R \quad \text{and} \quad m_1 T_1 = P_1 V_1 / R$$

and divide the energy equation with  $C_{V0}$  to solve for the change in mass

$$(P_2 V_1 - P_1 V_1) / R = (m_2 - m_1) (C_{P0} / C_{V0}) T_{in}$$

$$(m_2 - m_1) = (P_2 - P_1) V_1 / (R k T_{in})$$

$$= (1000 - 200) \times 0.25 / (0.2081 \times 1.667 \times 473.15) = 1.219 \text{ kg}$$

$$m_2 = 1.219 + 0.644 = 1.863 \text{ kg.}$$

$$T_2 = P_2 V_1 / (m_2 R) = 1000 \times 0.25 / (1.863 \times 0.2081) = 645 \text{ K} \quad \text{OK}$$

$$\text{Entropy Eq.9.12: } m_2 s_2 - m_1 s_1 = m_i s_i + 0 + {}_1S_{2 \text{ gen}}$$

$${}_1S_{2 \text{ gen}} = m_1 (s_2 - s_1) + (m_2 - m_1) (s_2 - s_i)$$

$$= m_1 \left[ C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \right] + (m_2 - m_1) \left[ C_p \ln \frac{T_2}{T_i} - R \ln \frac{P_2}{P_i} \right]$$

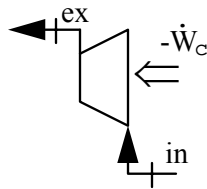
$$= 0.644 \left[ 0.52 \ln \frac{645}{373.15} - 0.2081 \ln \frac{1000}{200} \right]$$

$$+ 1.219 \left[ 0.52 \ln \frac{645}{473.15} - 0.2081 \ln \frac{1000}{1200} \right]$$

$$= -0.03242 + 0.24265 = \mathbf{0.21 \text{ kJ/K}}$$

## 9.170

Supercharging of an engine is used to increase the inlet air density so that more fuel can be added, the result of which is an increased power output. Assume that ambient air, 100 kPa and 27°C, enters the supercharger at a rate of 250 L/s. The supercharger (compressor) has an isentropic efficiency of 75%, and uses 20 kW of power input. Assume that the ideal and actual compressor have the same exit pressure. Find the ideal specific work and verify that the exit pressure is 175 kPa. Find the percent increase in air density entering the engine due to the supercharger and the entropy generation.



C.V.: Air in compressor (steady flow)

$$\text{Cont: } \dot{m}_{\text{in}} = \dot{m}_{\text{ex}} = \dot{m} = \dot{V}/v_{\text{in}} = 0.29 \text{ kg/s}$$

$$\text{Energy: } \dot{m}h_{\text{in}} - \dot{W} = \dot{m}h_{\text{ex}} \quad \text{Assume: } \dot{Q} = 0$$

$$\text{Entropy: } \dot{m}s_{\text{in}} + \dot{S}_{\text{gen}} = \dot{m}s_{\text{ex}}$$

$$v_{\text{in}} = \frac{RT_{\text{in}}}{P_{\text{in}}} = 0.8614 \text{ m}^3/\text{kg}, \quad s_{\text{Ti}}^{\circ} = 6.86975 \text{ kJ/kg K}, \quad h_{\text{in}} = 300.62 \text{ kJ/kg}$$

$$\eta_c = w_{\text{C s}}/w_{\text{C ac}} \Rightarrow -\dot{W}_s = -\dot{W}_{\text{AC}} \times \eta_c = 15 \text{ kW}$$

$$-w_{\text{C s}} = -\dot{W}_s/\dot{m} = 51.724 \text{ kJ/kg}, \quad -w_{\text{C ac}} = 68.966 \text{ kJ/kg}$$

$$\text{Table A.7: } h_{\text{ex s}} = h_{\text{in}} - w_{\text{C s}} = 300.62 + 51.724 = 352.3 \text{ kJ/kg}$$

