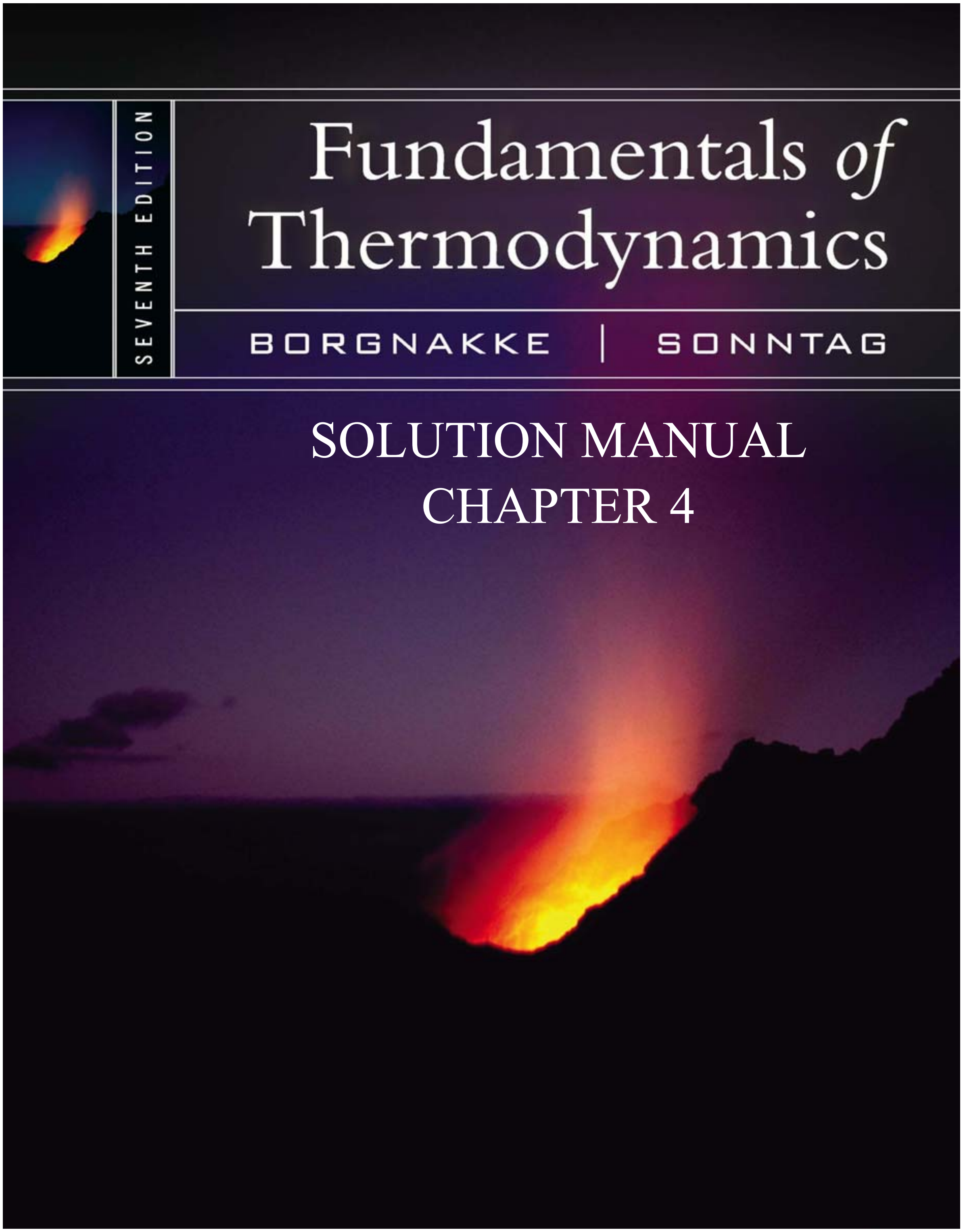

SEVENTH EDITION
Fundamentals of Thermodynamics
BORGNAKKE | SONNTAG
SOLUTION MANUAL
CHAPTER 4

CONTENT

In-Text Concept Questions

4.a

The electric company charges the customers per kW-hour. What is that in SI units?

Solution:

The unit kW-hour is a rate multiplied with time. For the standard SI units the rate of energy is in W and the time is in seconds. The integration in Eq.4.21 becomes

$$1 \text{ kW- hour} = 1000 \text{ W} \times 60 \frac{\text{min}}{\text{hour}} \text{hour} \times 60 \frac{\text{s}}{\text{min}} = 3\ 600\ 000 \text{ Ws}$$

$$= 3\ 600\ 000 \text{ J} = \mathbf{3.6\ MJ}$$

4.b

Torque and energy and work have the same units (N m). Explain the difference.

Solution:

Work = force × displacement, so units are N × m. Energy in transfer
Energy is stored, could be from work input 1 J = 1 N m
Torque = force × arm static, no displacement needed

4.c

What is roughly the relative magnitude of the work in the process 1-2c versus the process 1-2a shown in figure 4.8?

By visual inspection the area below the curve 1-2c is roughly 50% of the rectangular area below the curve 1-2a. To see this better draw a straight line from state 1 to point f on the axis. This curve has exactly 50% of the area below it.

4.d

Helium gas expands from 125 kPa, 350 K and 0.25 m^3 to 100 kPa in a polytropic process with n = 1.667. Is the work positive, negative or zero?

The boundary work is: $W = \int P\ dV$

P drops but does V go up or down?

The process equation is: $PV^n = C$

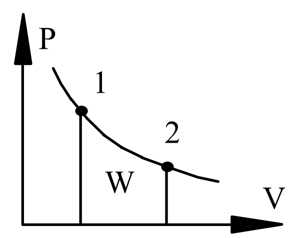
so we can solve for P to show it in a P-V diagram

$$P = CV^{-n}$$

as n = 1.667 the curve drops as V goes up we see

$$V_2 > V_1 \quad \text{giving} \quad dV > 0$$

and the work is then positive.

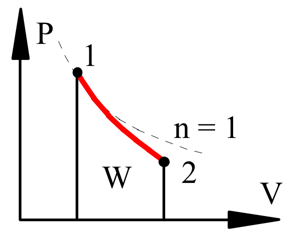


4.e

An ideal gas goes through an expansion process where the volume doubles. Which process will lead to the larger work output: an isothermal process or a polytropic process with *n* = 1.25?

The process equation is: $PV^n = C$

The polytropic process with n = 1.25 drops the pressure faster than the isothermal process with n = 1 and the area below the curve is then smaller.



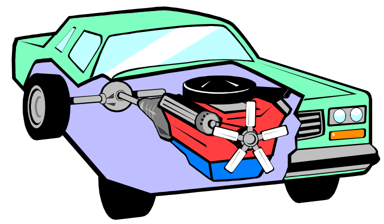
Concept Problems

4.1

A car engine is rated at 160 hp. What is the power in SI units?

Solution:

The horsepower is an older unit for power usually used for car engines. The conversion to standard SI units is given in Table A.1



1 hp = 0.7355 kW = 735.5 W

1 hp = 0.7457 kW for the UK horsepower

$$160 \text{ hp} = 160 \times 745.7 \text{ W} = 119\,312 \text{ W} = \mathbf{119.3\ kW}$$

4.2

Two engines provide the same amount of work to lift a hoist. One engine can provide 3 F in a cable and the other 1 F, What can you say about the motion of the point where the force F acts in the two engines?

Since the two work terms are the same we get

$$W = \int F\,dx = 3\,F\,x_1 = 1\,F\,x_2$$

$$x_2 = 3\,x_1$$

so the lower force has a larger displacement.

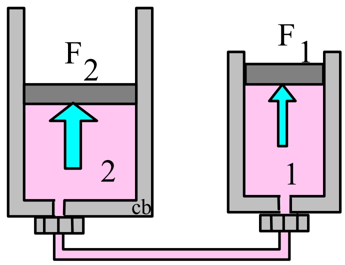
4.3

Two hydraulic piston/cylinders are connected through a hydraulic line so they have roughly the same pressure. If they have diameters of D_1 and $D_2 = 2D_1$ respectively, what can you say about the piston forces F_1 and F_2?

For each cylinder we have the total force as: $F = PA_{cyl} = P\ \pi\ D^2/4$

$$F_1 = PA_{cyl\ 1} = P\ \pi\ D_1^2/4$$

$$F_2 = PA_{cyl\ 2} = P\ \pi\ D_2^2/4 = P\ \pi\ 4\ D_1^2/4 = 4\ F_1$$



The forces are the total force acting up due to the cylinder pressure. There must be other forces on each piston to have a force balance so the pistons do not move.

4.4

Normally pistons have a flat head, but in diesel engines pistons can have bowls in them and protruding ridges. Does this geometry influence the work term?

The shape of the surface does not influence the displacement

$$dV = A_n \, dx$$

where A_n is the area projected to the plane normal to the direction of motion.

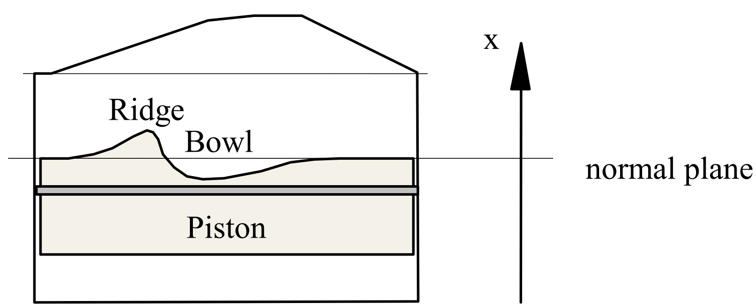
$$A_n = A_{cyl} = \pi D^2/4$$

Work is

$$dW = F \, dx = P \, dV = P \, A_n \, dx = P \, A_{cyl} \, dx$$

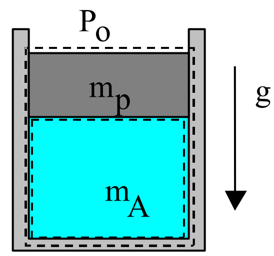
and thus unaffected by the surface shape.

Semi-spherical head is made to make room for larger valves.



4.5

CV A is the mass inside a piston-cylinder, CV B is that plus the piston, outside which is the standard atmosphere. Write the process equation and the work term for the two CVs.



Solution:

C.V. A: Process: $P = P_o + m_p g/A_{cyl} = C$,

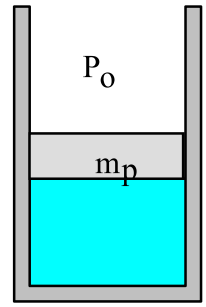
$$ {}_1W_2 = \int P\, dV = P_1 \int m\, dv = P_1 m(v_2 - v_1) $$

C.V. B: Process: $P = P_o = C$,

$$ {}_1W_2 = \int P_o\, dV = P_o \int m\, dv = P_o m(v_2 - v_1) $$

4.6

Assume a physical set-up as in Fig. P4.5. We now heat the cylinder. What happens to P, T and v (up, down or constant)? What transfers do we have for Q and W (pos., neg., or zero)?



Solution:

Process: $P = P_o + m_p g/A_{cyl} = C$

Heat in so T increases, v increases and Q is positive.

As the volume increases the work is positive. ${}_1W_2 = \int P\, dV$

4.7

For a buffer storage of natural gas (CH_4) a large bell in a container can move up and down keeping a pressure of 105 kPa inside. The sun then heats the container and the gas from 280 K to 300 K during 4 hours. What happens to the volume and what is the sign of the work term?

Solution

The process has constant pressure

Ideal gas: $PV = mRT$ as T increases then V **increases**

$_1W_2 = \int P \, dV > 0$ so **positive**.

4.8

A drag force on an object moving through a medium (like a car through air or a submarine through water) is $F_d = 0.225\ A\ \rho \mathbf{V}^2$. Verify the unit becomes Newton.