$$\Rightarrow T_{\text{ex s}} = 351.5 \text{ K}, \quad s_{\text{Te}}^{\circ} = 7.02830 \text{ kJ/kg K}$$

$$P_{\text{ex}} = P_{\text{in}} \times e^{(s_{\text{Te}}^{\circ} - s_{\text{Ti}}^{\circ})/R} = 100 \times \exp \left[ \frac{7.0283 - 6.86975}{0.287} \right]$$

$$= \mathbf{173.75 \text{ kPa}}$$

The actual exit state is

$$h_{\text{ex ac}} = h_{\text{in}} - w_{\text{C ac}} = 369.6 \text{ kJ/kg} \Rightarrow T_{\text{ex ac}} = 368.6 \text{ K}$$

$$v_{\text{ex}} = RT_{\text{ex}}/P_{\text{ex}} = 0.6088 \text{ m}^3/\text{kg}, \quad s_{\text{Tex ac}}^{\circ} = 7.0764$$

$$\rho_{\text{ex}}/\rho_{\text{in}} = v_{\text{in}}/v_{\text{ex}} = 0.8614/0.6088 = \mathbf{1.415 \text{ or } 41.5\% \text{ increase}}$$

$$s_{\text{gen}} = s_{\text{ex}} - s_{\text{in}} = 7.0764 - 6.86975 - 0.287 \ln\left(\frac{173.75}{100}\right) = \mathbf{0.0481 \text{ kJ/kg K}}$$

## 9.171

A rigid steel bottle,  $V = 0.25 \text{ m}^3$ , contains air at 100 kPa, 300 K. The bottle is now charged with air from a line at 260 K, 6 MPa to a bottle pressure of 5 MPa, state 2, and the valve is closed. Assume that the process is adiabatic, and the charge always is uniform. In storage, the bottle slowly returns to room temperature at 300 K, state 3. Find the final mass, the temperature  $T_2$ , the final pressure  $P_3$ , the heat transfer  ${}_1Q_3$  and the total entropy generation.

C.V. Bottle. Flow in, no work, no heat transfer.

$$\text{Continuity Eq.6.15: } m_2 - m_1 = m_{\text{in}} ;$$

$$\text{Energy Eq.6.16: } m_2 u_2 - m_1 u_1 = m_{\text{in}} h_{\text{in}}$$

State 1 and inlet: Table A.7,  $u_1 = 214.36 \text{ kJ/kg}$ ,  $h_{\text{in}} = 260.32 \text{ kJ/kg}$

$$m_1 = P_1 V / RT_1 = (100 \times 0.25) / (0.287 \times 300) = 0.290 \text{ kg}$$

$$m_2 = P_2 V / RT_2 = 5000 \times 0.25 / (0.287 \times T_2) = 4355.4 / T_2$$

Substitute into energy equation

$$u_2 + 0.00306 T_2 = 260.32$$

Now trial and error on  $T_2$

$$T_2 = 360 \Rightarrow \text{LHS} = 258.63 \text{ (low);}$$

$$T_2 = 370 \Rightarrow \text{LHS} = 265.88 \text{ (high)}$$

Interpolation  $T_2 = 362.3 \text{ K}$  (LHS = 260.3 OK)

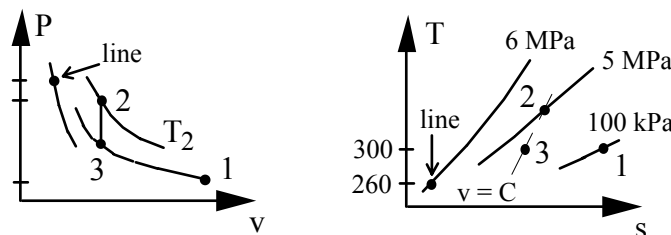
$$m_2 = 4355.4 / 362.3 = 12.022 \text{ kg}; P_3 = m_2 R T_3 / V = \mathbf{4140 \text{ kPa}}$$

Now use the energy equation from the beginning to the final state

$$\begin{aligned} {}_1Q_3 &= m_2 u_3 - m_1 u_1 - m_{\text{in}} h_{\text{in}} = (12.022 - 0.29) 214.36 - 11.732 \times 260.32 \\ &= \mathbf{-539.2 \text{ kJ}} \end{aligned}$$

Entropy equation from state 1 to state 3 with change in  $s$  from Eq.8.28

$$\begin{aligned} S_{\text{gen}} &= m_2 s_3 - m_1 s_1 - m_{\text{in}} s_{\text{in}} - {}_1Q_3 / T = m_2 (s_3 - s_{\text{in}}) - m_1 (s_1 - s_{\text{in}}) - {}_1Q_3 / T \\ &= 12.022 [6.8693 - 6.7256 - R \ln(4140/6000)] \\ &\quad - 0.29 [6.8693 - 6.7256 - R \ln(100/6000)] + 539.2/300 = \mathbf{4.423 \text{ kJ/K}} \end{aligned}$$



Problem could have been solved with constant specific heats from A.5 in which case we would get the energy explicit in  $T_2$  (no iterations).

## 9.172

A certain industrial process requires a steady 0.5 kg/s of air at 200 m/s, at the condition of 150 kPa, 300 K. This air is to be the exhaust from a specially designed turbine whose inlet pressure is 400 kPa. The turbine process may be assumed to be reversible and polytropic, with polytropic exponent  $n = 1.20$ .

- What is the turbine inlet temperature?
- What are the power output and heat transfer rate for the turbine?
- Calculate the rate of net entropy increase, if the heat transfer comes from a source at a temperature 100°C higher than the turbine inlet temperature.

Solution:

C.V. Turbine, this has heat transfer,  $PV^n = \text{Constant}$ ,  $n = 1.2$

Process polytropic Eq.8.37:  $T_e / T_i = (P_e / P_i)^{\frac{n-1}{n}} \Rightarrow T_i = 353.3 \text{ K}$

Energy Eq.6.12:  $\dot{m}_i(h + V^2/2)_{in} + \dot{Q} = \dot{m}_{ex}(h + V^2/2)_{ex} + \dot{W}_T$

Reversible shaft work in a polytropic process, Eq.9.14 and Eq.9.19:

$$w_T = -\int v \, dP + (V_i^2 - V_e^2)/2 = -\frac{n}{n-1}(P_e v_e - P_i v_i) + (V_i^2 - V_e^2)/2$$

$$= -\frac{n}{n-1}R(T_e - T_i) - V_e^2/2 = 71.8 \text{ kJ/kg}$$

$$\dot{W}_T = \dot{m}w_T = \mathbf{35.9 \text{ kW}}$$

Assume constant specific heat in the energy equation

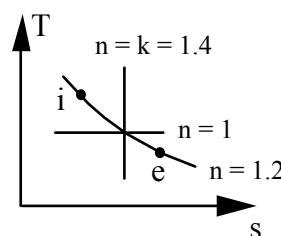
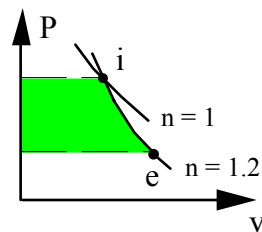
$$\dot{Q} = \dot{m}[C_p(T_e - T_i) + V_e^2/2] + \dot{W}_T = \mathbf{19.2 \text{ kW}}$$

Entropy Eq.9.7 or 9.23 with change in entropy from Eq.8.25:

$$dS_{net}/dt = \dot{S}_{gen} = \dot{m}(s_e - s_i) - \dot{Q}_H/T_H, \quad T_H = T_i + 100 = 453.3 \text{ K}$$

$$s_e - s_i = C_p \ln(T_e / T_i) - R \ln(P_e / P_i) = 0.1174 \text{ kJ/kg K}$$

$$dS_{net}/dt = 0.5 \times 0.1174 - 19.2/453.3 = 0.0163 \text{ kW/K}$$



## Problems solved with $P_r$ and $v_r$ functions



## 9.26

Do the previous problem using the air tables in A.7

The exit nozzle in a jet engine receives air at 1200 K, 150 kPa with negligible kinetic energy. The exit pressure is 80 kPa and the process is reversible and adiabatic. Use constant heat capacity at 300 K to find the exit velocity.