Solution:

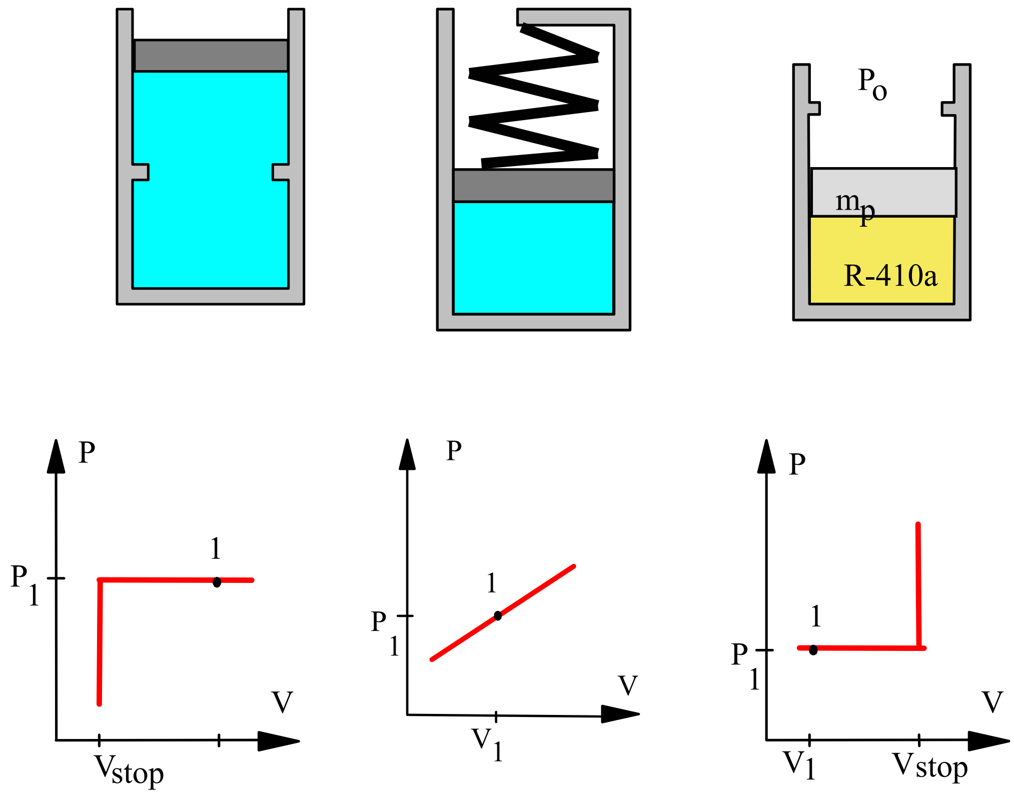
$$F_d = 0.225\ A\ \rho \mathbf{V}^2$$

$$\text{Units} = m^2 \times (kg/m^3) \times (m^2/ s^2) = kg\ m / s^2 = N$$

4.9

The sketch shows a physical situation, show the possible process in a P-v diagram.

a) b) c)



4.10

For the indicated physical set-up in a-b and c above write a process equation and the expression for work.

a) $P = P_1$ and $V \geq V_{stop}$ or $V = V_{stop}$ and $P \leq P_1$

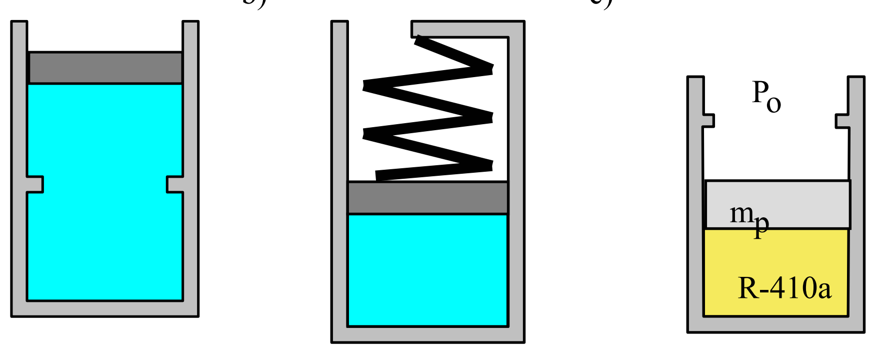
$${}_1W_2 = P_1(V_2 - V_1) \quad [\, P_1 = P_{float} \,]$$

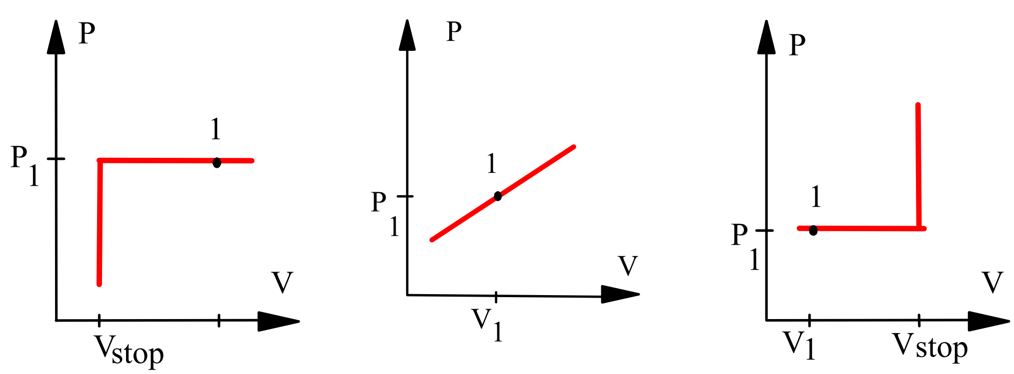
b) $P = A + BV$; $\quad {}_1W_2 = \frac{1}{2}(P_1 + P_2)(V_2 - V_1)$

c) $P = P_1$ and $V \leq V_{stop}$ or $V = V_{stop}$ and $P \geq P_1$

$${}_1W_2 = P_1(V_2 - V_1) \quad [\, P_1 = P_{float} \,]$$

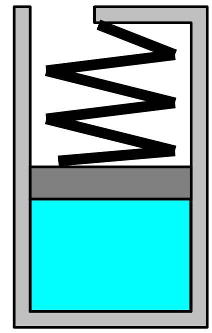
a) b) c)





4.11

Assume the physical situation as in Fig. P4.9b; what is the work term a, b, c or d?



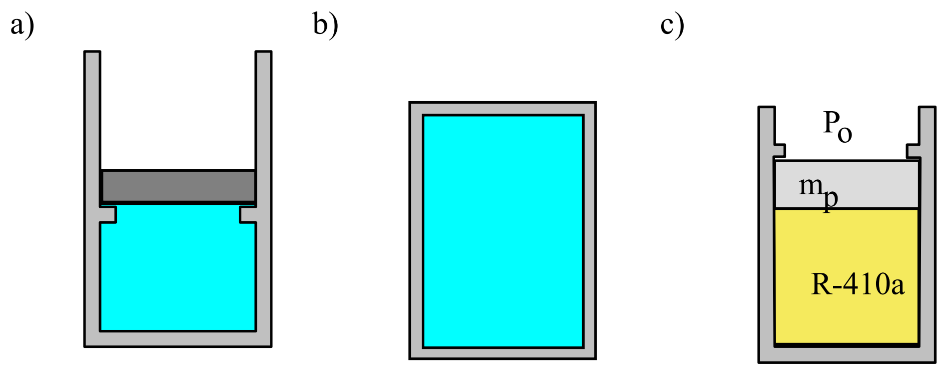
a: $_1w_2 = P_1(v_2 - v_1)$ b: $_1w_2 = v_1(P_2 - P_1)$

c: $_1w_2 = \frac{1}{2}(P_1 + P_2)(v_2 - v_1)$ d: $_1w_2 = \frac{1}{2}(P_1 - P_2)(v_2 + v_1)$

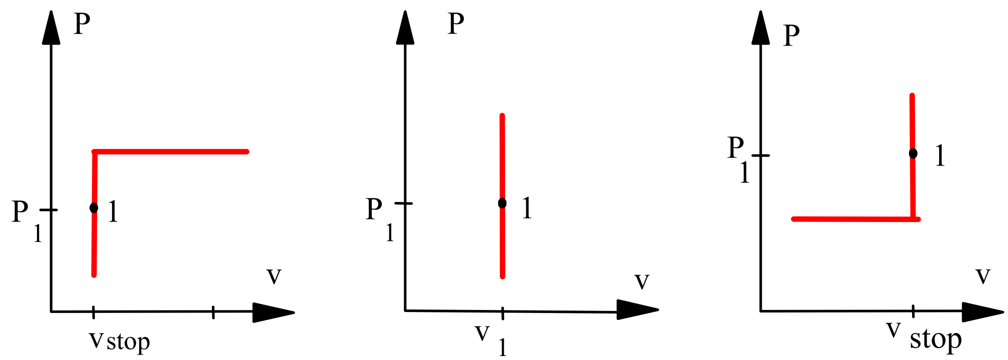
Solution: work term is formula c the area under the process curve in a P-v diagram.

4.12

The sketch in Fig. P4.12 shows a physical situation; show the possible process in a P-v diagram.



Solution:



4.13

What can you say about the beginning state of the R-410a in Fig. P4.9 versus the case in Fig. P4.12 for the same piston-cylinder?

For the case where the piston floats as in Fig. P4.9 the pressure of the R-410a must equal the equilibrium pressure that floats (balance forces on) the piston.

The situation in Fig. P4.12 is possible if the R-410a pressure equals or exceeds the float pressure.

4.14

Show how the polytropic exponent *n* can be evaluated if you know the end state properties, (P_1, V_1) and (P_2, V_2).

Polytropic process: $PV^n = C$

Both states must be on the process line: $P_2V_2^n = C = P_1V_1^n$

Take the ratio to get: $\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^n$

and then take ln of the ratio

$$\ln\left(\frac{P_1}{P_2}\right) = \ln\left(\frac{V_2}{V_1}\right)^n = n \ln\left(\frac{V_2}{V_1}\right)$$

now solve for the exponent n

$$n = \ln\left(\frac{P_1}{P_2}\right) \Big/ \ln\left(\frac{V_2}{V_1}\right)$$

4.15

A piece of steel has a conductivity of k = 15 W/mK and a brick has k = 1 W/mK. How thick a steel wall will provide the same insulation as a 10 cm thick brick?

The heat transfer due to conduction is from Eq. 4.18

$$\dot{Q} = -\,kA\,\frac{dT}{dx} \approx kA\,\frac{\Delta T}{\Delta x}$$

For the same area and temperature difference the heat transfers become the same for equal values of $(k / \Delta x)$ so

$$\left(\frac{k}{\Delta x}\right)_{brick} = \left(\frac{k}{\Delta x}\right)_{steel} \qquad =>$$

$$\Delta x_{steel} = \Delta x_{brick}\,\frac{k_{steel}}{k_{brick}} = 0.1\text{ m} \times \frac{15}{1} = \mathbf{1.5\ m}$$

4.16

A thermopane window, see Fig. 4.27, traps some gas between the two glass panes. Why is this beneficial?

The gas has a very low conductivity relative to a liquid or solid so the heat transfer for a given thickness becomes smaller. The gap is furthermore made so small that possible natural convection motion is reduced to a minimum. It becomes a trade off to minimize the overall heat transfer due to conduction and convection. Typically these windows can be manufactured with an E-glaze to reduce radiation loss (winter) or gain (summer).

4.17

On a chilly 10^{o}C fall day a house, 20^{o}C inside, loses 6 kW by heat transfer. What transfer happens on a 30^{o}C warm summer day assuming everything else is the same?

The heat transfer is $\dot{Q} = CA\ \Delta T$ where the details of the heat transfer is in the factor C. Assuming those details are the same then it is the temperature difference that changes the heat transfer so

$$\dot{Q} = CA\ \Delta T = 6\text{ kW} = CA\ (20 - 10) \quad \Rightarrow \quad CA = 0.6\ \frac{\text{kW}}{\text{K}}$$

Then

$$\dot{Q} = CA\ \Delta T = 0.6\ \frac{\text{kW}}{\text{K}} \times (20 - 30)\text{ K} = -6\text{ kW} \quad \text{(it goes in)}$$

Force displacement work

4.18

A piston of mass 2 kg is lowered 0.5 m in the standard gravitational field. Find the required force and work involved in the process.

Solution:

$$F = ma = 2\text{ kg} \times 9.80665\text{ m/s}^2 = \mathbf{19.61\ N}$$

$$W = \int F\,dx = F\int dx = F\,\Delta x = 19.61\text{ N} \times 0.5\text{ m} = \mathbf{9.805\ J}$$

4.19

A hydraulic cylinder of area 0.01 m^2 must push a 1000 kg arm and shovel 0.5 m straight up. What pressure is needed and how much work is done?

$F = mg = 1000 \text{ kg} \times 9.81 \text{ m/s}^2$
$= 9810 \text{ N} = PA$

$P = F/A = 9810 \text{ N}/ 0.01 \text{ m}^2$
$= 981\ 000 \text{ Pa} = \mathbf{981\ kPa}$

$W = \int F\ dx = F\ \Delta x = 9810 \text{ N} \times 0.5 \text{ m} = \mathbf{4905\ J}$

4.20

An escalator raises a 100 kg bucket of sand 10 m in 1 minute. Determine the total amount of work done during the process.

Solution:

The work is a force with a displacement and force is constant: F = mg

$$W = \int F\,dx = F \int dx = F\,\Delta x = 100\ \text{kg} \times 9.80665\ \text{m/s}^2 \times 10\ \text{m} = \mathbf{9807\ J}$$

4.21

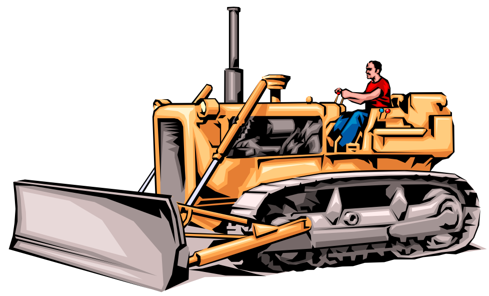
A bulldozer pushes 500 kg of dirt 100 m with a force of 1500 N. It then lifts the dirt 3 m up to put it in a dump truck. How much work did it do in each situation?

Solution:

$W = \int F\, dx = F\, \Delta x$

$= 1500\ \text{N} \times 100\ \text{m}$

$= 150\ 000\ \text{J} =$ **150 kJ**



$W = \int F\, dz = \int mg\, dz = mg\, \Delta Z$

$= 500\ \text{kg} \times\ 9.807\ \text{m/s}^2 \times 3\ \text{m}$

$= 14\ 710\ \text{J} =$ **14.7 kJ**

4.22

A hydraulic cylinder has a piston of cross sectional area 15 cm^2 and a fluid pressure of 2 MPa. If the piston is moved 0.25 m how much work is done?

Solution:

The work is a force with a displacement and force is constant: F = PA

$$W = \int F\, dx = \int PA\, dx = PA\, \Delta x$$

$$= 2000 \text{ kPa} \times 15 \times 10^{-4} \text{ m}^2 \times 0.25 \text{ m} = \mathbf{0.75\ kJ}$$

Units: $\text{kPa m}^2 \text{ m} = \text{kN m}^{-2} \text{ m}^2 \text{ m} = \text{kN m} = \text{kJ}$

4.23

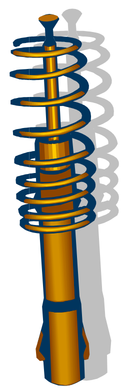
A linear spring, $F = k_s(x - x_0)$, with spring constant $k_s = 500$ N/m, is stretched until it is 100 mm longer. Find the required force and work input.