Solution:

C.V. Nozzle, Steady single inlet and exit flow, no work or heat transfer.

$$\text{Energy Eq.6.13: } h_i = h_e + \mathbf{V}_e^2/2 \quad (Z_i = Z_e)$$

$$\text{Entropy Eq.9.8: } s_e = s_i + \int dq/T + s_{\text{gen}} = s_i + 0 + 0$$

$$\text{Process: } q = 0, \quad s_{\text{gen}} = 0 \quad \text{as used above leads to } s_e = s_i$$

$$\text{Inlet state: } h_i = 1277.8 \text{ kJ/kg}, \quad P_{r,i} = 191.17$$

The constant  $s$  is done using the  $P_r$  function from A.7.2

$$P_{r,e} = P_{r,i} (P_e / P_i) = 191.17 (80/150) = 101.957$$

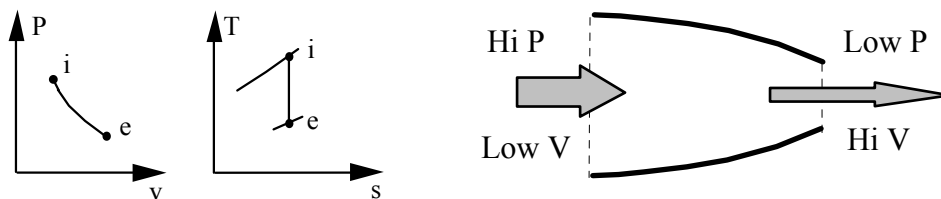
Interpolate in A.7  $\Rightarrow$

$$T_e = 1000 + 50 \frac{101.957 - 91.651}{111.35 - 91.651} = 1026.16 \text{ K}$$

$$h_e = 1046.2 + 0.5232 \times (1103.5 - 1046.2) = 1076.2 \text{ kJ/kg}$$

From the energy equation we have  $\mathbf{V}_e^2/2 = h_i - h_e$ , so then

$$\mathbf{V}_e = \sqrt{2(h_i - h_e)} = \sqrt{2(1277.8 - 1076.2) \text{ kJ/kg} \times 1000 \text{ J/kJ}} = \mathbf{635 \text{ m/s}}$$

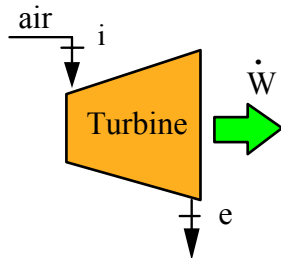


## 9.30

Air enters a turbine at 800 kPa, 1200 K, and expands in a reversible adiabatic process to 100 kPa. Calculate the exit temperature and the work output per kilogram of air, using

- The ideal gas tables, Table A.7
- Constant specific heat, value at 300 K from table A.5

Solution:



C.V. Air turbine.

Adiabatic:  $q = 0$ , reversible:  $s_{\text{gen}} = 0$

Energy Eq.6.13:  $w_T = h_i - h_e$ ,

Entropy Eq.9.8:  $s_e = s_i$

a) Table A.7:  $h_i = 1277.8 \text{ kJ/kg}$ ,  $P_{r,i} = 191.17$

The constant  $s$  process is done using the  $P_r$  function from A.7.2

$$\Rightarrow P_{r,e} = P_{r,i} (P_e / P_i) = 191.17 \left( \frac{100}{800} \right) = 23.896$$

Interpolate in A.7.1  $\Rightarrow T_e = \mathbf{705.7 \text{ K}}$ ,  $h_e = 719.7 \text{ kJ/kg}$

$$w = h_i - h_e = 1277.8 - 719.7 = \mathbf{558.1 \text{ kJ/kg}}$$

- b) Table A.5:  $C_{p0} = 1.004 \text{ kJ/kg K}$ ,  $R = 0.287 \text{ kJ/kg K}$ ,  $k = 1.4$ , then from Eq.8.32

$$T_e = T_i (P_e / P_i)^{\frac{k-1}{k}} = 1200 \left( \frac{100}{800} \right)^{0.286} = \mathbf{662.1 \text{ K}}$$

$$w = C_{p0}(T_i - T_e) = 1.004(1200 - 662.1) = \mathbf{539.8 \text{ kJ/kg}}$$

## 9.32

A compressor receives air at 290 K, 100 kPa and a shaft work of 5.5 kW from a gasoline engine. It should deliver a mass flow rate of 0.01 kg/s air to a pipeline. Find the maximum possible exit pressure of the compressor.

Solution:

C.V. Compressor, Steady single inlet and exit flows. Adiabatic:  $\dot{Q} = 0$ .

$$\text{Continuity Eq.6.11: } \dot{m}_i = \dot{m}_e = \dot{m},$$

$$\text{Energy Eq.6.12: } \dot{m}h_i = \dot{m}h_e + \dot{W}_C,$$

$$\text{Entropy Eq.9.8: } \dot{m}s_i + \dot{S}_{\text{gen}} = \dot{m}s_e \quad (\text{Reversible } \dot{S}_{\text{gen}} = 0)$$

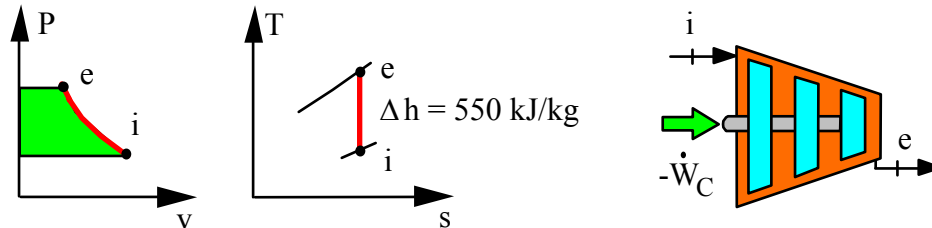
$$\dot{W}_C = \dot{m}w_c \Rightarrow -w_c = -\dot{W}/\dot{m} = 5.5/0.01 = 550 \text{ kJ/kg}$$

Use Table A.7,  $h_i = 290.43 \text{ kJ/kg}$ ,  $P_{r,i} = 0.9899$

$$h_e = h_i + (-w_c) = 290.43 + 550 = 840.43 \text{ kJ/kg}$$

$$\text{A.7 } \Rightarrow T_e = 816.5 \text{ K}, P_{r,e} = 41.717$$

$$P_e = P_i (P_{r,e}/P_{r,i}) = 100 \times (41.717/0.9899) = \mathbf{4214 \text{ kPa}}$$



## 9.50

An underground saltmine,  $100\,000\text{ m}^3$  in volume, contains air at  $290\text{ K}$ ,  $100\text{ kPa}$ . The mine is used for energy storage so the local power plant pumps it up to  $2.1\text{ MPa}$  using outside air at  $290\text{ K}$ ,  $100\text{ kPa}$ . Assume the pump is ideal and the process is adiabatic. Find the final mass and temperature of the air and the required pump work.