Solution:

$$F = k_s(x - x_0) = 500 \times 0.1 = \mathbf{50\ N}$$

$$W = \int F\, dx = \int k_s(x - x_0)d(x - x_0) = k_s(x - x_0)^2/2$$

$$= 500 \frac{N}{m} \times (0.1^2/2)\ m^2 = \mathbf{2.5\ J}$$



4.24

Two hydraulic cylinders maintain a pressure of 1200 kPa. One has a cross sectional area of 0.01 m^2 the other 0.03 m^2. To deliver a work of 1 kJ to the piston how large a displacement (V) and piston motion H is needed for each cylinder? Neglect P_{atm}.

Solution:

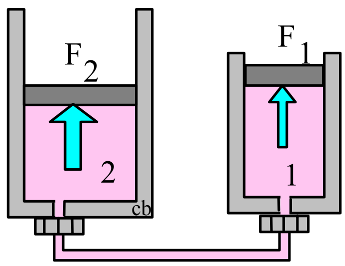
$$W = \int F\, dx = \int P\, dV = \int PA\, dx = PA * H = P\Delta V$$

$$\Delta V = \frac{W}{P} = \frac{1 \text{ kJ}}{1200 \text{ kPa}} = \mathbf{0.000\ 833\ m^3}$$

Both cases the height is $H = \Delta V/A$

$$H_1 = \frac{0.000833}{0.01} = \mathbf{0.0833\ m}$$

$$H_2 = \frac{0.000833}{0.03} = \mathbf{0.0278\ m}$$



4.25

Two hydraulic piston/cylinders are connected with a line. The master cylinder has an area of 5 cm^2 creating a pressure of 1000 kPa. The slave cylinder has an area of 3 cm^2. If 25 J is the work input to the master cylinder what is the force and displacement of each piston and the work out put of the slave cylinder piston?

Solution:

$$W = \int F_x \, dx = \int P \, dv = \int P \, A \, dx = P \, A \, \Delta x$$

$$\Delta x_{master} = \frac{W}{PA} = \frac{25}{1000 \times 5 \times 10^{-4}} = 0.05 \text{ m}$$

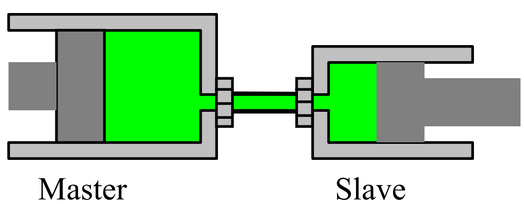
$$A\Delta x = \Delta V = 5 \times 10^{-4} \times 0.05 = 2.5 \times 10^{-5} \text{ m} = \Delta V_{slave} = A \, \Delta x \rightarrow$$

$$\Delta x_{slave} = \Delta V/A = 2.5 \times 10^{-5} / 3 \times 10^{-4} = 0.0083\,33 \text{ m}$$

$$F_{master} = P \, A = 1000 \times 5 \times 10^{-4} \times 10^{3} = \mathbf{500 \ N}$$

$$F_{slave} = P \, A = 1000 \times 10^{3} \times 3 \times 10^{-4} = \mathbf{300 \ N}$$

$$W_{slave} = F \, \Delta x = 300 \times 0.08333 = \mathbf{25 \ J}$$



4.26

The rolling resistance of a car depends on its weight as: $F = 0.006\ m\ g$. How long will a car of 1400 kg drive for a work input of 25 kJ?

Solution:

Work is force times distance so assuming a constant force we get

$$W = \int F\ dx = F\ x = 0.006\ mgx$$

Solve for x

$$x = \frac{W}{0.006\ mg} = \frac{25\ \text{kJ}}{0.006 \times 1400\ \text{kg} \times 9.807\ \text{m/s}^2} = \mathbf{303.5\ m}$$

4.27

The air drag force on a car is 0.225 A $\rho\mathbf{V}^2$. Assume air at 290 K, 100 kPa and a car frontal area of 4 m^2 driving at 90 km/h. How much energy is used to overcome the air drag driving for 30 minutes?

The formula involves density and velocity and work involves distance so:

$$\rho = \frac{1}{v} = \frac{P}{RT} = \frac{100}{0.287 \times 290} = 1.2015 \frac{kg}{m^3}$$

$$\mathbf{V} = 90 \frac{km}{h} = 90 \times \frac{1000}{3600} \frac{m}{s} = 25 \text{ m/s}$$

$$\Delta x = \mathbf{V}\, \Delta t = 25 \text{ m/s} \times 30 \text{ min} \times 60 \text{ s/min} = 45\,000 \text{ m}$$

Now

$$F = 0.225\, A\, \rho\, \mathbf{V}^2 = 0.225 \times 4 \times 1.2015 \times 25^2$$

$$= 675.8 \text{ m}^2 \frac{kg}{m^3} \times \frac{m^2}{s^2} = \mathbf{676\ N}$$

$$W = F\, \Delta x = 676 \text{ N} \times 45\,000 \text{ m} = 30\,420\,000 \text{ J} = \mathbf{30.42\ MJ}$$

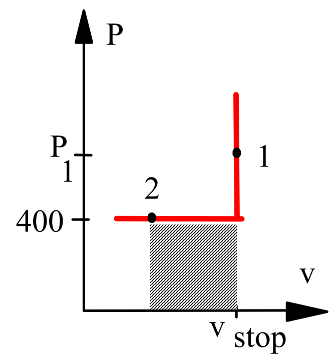
Boundary work simple 1 step process

4.28

The R-410a in Problem 4.12 c is at 1000 kPa, 50^{o}C with mass 0.1 kg. It is cooled so the volume is reduced to half the initial volume . The piston mass and gravitation is such that a pressure of 400 kPa will float the piston. Find the work in the process.

If the volume is reduced the piston must drop and thus float with P = 400 kPa. The process therefore follows a process curve shown in the P-V diagram.

Table B.4.2: $v_1 = 0.03320 \text{ m}^3\text{/kg}$



$_1W_2 = \int PdV = \text{area}$

$= P_{float} (V_2 - V_1) = -P_{float} V_1/2$

$= -400 \text{ kPa} \times 0.1 \text{ kg} \times 0.0332 \text{ m}^3\text{/kg} /2$

$= \mathbf{-0.664 \text{ kJ}}$

4.29

A steam radiator in a room at 25°C has saturated water vapor at 110 kPa flowing through it, when the inlet and exit valves are closed. What is the pressure and the quality of the water, when it has cooled to 25^{o}C? How much work is done?

Solution: Control volume radiator.

After the valve is closed no more flow, constant volume and mass.

1: $x_1 = 1$, $P_1 = 110$ kPa $\Rightarrow$ $v_1 = v_g = 1.566$ m^3/kg from Table B.1.2

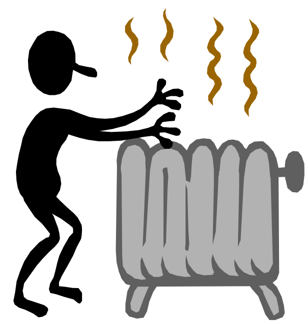
2: $T_2 = 25^oC$, ?

Process: $v_2 = v_1 = 1.566$ m^3/kg $= [0.001003 + x_2 \times 43.359]$ m^3/kg

$$x_2 = \frac{1.566 - 0.001003}{43.359} = \mathbf{0.0361}$$

State 2 : T_2 , x_2 From Table B.1.1 $\mathbf{P_2 = Psat = 3.169\ kPa}$

$${}_1W_2 = \int PdV = \mathbf{0}$$



4.30

A constant pressure piston cylinder contains 0.2 kg water as saturated vapor at 400 kPa. It is now cooled so the water occupies half the original volume. Find the work in the process.

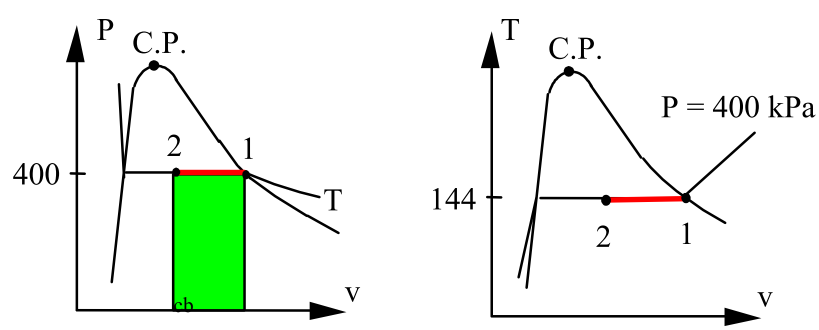
Solution:

Table B.1.2 $v_1 = 0.4625\ m^3/kg$ $V_1 = mv_1 = 0.0925\ m^3$

$v_2 = v_1/ 2 = 0.23125\ m^3/kg$ $V_2 = V_1 / 2 = 0.04625\ m^3$

Process: P = C so the work term integral is

$W = \int PdV = P(V_2\text{-}V_1) = 400\ kPa \times (0.04625 - 0.0925)\ m^3 =$ **-18.5 kJ**



4.31

Find the specific work in Problem 3.47.
Solution:

State 1 from Table B.1.2 at 200 kPa

$$v = v_f + x\, v_{fg} = 0.001061 + 0.25 \times 0.88467 = 0.22223\ m^3/kg$$

State 2 has same P from Table B.1.2 at 200 kPa

$$T_2 = T_{sat} + 20 = 120.23 + 20 = 140.23^oC$$

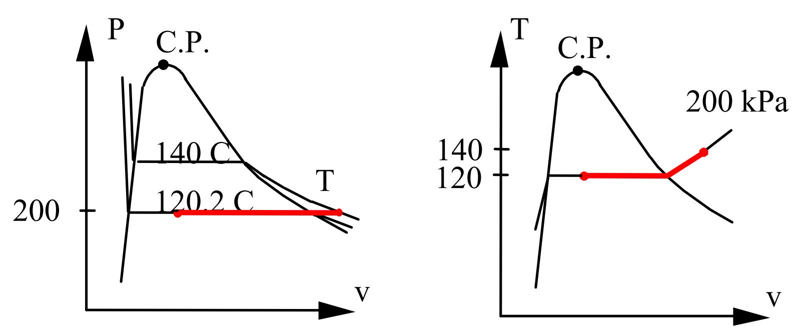
so state is superheated vapor

$$v_2 = 0.88573 + (0.95964 - 0.88573)\frac{20}{150 - 120.23} = 0.9354\ m^3/kg$$

Process P = C: $\quad {}_1w_2 = \int P\, dv = P_1(v_2 - v_1)$

$= 200\ (0.9354 - 0.22223)$

$=$ **142.6 kJ/kg**



4.32

A 400-L tank A, see figure P4.32, contains argon gas at 250 kPa, 30°C. Cylinder B, having a frictionless piston of such mass that a pressure of 150 kPa will float it, is initially empty. The valve is opened and argon flows into B and eventually reaches a uniform state of 150 kPa, 30°C throughout. What is the work done by the argon?

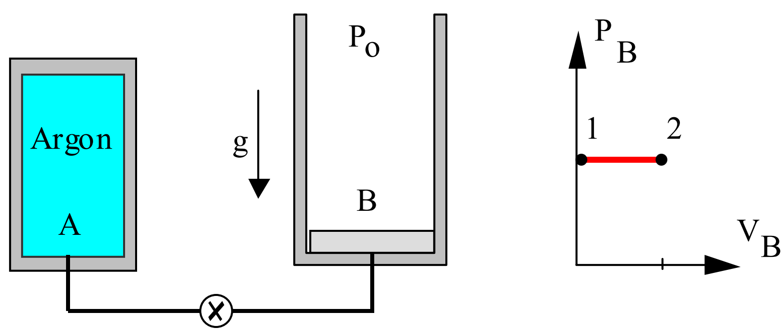
Solution:

Take C.V. as all the argon in both A and B. Boundary movement work done in cylinder B against constant external pressure of 150 kPa. Argon is an ideal gas, so write out that the mass and temperature at state 1 and 2 are the same

$$P_{A1}V_A = m_A RT_{A1} = m_A RT_2 = P_2(V_A + V_{B2})$$

$$=> \quad V_{B2} = \frac{250 \times 0.4}{150} - 0.4 = 0.2667 \text{ m}^3$$

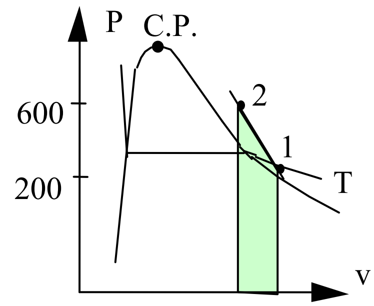
$${}_1W_2 = \int_1^2 P_{ext} dV = P_{ext}(V_{B2} - V_{B1}) = 150 \text{ kPa } (0.2667 - 0) \text{ m}^3 = \mathbf{40 \ kJ}$$



Notice there is a pressure loss in the valve so the pressure in B is always 150 kPa while the piston floats.

4.33

A piston cylinder contains 1.5 kg water at 200 kPa, 150^{o}C. It is now heated in a process where pressure is linearly related to volume to a state of 600 kPa, 350^{o}C. Find the final volume and the work in the process.



Take as CV the 1.5 kg of water.

$$m_2 = m_1 = m \;;$$

Process Eq.: $P = A + BV$ (linearly in V)

State 1: (P, T) => $v_1 = 0.95964$ m^3/kg,

State 2: (P, T) => $v_2 = 0.47424$ m^3/kg, $V_2 = mv_2 = 0.7114$ m^3

From process eq.:

$$_1W_2 = \int P\, dV = \text{area} = \frac{m}{2} (P_1 + P_2)(v_2 - v_1)$$

$$= \frac{1.5}{2} \text{ kg } (200 + 600) \text{ kPa } (0.47424 - 0.95964) \text{ m}^3\text{/kg} = \mathbf{-291.24\ kJ}$$

Notice volume is reduced so work is negative.

4.34

A cylinder fitted with a frictionless piston contains 5 kg of superheated refrigerant R-134a vapor at 1000 kPa, 140°C. The setup is cooled at constant pressure until the R-134a reaches a quality of 25%. Calculate the work done in the process.

Solution:

Constant pressure process boundary work. State properties from Table B.5.2

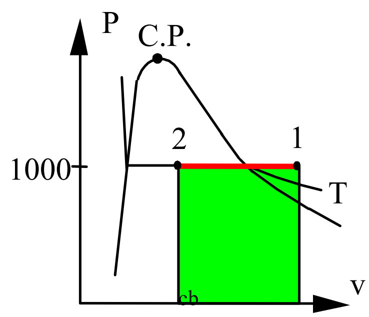
State 1: $v = 0.03150 \text{ m}^3/\text{kg}$,

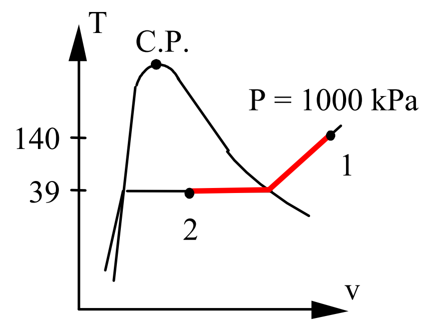
State 2: $v = 0.000871 + 0.25 \times 0.01956 = 0.00576 \text{ m}^3/\text{kg}$

Interpolated to be at 1000 kPa, numbers at 1017 kPa could have been used in which case: $v = 0.00566 \text{ m}^3/\text{kg}$

$${}_1W_2 = \int P\, dV = P\,(V_2\text{-}V_1) = mP\,(v_2\text{-}v_1)$$

$$= 5 \times 1000\,(0.00576 - 0.03150) = \mathbf{-128.7\ kJ}$$





4.35

A piston cylinder contains air at 600 kPa, 290 K and a volume of 0.01 m^3. A constant pressure process gives 54 kJ of work out. Find the final volume and temperature of the air.

Solution:

$$W = \int P\ dV = P\ \Delta V$$

$$\Delta V = W/P = \frac{54}{600} = 0.09\ m^3$$

$$V_2 = V_1 + \Delta V = 0.01 + 0.09 = 0.1\ m^3$$

Assuming ideal gas, PV = mRT, then we have

$$T_2 = \frac{P_2 V_2}{mR} = \frac{P_2 V_2}{P_1 V_1} T_1 = \frac{V_2}{V_1} T_1 = \frac{0.1}{0.01} \times 290 = \mathbf{2900\ K}$$

4.36

A piston/cylinder has 5 m of liquid 20°C water on top of the piston (m = 0) with cross-sectional area of 0.1 m^2, see Fig. P2.56. Air is let in under the piston that rises and pushes the water out over the top edge. Find the necessary work to push all the water out and plot the process in a P-V diagram.

Solution:

$$P_1 = P_o + \rho g H$$

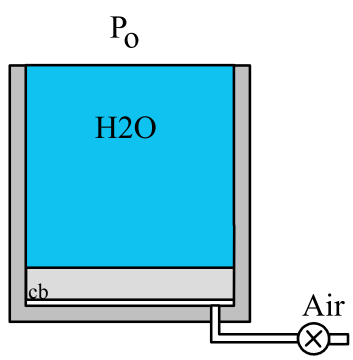
$$= 101.32 + 997 \times 9.807 \times 5 / 1000 = 150.2 \text{ kPa}$$

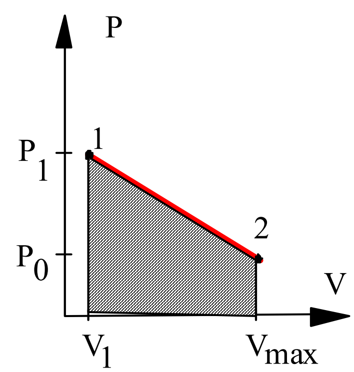
$$\Delta V = H \times A = 5 \times 0.1 = 0.5 \text{ m}^3$$

$${}_1W_2 = \text{AREA} = \int P\, dV = ½ (P_1 + P_o)(V_{max} - V_1)$$

$$= ½ (150.2 + 101.32) \text{ kPa} \times 0.5 \text{ m}^3$$

$$\mathbf{= 62.88\ kJ}$$





4.37

Saturated water vapor at 200 kPa is in a constant pressure piston cylinder. At this state the piston is 0.1 m from the cylinder bottom and cylinder area is 0.25 m^2. The temperature is then changed to 200°C. Find the work in the process.

Solution:

State 1 from B.1.2 (P, x): $v_1 = v_g = 0.8857\ m^3/kg$ (also in B.1.3)

State 2 from B.1.3 (P, T): $v_2 = 1.0803\ m^3/kg$

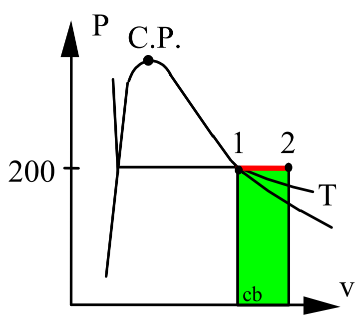
Since the mass and the cross sectional area is the same we get

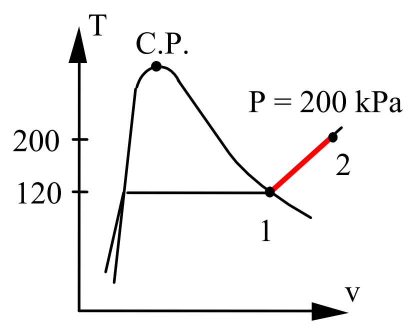
$$h_2 = \frac{v_2}{v_1} \times h_1 = \frac{1.0803}{0.8857} \times 0.1 = 0.122\ m$$

Process: P = C so the work integral is

$$W = \int PdV = P(V_2 - V_1) = PA\ (h_2 - h_1)$$

$$W = 200\ kPa \times 0.25\ m^2 \times (0.122 - 0.1)\ m = \mathbf{1.1\ kJ}$$





4.38

A piston cylinder contains 1 kg of liquid water at 20°C and 300 kPa, as shown in Fig. P4.38. There is a linear spring mounted on the piston such that when the water is heated the pressure reaches 3 MPa with a volume of 0.1 m^3.

a) Find the final temperature

b) Plot the process in a P-v diagram.

c) Find the work in the process.

Solution:

Take CV as the water. This is a constant mass:

$$m_2 = m_1 = m \;;$$

State 1: Compressed liquid, take saturated liquid at same temperature.

B.1.1: $v_1 = v_f(20) = 0.001002 \text{ m}^3/\text{kg}$,

State 2: $v_2 = V_2/m = 0.1/1 = 0.1 \text{ m}^3/\text{kg}$ and P = 3000 kPa from B.1.3

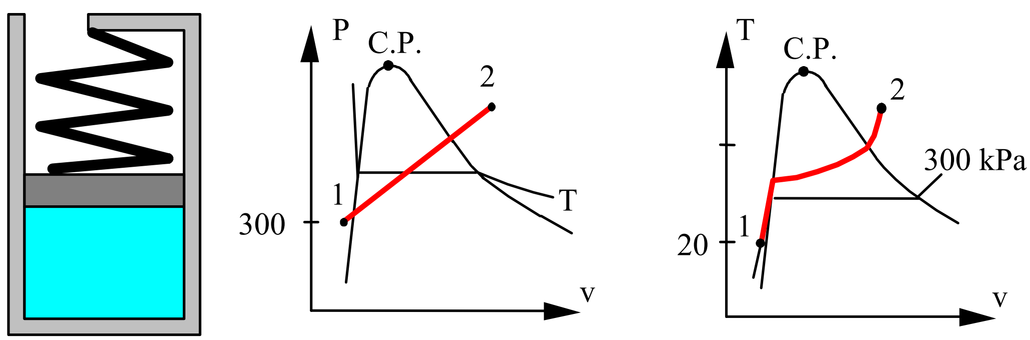
=> Superheated vapor close to T = 400°C

Interpolate: $T_2 = \mathbf{404^oC}$

Work is done while piston moves at linearly varying pressure, so we get:

$$_1W_2 = \int P\, dV = \text{area} = P_{avg}\,(V_2 - V_1) = \frac{1}{2}(P_1 + P_2)(V_2 - V_1)$$

$$= 0.5\,(300 + 3000)(0.1 - 0.001) = \mathbf{163.35\ kJ}$$



4.39

Find the specific work in Problem 3.53 for the case the volume is reduced.

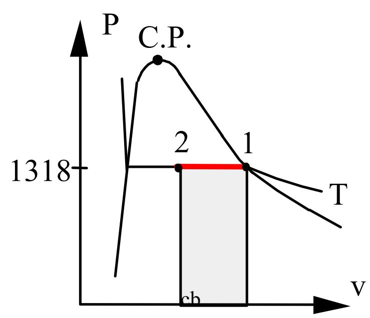
Saturated vapor R-134a at 50°C changes volume at constant temperature. Find the new pressure, and quality if saturated, if the volume doubles. Repeat the question for the case the volume is reduced to half the original volume.

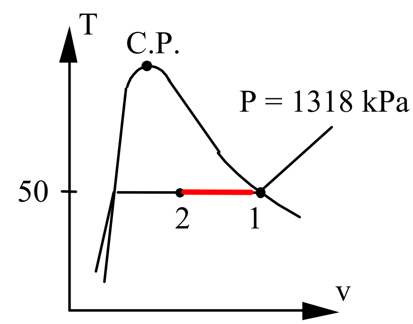
Solution:

R-134a 50°C

Table B.4.1: $v_1 = v_g = 0.01512 \text{ m}^3/\text{kg}$, $v_2 = v_1 / 2 = 0.00756 \text{ m}^3/\text{kg}$

$_1W_2 = \int PdV = 1318.1 \text{ kPa } (0.00756 - 0.01512) \text{ m}^3/\text{kg} = \mathbf{-9.96 \text{ kJ/kg}}$





4.40

A piston/cylinder contains 1 kg water at 20°C with volume 0.1 m^3. By mistake someone locks the piston preventing it from moving while we heat the water to saturated vapor. Find the final temperature, volume and the process work.

Solution

1: $v_1 = V/m = 0.1\ m^3/1\ kg = 0.1\ m^3/kg$ (two-phase state)

2: Constant volume: $v_2 = v_g = v_1$

$$V_2 = V_1 = \mathbf{0.1\ m^3}$$

$$_1W_2 = \int P\, dV = \mathbf{0}$$

State 2: $(v_2, x_2 = 1)$

$$T_2 = T_{sat} = 210 + 5\,\frac{0.1 - 0.10324}{0.09361 - 0.10324} = \mathbf{211.7°C}$$

4.41

Ammonia (0.5 kg) is in a piston cylinder at 200 kPa, -10°C is heated in a process where the pressure varies linear with the volume to a state of 120°C, 300 kPa. Find the work the ammonia gives out in the process.

Solution:

Take CV as the Ammonia, constant mass.

Continuity Eq.: $m_2 = m_1 = m$;

Process: $P = A + BV$ (linear in V)

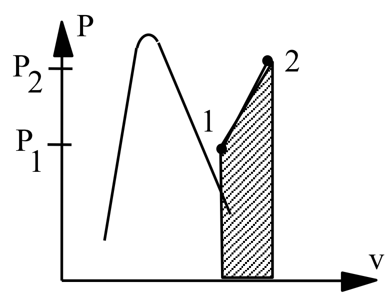
State 1: Superheated vapor $v_1 = 0.6193\ m^3/kg$

State 2: Superheated vapor $v_2 = 0.63276\ m^3/kg$

Work is done while piston moves at increasing pressure, so we get

$$_1W_2 = \int P\,dV = \text{area} = P_{avg}\,(V_2 - V_1) = \frac{1}{2}(P_1 + P_2)m(v_2 - v_1)$$

$$= ½(200 + 300) \times 0.5\,(0.63276 - 0.6193) = \mathbf{1.683\ kJ}$$



4.42

Air in a spring loaded piston/cylinder has a pressure that is linear with volume, $P = A + BV$. With an initial state of P = 150 kPa, V = 1 L and a final state of 800 kPa and volume 1.5 L it is similar to the setup in Problem 4.38. Find the work done by the air.

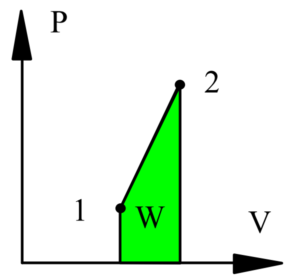
Solution:

Knowing the process equation: P = A + BV giving a linear variation of pressure versus volume the straight line in the P-V diagram is fixed by the two points as state 1 and state 2. The work as the integral of PdV equals the area under the process curve in the P-V diagram.

State 1: $P_1 = 150$ kPa $V_1 = 1\text{ L} = 0.001\text{ m}^3$

State 2: $P_2 = 800$ kPa $V_2 = 1.5\text{ L} = 0.0015\text{ m}^3$

Process: P = A + BV linear in V



$$\Rightarrow \quad {}_1W_2 = \int_1^2 P\,dV = \left(\frac{P_1 + P_2}{2}\right)(V_2 - V_1)$$

$$= \frac{1}{2}(150 + 800)\text{ kPa }(1.5 - 1) \times 0.001\text{ m}^3 = \mathbf{0.2375\ kJ}$$

4.43

Air, 3 kg, is in a piston cylinder similar to Fig. P.4.5 at 27^{o}C, 300 kPa. It is now heated to 500 K. Plot the process path in a P-v diagram, and find the work in the process.