Solution:

C.V. The mine volume and the pump

$$\text{Continuity Eq.6.15: } m_2 - m_1 = m_{\text{in}}$$

$$\text{Energy Eq.6.16: } m_2 u_2 - m_1 u_1 = {}_1Q_2 - {}_1W_2 + m_{\text{in}} h_{\text{in}}$$

$$\text{Entropy Eq.9.12: } m_2 s_2 - m_1 s_1 = \int dQ/T + {}_1S_2 \text{ gen} + m_{\text{in}} s_{\text{in}}$$

$$\text{Process: Adiabatic } {}_1Q_2 = 0, \text{ Process ideal } {}_1S_2 \text{ gen} = 0, s_1 = s_{\text{in}}$$

$$\Rightarrow m_2 s_2 = m_1 s_1 + m_{\text{in}} s_{\text{in}} = (m_1 + m_{\text{in}}) s_1 = m_2 s_1 \Rightarrow s_2 = s_1$$

$$\text{Constant } s \Rightarrow P_{r2} = P_{r1} (P_2 / P_1) = 0.9899 \left( \frac{2100}{100} \right) = 20.7879$$

$$\text{A.7.2 } \Rightarrow T_2 = \mathbf{680\text{ K}}, u_2 = 496.94\text{ kJ/kg}$$

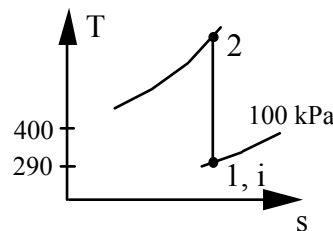
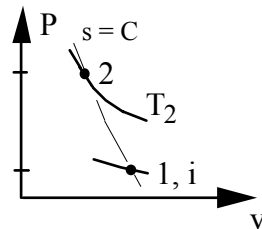
$$m_1 = P_1 V_1 / RT_1 = 100 \times 10^5 / (0.287 \times 290) = 1.20149 \times 10^5\text{ kg}$$

$$m_2 = P_2 V_2 / RT_2 = 100 \times 21 \times 10^5 / (0.287 \times 680) = \mathbf{10.760 \times 10^5\text{ kg}}$$

$$\Rightarrow m_{\text{in}} = 9.5585 \times 10^5\text{ kg}$$

$${}_1W_2 = m_{\text{in}} h_{\text{in}} + m_1 u_1 - m_2 u_2$$

$$= m_{\text{in}}(290.43) + m_1(207.19) - m_2(496.94) = \mathbf{-2.322 \times 10^8\text{ kJ}}$$



## 9.72

Calculate the air temperature and pressure at the stagnation point right in front of a meteorite entering the atmosphere ( $-50\text{ }^{\circ}\text{C}$ ,  $50\text{ kPa}$ ) with a velocity of  $2000\text{ m/s}$ . Do this assuming air is incompressible at the given state and repeat for air being a compressible substance going through an adiabatic compression.

Solution:

$$\text{Kinetic energy: } \quad \frac{1}{2} \mathbf{V}^2 = \frac{1}{2} (2000)^2/1000 = 2000\text{ kJ/kg}$$

$$\text{Ideal gas: } \quad v_{\text{atm}} = RT/P = 0.287 \times 223/50 = 1.28\text{ m}^3/\text{kg}$$

a) incompressible

$$\text{Energy Eq.6.13: } \quad \Delta h = \frac{1}{2} \mathbf{V}^2 = 2000\text{ kJ/kg}$$

If A.5  $\Delta T = \Delta h/C_p = 1992\text{ K}$  unreasonable, too high for that  $C_p$

$$\text{Use A.7: } \quad h_{\text{st}} = h_o + \frac{1}{2} \mathbf{V}^2 = 223.22 + 2000 = 2223.3\text{ kJ/kg}$$

$$T_{\text{st}} = 1977\text{ K}$$

Bernoulli (incompressible) Eq.9.17:

$$\Delta P = P_{\text{st}} - P_o = \frac{1}{2} \mathbf{V}^2/v = 2000/1.28 = 1562.5\text{ kPa}$$

$$P_{\text{st}} = 1562.5 + 50 = 1612.5\text{ kPa}$$

b) compressible

$$T_{\text{st}} = 1977\text{ K} \quad \text{the same energy equation.}$$

From A.7.2: Stagnation point  $P_{r\text{st}} = 1580.3$ ; Free  $P_{r_o} = 0.39809$