Solution:

Ideal gas $PV = mRT$

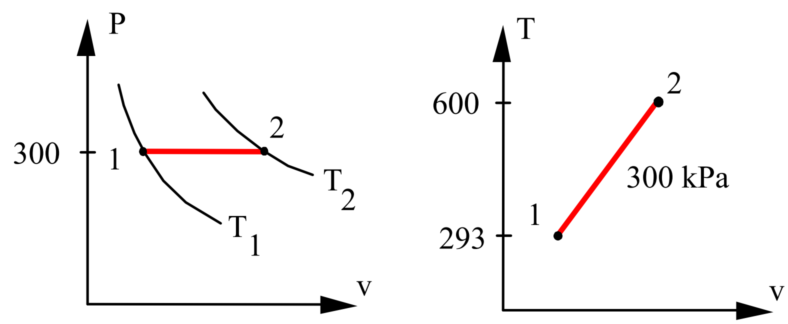
State 1: T_1, P_1 ideal gas so $P_1V_1 = mRT_1$

$V_1 = mR\,T_1 / P_1 = 3 \times 0.287 \times 293.15/300 = 0.8413\ m^3$

State 2: $T_2, P_2 = P_1$ and ideal gas so $P_2V_2 = mRT_2$

$V_2 = mR\,T_2 / P_2 = 3 \times 0.287 \times 600/300 = \mathbf{1.722\ m^3}$

${}_1W_2 = \int PdV = P\,(V_2 - V_1) = 300\,(1.722 – 0.8413) = \mathbf{264.2\ kJ}$

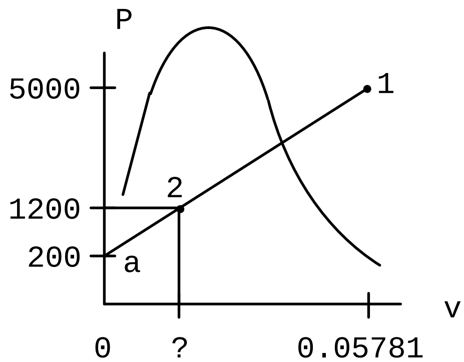


4.44

Find the work for Problem 3.62.

A piston/cylinder arrangement is loaded with a linear spring and the outside atmosphere. It contains water at 5 MPa, 400°C with the volume being 0.1 m^3. If the piston is at the bottom, the spring exerts a force such that P_{lift} = 200 kPa. The system now cools until the pressure reaches 1200 kPa. Find the mass of water, the final state (T_2, v_2) and plot the P–v diagram for the process.

Solution :



1: 5 MPa, 400°C $\Rightarrow$ v_1= 0.05781 m^3/kg

m = V/v_1 = 0.1/0.05781 = 1.73 kg

Straight line: $P = P_a + Cv$

$$v_2 = v_1 \frac{P_2 - P_a}{P_1 - P_a} = \mathbf{0.01204\ m^3/kg}$$

$v_2 < v_g$(1200 kPa) so two-phase T_2 = **188°C**

$$\Rightarrow\ x_2 = \frac{v_2 - 0.001139}{0.1622} = 0.0672$$

The P-V coordinates for the two states are then:

(P_1 = 5 MPa, V_1 = 0.1 m^3), (P_2 = 1200 kPa, V_2 = mv_2 = 0.02083 m^3)

P vs. V is linear so $\ _1W_2 = \int PdV = \frac{1}{2}(P_1 + P_2)(V_2 - V_1)$

$$= \frac{1}{2}(5000 + 1200)(0.02083 - 0.1)\ \text{kPa-m}^3$$

$$= \mathbf{-245.4\ kJ}$$

4.45

Heat transfer to a block of 1.5 kg ice at -10^{o}C melts it to liquid at 10^{o}C in a kitchen. How much work does the water gives out?

Work is done against the atmosphere due to volume change in the process.

State 1: Compressed solid, B.1.5, $v_1 = 0.0010891$ m^3/kg

State 2: Compressed liquid B.1.1 $v_2 = 0.001000$ m^3/kg

$${}_1W_2 = \int PdV = P_o (V_2 - V_1) = P_o\, m\, (v_2 - v_1)$$

$$= 101.325 \text{ kPa} \times 1.5 \text{ kg} \times (0.001 - 0.0010891) \text{ m}^3\text{/kg}$$

$$= \mathbf{-\ 0.0135\ kJ}$$

Notice the work is negative, the volume is reduced!

4.46

A piston cylinder contains 0.5 kg air at 500 kPa, 500 K. The air expands in a process so P is linearly decreasing with volume to a final state of 100 kPa, 300 K. Find the work in the process.

Solution:

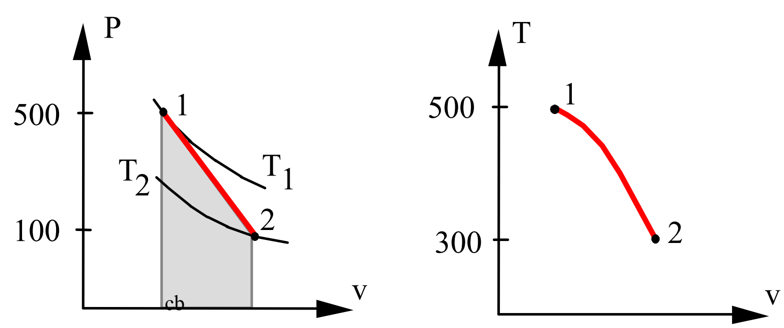
Process: $P = A + BV$ (linear in V, decreasing means B is negative)

From the process: $_1W_2 = \int PdV = \text{AREA} = \frac{1}{2}(P_1 + P_2)(V_2 - V_1)$

$V_1 = mR\,T_1/P_1 = 0.5 \times 0.287 \times (500/500) = 0.1435 \text{ m}^3$

$V_2 = mR\,T_2/P_2 = 0.5 \times 0.287 \times (300/100) = 0.4305 \text{ m}^3$

$_1W_2 = \frac{1}{2} \times (500 + 100) \text{ kPa} \times (0.4305 - 0.1435) \text{ m}^3 =$ **86.1 kJ**



Polytropic process

4.47

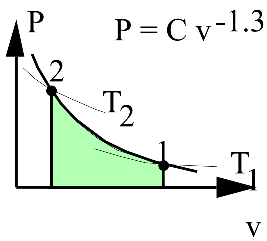
A nitrogen gas goes through a polytropic process with n = 1.3 in a piston/cylinder. It starts out at 600 K, 600 kPa and ends at 800 K. Is the work positive, negative or zero?

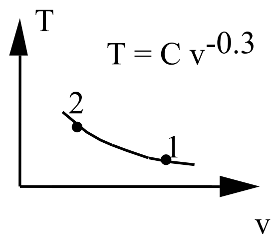
The work is a boundary work so it is

$$W = \int P dV = \int P m\, dv = \text{AREA}$$

so the sign depends on the sign for dV (or dv). The process looks like the following

The actual process is on a steeper curve than n = 1.





As the temperature increases we notice the volume decreases so

$$dv < 0 \quad \Rightarrow \quad W < 0$$

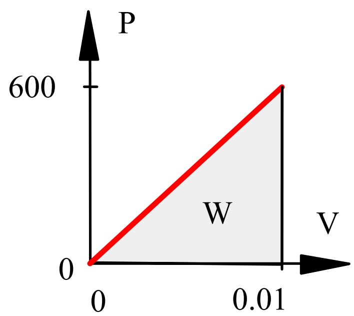
Work is **negative** and goes into the nitrogen gas.

4.48

Consider a mass going through a polytropic process where pressure is directly proportional to volume (n = – 1). The process start with *P* = 0, *V* = 0 and ends with *P* = 600 kPa, *V* = 0.01 m^3. Find the boundary work done by the mass.

Solution:

The setup has a pressure that varies linear with volume going through the initial and the final state points. The work is the area below the process curve.



$$W = \int PdV = \text{AREA}$$

$$= \frac{1}{2}(P_1 + P_2)(V_2 - V_1)$$

$$= \frac{1}{2}(P_2 + 0)(V_2 - 0)$$

$$= \frac{1}{2} P_2 V_2 = \frac{1}{2} \times 600 \times 0.01 = \mathbf{3\ kJ}$$

4.49

Helium gas expands from 125 kPa, 350 K and 0.25 m^3 to 100 kPa in a polytropic process with n = 1.667. How much work does it give out?

Solution:

Process equation: $PV^n = \text{constant} = P_1V_1^n = P_2V_2^n$

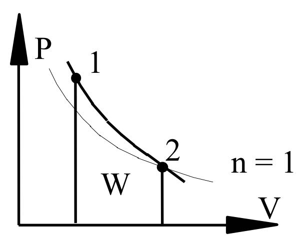
Solve for the volume at state 2

$$V_2 = V_1\ (P_1/P_2)^{1/n} = 0.25 \times \left(\frac{125}{100}\right)^{0.6} = 0.2852\ m^3$$

Work from Eq.4.4

$$_1W_2 = \frac{P_2V_2 - P_1\ V_1}{1-n} = \frac{100\times 0.2852 - 125\times 0.25}{1 - 1.667}\ \text{kPa m}^3 = \mathbf{4.09\ kJ}$$

The actual process is on a steeper curve than n = 1.



4.50

Air at 1500 K, 1000 kPa expands in a polytropic process, n = 1.5, to a pressure of 200 kPa. How cold does the air become and what is the specific work out?

Process equation: $PV^n = \text{constant} = P_1V_1^n = P_2V_2^n$

Solve for the temperature at state 2 by using ideal gas (PV = mRT)

$$\frac{T_2}{T_1} = \frac{P_2V_2}{P_1V_1} = \frac{P_2}{P_1} \times \left(\frac{P_1}{P_2}\right)^{1/n} = \left(\frac{P_2}{P_1}\right)^{(n-1)/n}$$

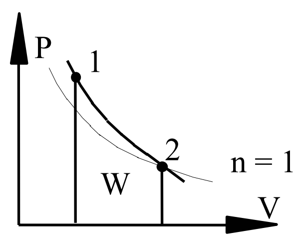
$$T_2 = T_1\ (P_2/P_1)^{(n-1)/n} = 1500 \times \left(\frac{200}{1000}\right)^{(1.5-1)/1.5} = \mathbf{877.2\ K}$$

Work from Eq.4.5

$${}_1w_2 = \frac{P_2\, v_2 - P_1\, v_1}{1-n} = \frac{R(P_2 - T_1)}{1-n}$$

$$= \frac{0.287(877.2 - 1500)}{1 - 1.5} = \mathbf{357.5\ kJ/kg}$$

The actual process is on a steeper curve than n = 1.



4.51

The piston/cylinder shown in Fig. P4.51 contains carbon dioxide at 300 kPa, 100°C with a volume of 0.2 m^3. Mass is added at such a rate that the gas compresses according to the relation $PV^{1.2}$ = constant to a final temperature of 200°C. Determine the work done during the process.

Solution:

From Eq. 4.4 for the polytopic process $PV^n = \text{const}$ ($n \neq 1$)

$$ {}_1W_2 = \int_1^2 PdV = \frac{P_2V_2 - P_1V_1}{1 - n} $$

Assuming ideal gas, PV = mRT

$$ {}_1W_2 = \frac{mR(T_2 - T_1)}{1 - n}, $$

But $\quad mR = \frac{P_1V_1}{T_1} = \frac{300 \times 0.2}{373.15} \frac{\text{kPa m}^3}{\text{K}} = 0.1608 \text{ kJ/K}$

$$ {}_1W_2 = \frac{0.1608(473.2 - 373.2)}{1 - 1.2} \frac{\text{kJ K}}{\text{K}} = \mathbf{-80.4\ kJ} $$

4.52

Air goes through a polytropic process from 125 kPa, 325 K to 300 kPa and 500 K. Find the polytropic exponent n and the specific work in the process.

Solution:

Process: $Pv^n = \text{Const} = P_1 v_1^n = P_2\, v_2^n$

Ideal gas $Pv = RT$ so

$$v_1 = \frac{RT}{P} = \frac{0.287 \times 325}{125} = 0.7462 \text{ m}^3\text{/kg}$$

$$v_2 = \frac{RT}{P} = \frac{0.287 \times 500}{300} = 0.47833 \text{ m}^3\text{/kg}$$

From the process equation

$$(P_2/\, P_1) = (v_1/\, v_2)^n \quad => \quad \ln(P_2/\, P_1) = n \ln(v_1/\, v_2)$$

$$n = \ln(P_2/\, P_1) \,/\, \ln(v_1/\, v_2) = \frac{\ln 2.4}{\ln 1.56} = \mathbf{1.969}$$

The work is now from Eq.4.4 per unit mass

$$_1w_2 = \frac{P_2v_2\text{-}P_1v_1}{1\text{-}n} = \frac{R(T_2 - T_1)}{1\text{-}n} = \frac{0.287(500 - 325)}{1\text{-}1.969} = \mathbf{-51.8\ kJ/kg}$$

4.53

A gas initially at 1 MPa, 500°C is contained in a piston and cylinder arrangement with an initial volume of 0.1 m^3. The gas is then slowly expanded according to the relation *PV* = constant until a final pressure of 100 kPa is reached. Determine the work for this process.

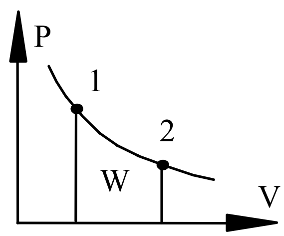
Solution:

By knowing the process and the states 1 and 2 we can find the relation between the pressure and the volume so the work integral can be performed.