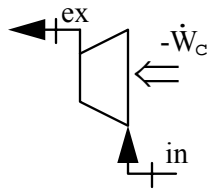
$$\begin{aligned} P_{\text{st}} &= P_o \times \frac{P_{r\text{st}}}{P_{r_o}} = 50 \times \frac{1580.3}{0.39809} \\ &= \mathbf{198\ 485\text{ kPa}} \end{aligned}$$



Notice that this is highly compressible,  $v$  is not constant.

## 9.170

Supercharging of an engine is used to increase the inlet air density so that more fuel can be added, the result of which is an increased power output. Assume that ambient air, 100 kPa and 27°C, enters the supercharger at a rate of 250 L/s. The supercharger (compressor) has an isentropic efficiency of 75%, and uses 20 kW of power input. Assume that the ideal and actual compressor have the same exit pressure. Find the ideal specific work and verify that the exit pressure is 175 kPa. Find the percent increase in air density entering the engine due to the supercharger and the entropy generation.



C.V.: Air in compressor (steady flow)

$$\text{Cont: } \dot{m}_{\text{in}} = \dot{m}_{\text{ex}} = \dot{m} = \dot{V}/v_{\text{in}} = 0.29 \text{ kg/s}$$

$$\text{Energy: } \dot{m}h_{\text{in}} - \dot{W} = \dot{m}h_{\text{ex}} \quad \text{Assume: } \dot{Q} = 0$$

$$\text{Entropy: } \dot{m}s_{\text{in}} + \dot{S}_{\text{gen}} = \dot{m}s_{\text{ex}}$$

$$\text{Inlet state: } v_{\text{in}} = RT_{\text{in}}/P_{\text{in}} = 0.8614 \text{ m}^3/\text{kg}, \quad P_{r \text{ in}} = 1.1167$$

$$\eta_c = w_{C s}/w_{C ac} \Rightarrow -\dot{W}_S = -\dot{W}_{AC} \times \eta_c = 15 \text{ kW}$$

$$-w_{C s} = -\dot{W}_S/\dot{m} = 51.724 \text{ kJ/kg}, \quad -w_{C ac} = 68.966 \text{ kJ/kg}$$

$$\text{Table A.7: } h_{\text{ex s}} = h_{\text{in}} - w_{C s} = 300.62 + 51.724 = 352.3 \text{ kJ/kg}$$

$$\Rightarrow T_{\text{ex s}} = 351.5 \text{ K}, \quad P_{r \text{ ex}} = 1.949$$

$$P_{\text{ex}} = P_{\text{in}} \times P_{r \text{ ex}}/P_{r \text{ in}} = 100 \times 1.949 / 1.1167 = \mathbf{174.5 \text{ kPa}}$$

The actual exit state is

$$h_{\text{ex ac}} = h_{\text{in}} - w_{C ac} = 369.6 \text{ kJ/kg} \quad \Rightarrow T_{\text{ex ac}} = 368.6 \text{ K}$$

$$v_{\text{ex}} = RT_{\text{ex}}/P_{\text{ex}} = 0.606 \text{ m}^3/\text{kg}$$

$$\rho_{\text{ex}}/\rho_{\text{in}} = v_{\text{in}}/v_{\text{ex}} = 0.8614/0.606 = \mathbf{1.42 \text{ or } 42 \% \text{ increase}}$$

$$S_{\text{gen}} = s_{\text{ex}} - s_{\text{in}} = 7.0767 - 6.8693 - 0.287 \ln(174/100) = \mathbf{0.0484 \text{ kJ/kg K}}$$