Process: PV = C ⇒ $V_2 = P_1V_1/P_2 = 1000 \times 0.1/100 = 1\ m^3$

For this process work is integrated to Eq.4.5

$$_1W_2 = \int P\,dV = \int CV^{-1}dV = C\ \ln(V_2/V_1)$$



$$_1W_2 = P_1V_1\ \ln\frac{V_2}{V_1}$$

$$= 1000\ \text{kPa} \times 0.1\ \ m^3 \times \ln\ (1/0.1)$$

$$= \mathbf{230.3\ kJ}$$

4.54

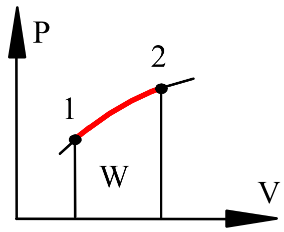
A balloon behaves so the pressure is $P = C_2 V^{1/3}$, $C_2 = 100$ kPa/m. The balloon is blown up with air from a starting volume of 1 m^3 to a volume of 3 m^3. Find the final mass of air assuming it is at 25°C and the work done by the air.

Solution:

The process is polytropic with exponent $n = -1/3$.

$$P_1 = C_2 V^{1/3} = 100 \times 1^{1/3} = 100 \text{ kPa}$$

$$P_2 = C_2 V^{1/3} = 100 \times 3^{1/3} = 144.22 \text{ kPa}$$



$$_1W_2 = \int P\, dV = \frac{P_2V_2 - P_1V_1}{1 - n} \qquad \text{(Equation 4.4)}$$

$$= \frac{144.22 \times 3 - 100 \times 1}{1 - (-1/3)} \text{ kPa-m}^3 = \mathbf{249.5\ kJ}$$

$$m_2 = \frac{P_2V_2}{RT_2} = \frac{144.22 \times 3}{0.287 \times 298} \frac{\text{kPa-m}^3}{\text{kJ/kg}} = \mathbf{5.056\ kg}$$

4.55

A piston cylinder contains 0.1 kg nitrogen at 100 kPa, 27^{o}C and it is now compressed in a polytropic process with n = 1.25 to a pressure of 250 kPa. What is the work involved?

Take CV as the nitrogen. $m_2 = m_1 = m$;

Process Eq.: $Pv^n = \text{Constant}$ (polytropic)

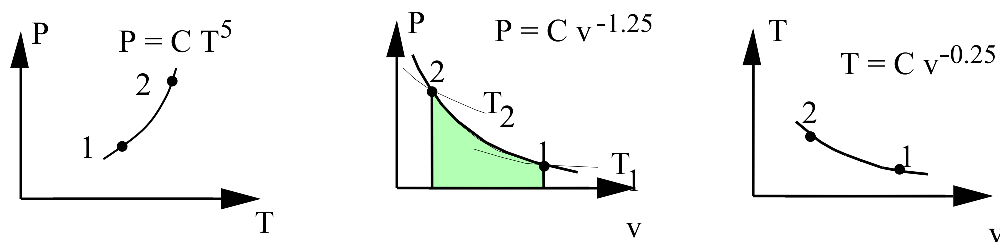
From the ideal gas law and the process equation we can get:

State 2: $T_2 = T_1 \left(P_2 / P_1\right)^{\frac{n-1}{n}} = 300.15 \left(\frac{250}{100}\right)^{\frac{0.25}{1.25}} = 360.5 \text{ K}$

From process eq.:

$${}_1W_2 = \int P\, dV = \text{area} = \frac{m}{1-n}(P_2v_2 - P_1v_1) = \frac{mR}{1-n}(T_2 - T_1)$$

$$= \frac{0.1 \times 0.2968}{1 - 1.25}(360.5 - 300.15) = \mathbf{-7.165\ kJ}$$



4.56

A piston cylinder contains 0.1 kg air at 100 kPa, 400 K which goes through a polytropic compression process with n = 1.3 to a pressure of 300 kPa. How much work has the air done in the process?

Solution:

Process: Pv^n = Const.

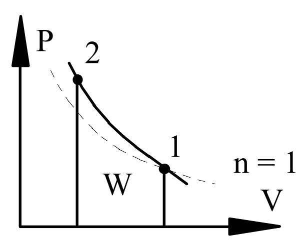
$$T_2 = T_1 \,(P_2 V_2 / P_1V_1) = T_1 \,(P_2 / P_1)(P_1 / P_2)^{1/n}$$

$$= 400 \times (300/100)^{(1 - 1/1.3)} = 515.4 \text{ K}$$

Work term is already integrated giving Eq.4.4

$${}_1W_2 = \frac{1}{1-n}(P_2 V_2 - P_1V_1) = \frac{mR}{1-n}(T_2 - T_1) \qquad \text{Since Ideal gas,}$$

$$= \frac{0.1 \times 0.287}{1 - 1.3} \times (515.4 - 400) = \mathbf{-11\ kJ}$$



4.57

A balloon behaves such that the pressure inside is proportional to the diameter squared. It contains 2 kg of ammonia at 0°C, 60% quality. The balloon and ammonia are now heated so that a final pressure of 600 kPa is reached. Considering the ammonia as a control mass, find the amount of work done in the process.

Solution:

Process : $P \propto D^2$, with $V \propto D^3$ this implies $P \propto D^2 \propto V^{2/3}$ so

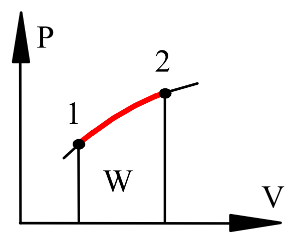
$PV^{-2/3}$ = constant, which is a polytropic process, $n = -2/3$

From table B.2.1: $V_1 = mv_1 = 2(0.001566 + 0.6 \times 0.28783) = 0.3485 \text{ m}^3$

$$V_2 = V_1 \left(\frac{P_2}{P_1}\right)^{3/2} = 0.3485 \left(\frac{600}{429.3}\right)^{3/2} = 0.5758 \text{ m}^3$$

$${}_1W_2 = \int P\, dV = \frac{P_2V_2 - P_1V_1}{1 - n} \qquad \text{(Equation 4.4)}$$

$$= \frac{600 \times 0.5758 - 429.3 \times 0.3485}{1 - (-2/3)} \text{ kPa-m}^3 = \mathbf{117.5\ kJ}$$



4.58

Consider a piston cylinder with 0.5 kg of R-134a as saturated vapor at -10°C. It is now compressed to a pressure of 500 kPa in a polytropic process with n = 1.5. Find the final volume and temperature, and determine the work done during the process.

Solution:

Take CV as the R-134a which is a control mass. $m_2 = m_1 = m$

Process: $Pv^{1.5}$ = constant until P = 500 kPa

1: (T, x) $v_1 = 0.09921$ m^3/kg, $P = P_{sat} = 201.7$ kPa from Table B.5.1

2: (P, process) $v_2 = v_1 (P_1/P_2)^{(1/1.5)}$

$$= 0.09921 \times (201.7/500)^{2/3} = \mathbf{0.05416} \text{ m}^3\text{/kg}$$

Given (P, v) at state 2 from B.5.2 it is superheated vapor at $\mathbf{T_2 = 79°C}$

Process gives $P = C\, v^{-1.5}$, which is integrated for the work term, Eq.(4.4)

$$_1W_2 = \int P\, dV = \frac{m}{1 - 1.5} (P_2v_2 - P_1v_1)$$

$$= \frac{2}{-0.5} \text{ kg} \times (500 \times 0.05416 - 201.7 \times 0.09921) \text{ kPa-m}^3\text{/kg}$$

$$= \mathbf{-7.07\ kJ}$$

4.59

A piston/cylinder contains water at 500°C, 3 MPa. It is cooled in a polytropic process to 200°C, 1 MPa. Find the polytropic exponent and the specific work in the process.

Solution:

Polytropic process: $Pv^n = C$

Both states must be on the process line: $P_2 v_2^n = C = P_1 v_1^n$

Take the ratio to get: $\frac{P_1}{P_2} = \left(\frac{v_2}{v_1}\right)^n$

and then take ln of the ratio: $\ln\left(\frac{P_1}{P_2}\right) = \ln\left(\frac{v_2}{v_1}\right)^n = n \ln\left(\frac{v_2}{v_1}\right)$

now solve for the exponent n

$$n = \ln\left(\frac{P_1}{P_2}\right) \Big/ \ln\left(\frac{v_2}{v_1}\right) = \frac{1.0986}{0.57246} = \mathbf{1.919}$$

Table B.1.3 $v_1 = 0.11619\ m^3/kg$, $v_2 = 0.20596\ m^3/kg$

$$_1w_2 = \int P\, dv = \frac{P_2 v_2 - P_1 v_1}{1 - n} \qquad \text{(Equation 4.4)}$$

$$= \frac{1000 \times 0.20596 - 3000 \times 0.11619}{1 - 1.919}\ \text{kPa-m}^3\text{/kg}$$

$= \mathbf{155.2\ kJ/kg}$

4.60

A spring loaded piston/cylinder assembly contains 1 kg water at 500°C, 3 MPa. The setup is such that the pressure is proportional to volume, P = CV. It is now cooled until the water becomes saturated vapor. Sketch the P-v diagram and find the final state by iterations and the work in the process.

Solution :

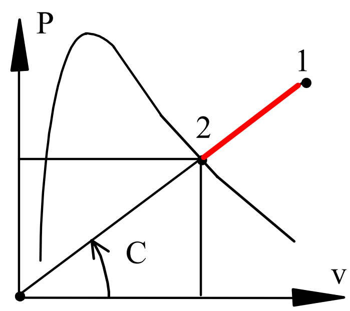
State 1: Table B.1.3: $v_1 = 0.11619 \text{ m}^3/\text{kg}$

Process: m is constant and $P = C_0V = C_0m\, v = C\, v$

polytropic process with n = −1

$P = Cv \quad \Rightarrow \quad C = P_1/v_1 = 3000/0.11619 = 25820 \text{ kPa kg/m}^3$

State 2: $x_2 = 1$ & $P_2 = Cv_2$ (on process line)



Trial & error on T_{2sat} or P_{2sat}:

Here from B.1.2:

at 2 MPa $v_g = 0.09963 \Rightarrow C = P/v_g = 20074$ (low)

2.5 MPa $v_g = 0.07998 \Rightarrow C = P/v_g = 31258$ (high)

2.25 MPa $v_g = 0.08875 \Rightarrow C = P/v_g = 25352$ (low)

Now interpolate to match the right slope C:

$$P_2 = 2250 + 250\,\frac{25\,820 - 25\,352}{31\,258 - 25\,352} = 2270 \text{ kPa},$$

$$v_2 = P_2/C = 2270/25820 = 0.0879 \text{ m}^3/\text{kg}$$

P is linear in V so the work becomes (area in P-v diagram)

$$_1W_2 = \int P\, dv = m\,\frac{1}{2}(P_1 + P_2)(v_2 - v_1)$$

$$= 1 \text{ kg} \times \frac{1}{2}(3000 + 2270)(0.0879 - 0.11619) \text{ kPa-m}^3$$

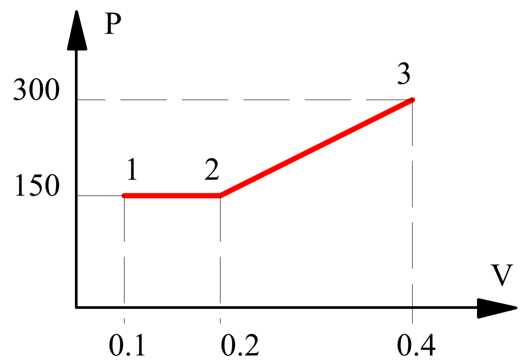
$$= \mathbf{-\ 74.5\ kJ}$$

4.61

Consider a two-part process with an expansion from 0.1 to 0.2 m^3 at a constant pressure of 150 kPa followed by an expansion from 0.2 to 0.4 m^3 with a linearly rising pressure from 150 kPa ending at 300 kPa. Show the process in a P-V diagram and find the boundary work.

Solution:

By knowing the pressure versus volume variation the work is found. If we plot the pressure versus the volume we see the work as the area below the process curve.



$$_1W_3 = {}_1W_2 + {}_2W_3 = \int_1^2 PdV + \int_2^3 PdV$$

$$= P_1 (V_2 – V_1) + \frac{1}{2} (P_2 + P_3)(V_3\text{-}V_2)$$

$$= 150 \text{ kPa } (0.2 – 1.0) \text{ m}^3 + \frac{1}{2} (150 + 300) \text{ kPa } (0.4 – 0.2) \text{ m}^3$$

$$= (15 + 45) \text{ kJ} = \mathbf{60\ kJ}$$

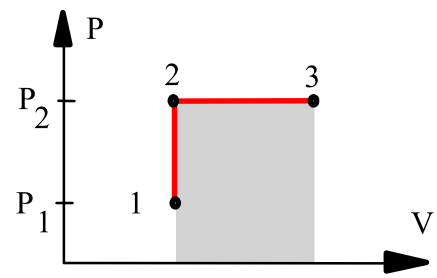
4.62

A helium gas is heated at constant volume from a state of 100 kPa, 300 K to 500 K. A following process expands the gas at constant pressure to three times the initial volume. What is the specific work in the combined process?

The two processes are:

1 -> 2: Constant volume $V_2 = V_1$

2 -> 3: Constant pressure $P_3 = P_2$



Use ideal gas approximation for helium.

State 1: T, P => $v_1 = RT_1/P_1$

State 2: $V_2 = V_1$ => $P_2 = P_1 (T_2/T_1)$

State 3: $P_3 = P_2$ => $V_3 = 3V_2$; $T_3 = T_2\, v_3/v_2 = 500 \times 3 = 1500$ K

We find the work by summing along the process path.

$$_1w_3 = {}_1w_2 + {}_2w_3 = {}_2w_3 = P_3(v_3 - v_2) = R(T_3 - T_2)$$

$$= 2.0771 \text{ kJ/kg-K} \times (1500 - 500) \text{ K} = \mathbf{2077\ kJ/kg}$$

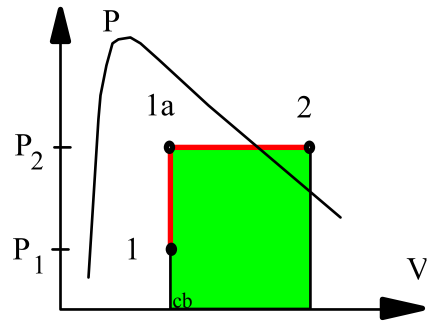
4.63

Find the work in Problem 3.59.

Ammonia at 10°C with a mass of 10 kg is in a piston cylinder arrangement with an initial volume of 1 m³. The piston initially resting on the stops has a mass such that a pressure of 900 kPa will float it. The ammonia is now slowly heated to 50°C. Find the work in the process.

C.V. Ammonia, constant mass.

Process: V = constant unless $P = P_{float}$



State 1: T = 10°C, $v_1 = \frac{V}{m} = \frac{1}{10} = 0.1\ m^3/kg$

From Table B.2.1 $v_f < v < v_g$

$x_1 = (v - v_f)/v_{fg} = (0.1 - 0.0016)/0.20381$

$= 0.4828$

State 1a: P = 900 kPa, $v = v_1 = 0.1 < v_g$ at 900 kPa

This state is two-phase $T_{1a} = 21.52°C$

Since $T_2 > T_{1a}$ then $v_2 > v_{1a}$

State 2: 50°C and on line(s) means 900 kPa which is superheated vapor.

From Table B.2.2 linear interpolation between 800 and 1000 kPa:

$v_2 = 0.1648\ m^3/kg, \quad V_2 = mv_2 = 1.648\ m^3$

$_1W_2 = \int P\,dV = P_{float}(V_2 - V_1) = 900\ kPa\ (1.648 - 1.0)\ m^3$

$=$ **583.2 kJ**

4.64

A piston/cylinder arrangement shown in Fig. P4.64 initially contains air at 150 kPa, 400°C. The setup is allowed to cool to the ambient temperature of 20°C.

a. Is the piston resting on the stops in the final state? What is the final pressure in the cylinder?

b. What is the specific work done by the air during this process?

Solution:

State 1: $P_1 = 150$ kPa, $T_1 = 400°C = 673.2$ K

State 2: $T_2 = T_0 = 20°C = 293.2$ K

For all states air behave as an ideal gas.

a) If piston at stops at 2, $V_2 = V_1/2$ and pressure less than $P_{lift} = P_1$

$$\Rightarrow P_2 = P_1 \times \frac{V_1}{V_2} \times \frac{T_2}{T_1} = 150 \text{ kPa} \times 2 \times \frac{293.2}{673.2} = 130.7 \text{ kPa} < P_1$$

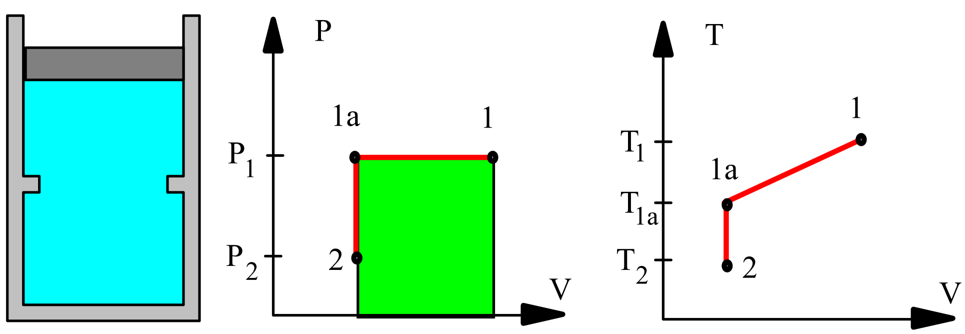
$\Rightarrow$ Piston is resting on stops at state 2.

b) Work done while piston is moving at constant $P_{ext} = P_1$.

$${}_1W_2 = \int P_{ext}\, dV = P_1 (V_2 - V_1) \;;\quad V_2 = \frac{1}{2} V_1 = \frac{1}{2} m\ RT_1/P_1$$

$${}_1w_2 = {}_1W_2/m = RT_1 \left(\frac{1}{2} - 1\right) = -\frac{1}{2} \times 0.287 \text{ kJ/kg-K} \times 673.2 \text{ K}$$

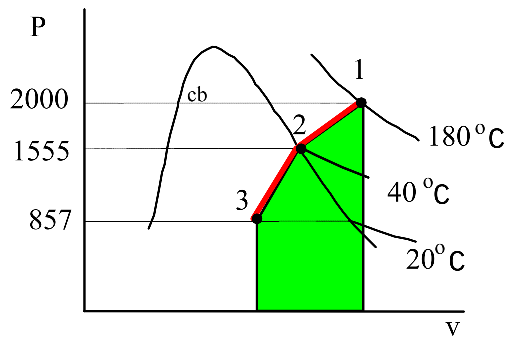
$$= \mathbf{-96.6\ kJ/kg}$$



4.65

A cylinder containing 1 kg of ammonia has an externally loaded piston. Initially the ammonia is at 2 MPa, 180°C and is now cooled to saturated vapor at 40°C, and then further cooled to 20°C, at which point the quality is 50%. Find the total work for the process, assuming a piecewise linear variation of *P* versus *V*.

Solution:



State 1: (T, P) Table B.2.2

$v_1 = 0.10571 \ m^3/kg$

State 2: (T, x) Table B.2.1 sat. vap.

$P_2 = 1555$ kPa,

$v_2 = 0.08313 \ m^3/kg$

State 3: (T, x) $P_3 = 857$ kPa, $v_3 = (0.001638 + 0.14922)/2 = 0.07543 \ m^3/kg$

Sum the work as two integrals each evaluated by the area in the P-v diagram.

$${}_1W_3 = \int_1^3 PdV \approx \left(\frac{P_1 + P_2}{2}\right) m(v_2 - v_1) + \left(\frac{P_2 + P_3}{2}\right) m(v_3 - v_2)$$

$$= \frac{2000 + 1555}{2} \text{ kPa} \times 1 \text{ kg} \times (0.08313 - 0.10571) \ m^3/kg$$

$$+ \frac{1555 + 857}{2} \text{ kPa} \times 1 \text{ kg} \times (0.07543 - 0.08313) \ m^3/kg$$

= **-49.4 kJ**

4.66

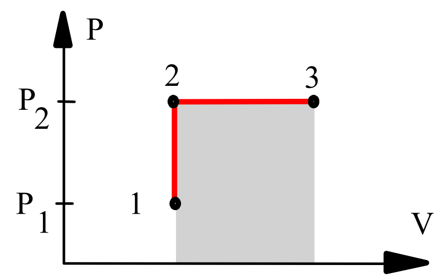
A piston cylinder has 1.5 kg of air at 300 K and 150 kPa. It is now heated up in a two step process. First constant volume to 1000 K (state 2) then followed by a constant pressure process to 1500 K, state 3. Find the final volume and the work in the process.

Solution:

The two processes are:

1 -> 2: Constant volume $V_2 = V_1$

2 -> 3: Constant pressure $P_3 = P_2$



Use ideal gas approximation for air.

State 1: T, P => $V_1 = mRT_1/P_1 = 1.5 \times 0.287 \times 300/150 = 0.861 \text{ m}^3$

State 2: $V_2 = V_1$ => $P_2 = P_1 (T_2/T_1) = 150 \times 1000/300 = 500 \text{ kPa}$

State 3: $P_3 = P_2$ => $V_3 = V_2 (T_3/T_2) = 0.861 \times 1500/1000 = \mathbf{1.2915 \ m^3}$

We find the work by summing along the process path.

$$_1W_3 = {}_1W_2 + {}_2W_3 = {}_2W_3 = P_3(V_3 - V_2)$$

$$= 500 \text{ kPa} \times (1.2915 - 0.861) \text{ m}^3 = \mathbf{215.3 \ kJ}$$

4.67

The refrigerant R-410a is contained in a piston/cylinder as shown in Fig. P4.67, where the volume is 11 L when the piston hits the stops. The initial state is –30°C, 150 kPa with a volume of 10 L. This system is brought indoors and warms up to 15°C.

a. Is the piston at the stops in the final state?

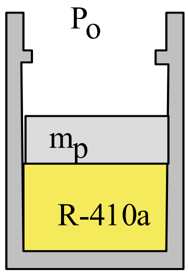
b. Find the work done by the R-410a during this process.

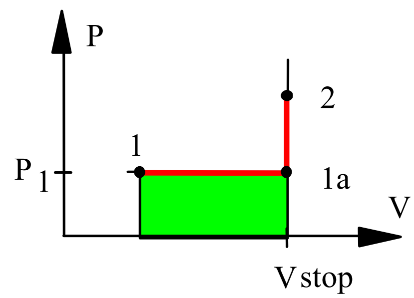
Solution:

Initially piston floats, $V < V_{stop}$ so the piston moves at constant $P_{ext} = P_1$ until it reaches the stops or 15°C, whichever is first.

a) From Table B.4.2: $v_1 = 0.17732\ m^3/kg$,

$$m = V/v = \frac{0.010}{0.17732} = 0.056395\ kg$$





Check the temperature at state 1a: $P_{1a} = 150$ kPa, $v = V_{stop}/m$.

$$v_{1a} = V/m = \frac{0.011}{0.056395} = 0.19505\ m^3/kg \quad \Rightarrow \quad T_{1a} = -9.4°C\ \ \&\ T_2 = 15°C$$

Since $T_2 > T_{1a}$ then it follows that $P_2 > P_1$ and the piston is against stop.

b) Work done at constant $P_{ext} = P_1$.

$${}_1W_2 = \int P_{ext}\, dV = P_{ext}(V_2 - V_1) = 150(0.011 - 0.010) = \mathbf{0.15\ kJ}$$

4.68

A piston/cylinder contains 1 kg of liquid water at 20°C and 300 kPa. Initially the piston floats, similar to the setup in Problem 4.67, with a maximum enclosed volume of 0.002 m^3 if the piston touches the stops. Now heat is added so a final pressure of 600 kPa is reached. Find the final volume and the work in the process.

Solution:

Take CV as the water which is a control mass: $m_2 = m_1 = m$;

Table B.1.1: 20°C => $P_{sat} = 2.34$ kPa

State 1: Compressed liquid $v = v_f(20) = 0.001002\ m^3/kg$

State 1a: $v_{stop} = 0.002\ m^3/kg$, 300 kPa

State 2: Since $P_2 = 600$ kPa $> P_{lift}$ then piston is pressed against the stops

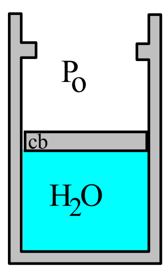
$$v_2 = v_{stop} = 0.002\ m^3/kg \text{ and } V = 0.002\ m^3$$

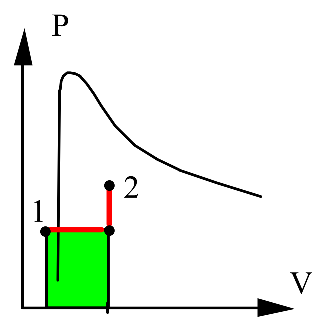
For the given P : $v_f < v < v_g$ so 2-phase $T = T_{sat} = 158.85$ °C

Work is done while piston moves at P_{lift} = constant = 300 kPa so we get

$$_1W_2 = \int P\, dV = m\, P_{lift}(v_2 - v_1) = 1 \text{ kg} \times 300 \text{ kPa} \times (0.002 - 0.001002)\ m^3$$

$$= \mathbf{0.30\ kJ}$$





4.69

A piston/cylinder contains 50 kg of water at 200 kPa with a volume of 0.1 m^3. Stops in the cylinder restricts the enclosed volume to 0.5 m^3, similar to the setup in Problem 4.67. The water is now heated to 200°C. Find the final pressure, volume and the work done by the water.

Solution:

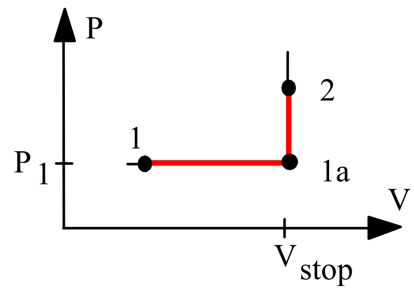
Initially the piston floats so the equilibrium lift pressure is 200 kPa

1: 200 kPa, $v_1 = 0.1/50 = 0.002$ m^3/kg,

2: 200°C, on line

Check state 1a:

$$v_{stop} = 0.5/50 = 0.01 \; m^3/kg$$



=>

Table B.1.2: 200 kPa , $v_f < v_{stop} < v_g$

State 1a is two phase at 200 kPa and $T_{stop} \approx 120.2$ °C so as $T_2 > T_{stop}$ the state is higher up in the P-V diagram with

$$v_2 = v_{stop} < v_g = 0.127 \; m^3/kg \; \text{(at 200°C)}$$

State 2 two phase => $P_2 = P_{sat}(T_2) =$ **1.554 MPa**, $V_2 = V_{stop} =$ **0.5 m^3**

$${}_1W_2 = {}_1W_{stop} = 200 \text{ kPa} \times (0.5 - 0.1) \text{ m}^3 = \mathbf{80 \text{ kJ}}$$

4.70

A piston/cylinder assembly (Fig. P4.70) has 1 kg of R-134a at state 1 with 110°C, 600 kPa, and is then brought to saturated vapor, state 2, by cooling while the piston is locked with a pin. Now the piston is balanced with an additional constant force and the pin is removed. The cooling continues to a state 3 where the R-134a is saturated liquid. Show the processes in a P-V diagram and find the work in each of the two steps, 1 to 2 and 2 to 3.

Solution :

CV R-134a This is a control mass.

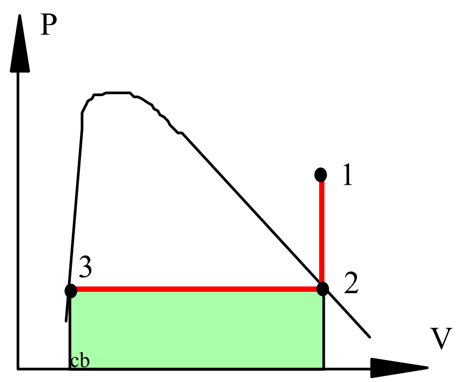
Properties from table B.5.1 and 5.2

State 1: (T,P) B.5.2 => $v = 0.04943\ m^3/kg$

State 2: given by fixed volume $v_2 = v_1$ and $x_2 = 1.0$ so from B.5.1

$$v_2 = v_1 = v_g = 0.04943\ m^3/kg \quad => \quad T = 10°C$$

State 3 reached at constant P (F = constant) $v_3 = v_f = 0.000794\ m^3/kg$



Since no volume change from 1 to 2 => $\mathbf{{}_1W_2 = 0}$

$${}_2W_3 = \int P\, dV = P(V_3 - V_2) = mP(v_3 - v_2) \qquad \text{Constant pressure}$$

$$= 415.8\ kPa \times (0.000794 - 0.04943)\ m^3/kg \times 1\ kg$$

$$= \mathbf{-20.22\ kJ}$$

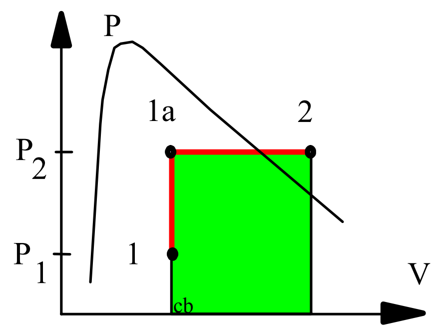
4.71

Find the work in Problem 3.84.

Ammonia at 10°C with a mass of 10 kg is in a piston cylinder arrangement with an initial volume of 1 m^3. The piston initially resting on the stops has a mass such that a pressure of 900 kPa will float it. The ammonia is now slowly heated to 50°C. Find the work in the process.

C.V. Ammonia, constant mass.

Process: V = constant unless $P = P_{float}$



State 1: T = 10°C, $v_1 = \frac{V}{m} = \frac{1}{10} = 0.1\ m^3/kg$

From Table B.2.1 $v_f < v < v_g$

$x_1 = (v - v_f)/v_{fg} = (0.1 - 0.0016)/0.20381$
$= 0.4828$

State 1a: P = 900 kPa, $v = v_1 = 0.1 < v_g$ at 900 kPa

This state is two-phase $T_{1a} = 21.52°C$

Since $T_2 > T_{1a}$ then $v_2 > v_{1a}$

State 2: 50°C and on line(s) means 900 kPa which is superheated vapor.

From Table B.2.2 linear interpolation between 800 and 1000 kPa:

$v_2 = 0.1648\ m^3/kg, \quad V_2 = mv_2 = 1.648\ m^3$

${}_1W_2 = \int P\, dV = P_{float} (V_2 - V_1) = 900\ kPa \times (1.648 - 1.0)\ m^3$
$=$ **583.2 kJ**

4.72

10 kg of water in a piston cylinder arrangement exists as saturated liquid/vapor at 100 kPa, with a quality of 50%. It is now heated so the volume triples. The mass of the piston is such that a cylinder pressure of 200 kPa will float it, see Fig. P4.72.

a) Find the final temperature and volume of the water.

b) Find the work given out by the water.

Solution:

Take CV as the water $m_2 = m_1 = m$;

Process: v = constant until $P = P_{lift}$ then P is constant.

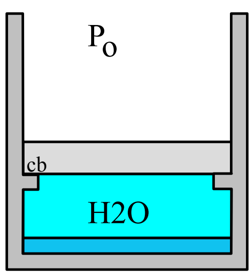
State 1: $v_1 = v_f + x\ v_{fg} = 0.001043 + 0.5 \times 1.69296 = 0.8475\ m^3/kg$

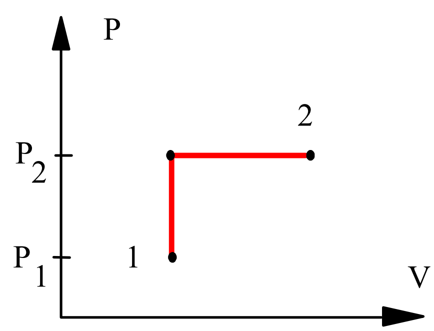
State 2: $v_2,\ P_2 \le P_{lift}$ => $v_2 = 3 \times 0.8475 = 2.5425\ m^3/kg$;

$T_2 =$ **829°C ;** $V_2 = m\ v_2 =$ **25.425 m³**

$_1W_2 = \int P\ dV = P_{lift} \times (V_2 - V_1)$

$= 200\ kPa \times 10\ kg \times (2.5425 – 0.8475)\ m^3/kg =$ **3390 kJ**





4.73

A piston cylinder setup similar to Problem 4.72 contains 0.1 kg saturated liquid and vapor water at 100 kPa with quality 25%. The mass of the piston is such that a pressure of 500 kPa will float it. The water is heated to 300°C. Find the final pressure, volume and the work, ${}_1W_2$.

Solution:

Take CV as the water: $m_2 = m_1 = m$

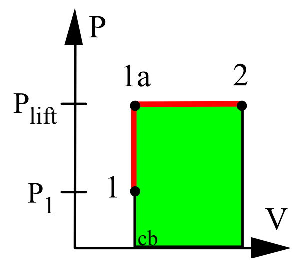
Process: v = constant until $P = P_{lift}$

To locate state 1: Table B.1.2

$v_1 = 0.001043 + 0.25\times1.69296 = 0.42428 \text{ m}^3/\text{kg}$

1a: $v_{1a} = v_1 = 0.42428 \text{ m}^3/\text{kg} > v_g$ at 500 kPa

so state 1a is superheated vapor $T_{1a} = 200°\text{C}$



State 2 is 300°C so heating continues after state 1a to 2 at constant P =>

2: T_2, $P_2 = P_{lift}$ = **500 kPa** => Table B.1.3 $v_2 = 0.52256 \text{ m}^3/\text{kg}$;

$V_2 = mv_2 = \mathbf{0.05226\ m^3}$

${}_1W_2 = P_{lift}(V_2 - V_1) = 500 \text{ kPa} \times (0.05226 - 0.04243) \text{ m}^3 = \mathbf{4.91\ kJ}$

4.74

A piston cylinder contains air at 1000 kPa, 800 K with a volume of 0.05 m^3. The piston is pressed against the upper stops and it will float at a pressure of 750 kPa. Then the air is cooled to 400 K. What is the process work?

We need to find state 2. Let us see if we proceed past state 1a during the cooling.

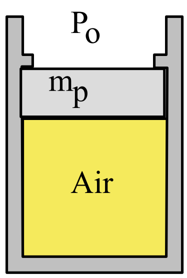
$$T_{1a} = T_1 \, P_{float} / P_1 = 800 \times 750 / 100 = 600 \text{ K}$$

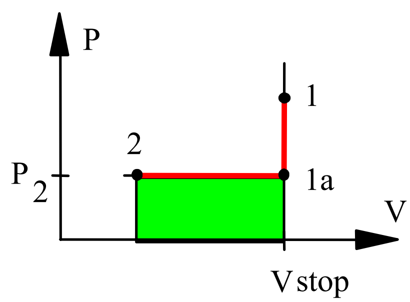
so we do cool below T_{1a}. That means the piston is floating. Write the ideal gas law for state 1 and 2 to get

$$V_2 = \frac{mRT_2}{P_2} = \frac{P_1V_1T_2}{P_2T_1} = \frac{1000 \times 0.05 \times 400}{750 \times \ 800} = 0.0333 \text{ m}^3$$

$$_1W_2 = {}_{1a}W_2 = \int P \, dV = P_2 \, (V_2 - V_1)$$

$$= 750 \text{ kPa} \times (0.0333 - 0.05) \text{ m}^3 = \mathbf{-12.5 \ kJ}$$





Other types of work and general concepts

4.75

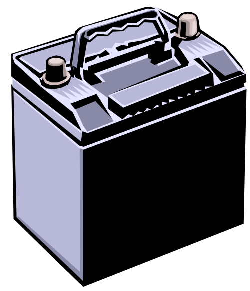
Electric power is volts times ampere (P = V i). When a car battery at 12 V is charged with 6 amp for 3 hours how much energy is delivered?

Solution:

$$W = \int \dot{W}\, dt = \dot{W}\, \Delta t = V\, i\, \Delta t$$

$$= 12\ V \times 6\ Amp \times 3 \times 3600\ s$$

$$= 777\ 600\ J = \mathbf{777.6\ kJ}$$



Remark: Volt times ampere is also watts, 1 W = 1 V × 1 Amp. = 1 J/s

4.76

A copper wire of diameter 2 mm is 10 m long and stretched out between two posts. The normal stress (pressure) $\sigma = E(L - L_o)/L_o$, depends on the length L versus the un-stretched length L_o and Young's modulus $E = 1.1 \times 10^6$ kPa. The force is $F = A\sigma$ and measured to be 110 N. How much longer is the wire and how much work was put in?

Solution:

$F = As = AE\ \Delta L/ L_o$ and $\Delta L = FL_o /AE$

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4} \times 0.002^2 = 3.142 \times 10^{-6}\ m^2$$

$$\Delta L = \frac{110 \times 10}{3.142\times10^{-6} \times 1.1 \times 10^6 \times 10^3} = \mathbf{0.318\ m}$$

$$_1W_2 = \int F\ dx = \int A\ s\ dx = \int AE \frac{x}{L_o}\ dx$$

$$= \frac{AE}{L_o} ½\ x^2 \qquad \text{where } x = L - L_o$$

$$= \frac{3.142\times10^{-6} \times 1.1 \times 10^6 \times 10^3}{10} \times ½ \times 0.318^2 = \mathbf{17.47\ J}$$

4.77

A 0.5-m-long steel rod with a 1-cm diameter is stretched in a tensile test. What is the required work to obtain a relative strain of 0.1%? The modulus of elasticity of steel is 2×10^8 kPa.

Solution :

$$-{}_1W_2 = \frac{AEL_0}{2} (e)^2, \quad A = \frac{\pi}{4} (0.01)^2 = 78.54 \times 10^{-6} \text{ m}^2$$

$$-{}_1W_2 = \frac{78.54 \times 10^{-6} \times 2 \times 10^8 \times 0.5}{2} \text{ m}^2\text{-kPa-m } (10^{-3})^2$$

$$= \mathbf{3.93 \ J}$$

4.78

A soap bubble has a surface tension of $S = 3 \times 10^{-4}$ N/cm as it sits flat on a rigid ring of diameter 5 cm. You now blow on the film to create a half sphere surface of diameter 5 cm. How much work was done?

$$_1W_2 = \int F\ dx = \int S\ dA = S\,\Delta A$$

$$= 2 \times S \times \left(\frac{\pi}{2} D^2 - \frac{\pi}{4} D^2\right)$$

$$= 2 \times 3 \times 10^{-4}\ \text{N/cm} \times 100\ \text{cm/m} \times \frac{\pi}{2}\, 0.05^2\ \text{m}^2\ (1 - 0.5)$$

$$= \mathbf{1.18 \times 10^{-4}\ J}$$

Notice the bubble has 2 surfaces.

$$A_1 = \frac{\pi}{4} D^2,$$

$$A_2 = ½\, \pi\, D^2$$

4.79

A film of ethanol at 20°C has a surface tension of 22.3 mN/m and is maintained on a wire frame as shown in Fig. P4.79. Consider the film with two surfaces as a control mass and find the work done when the wire is moved 10 mm to make the film 20 × 40 mm.

Solution :

Assume a free surface on both sides of the frame, i.e., there are two surfaces 20 × 30 mm

$$W = -\int \mathcal{S}\, dA = -22.3\times10^{-3}\ \text{N/m} \times 2\ (800 - 600)\times10^{-6}\ \text{m}^2$$

$$= -8.92\times10^{-6}\ \text{J} = \mathbf{-8.92\ \mu J}$$

4.80

Assume we fill a spherical balloon from a bottle of helium gas. The helium gas provides work $\int$ PdV that stretches the balloon material $\int$ S dA and pushes back the atmosphere $\int P_o$ dV. Write the incremental balance for $dW_{helium} = dW_{stretch} + dW_{atm}$ to establish the connection between the helium pressure, the surface tension S and P_o as a function of radius.

$$W_{He} = \int P\,dV = \int S\,dA + \int P_o\,dV$$

$$dW_{He} = P\,dV = S\,dA + P_o\,dV$$

$$dV = d\left(\frac{\pi}{6} D^3\right) = \frac{\pi}{6} \times 3D^2\,dD$$

$$dA = d\left(2 \times \pi \times D^2\right) = 2\pi\,(2D)\,dD$$

$$P\,\frac{\pi}{2} D^2\,dD = S\,(4\pi) D\,dD + P_o \frac{\pi}{2} D^2\,dD$$

Divide by $\frac{\pi}{2} D^2$ to recognize

$$\mathbf{P_{He} = P_o + 8\,\frac{S}{D}}$$

4.81

Assume a balloon material with a constant surface tension of S = 2 N/m. What is the work required to stretch a spherical balloon up to a radius of $r = 0.5$ m? Neglect any effect from atmospheric pressure.

Assume the initial area is small, and that we have 2 surfaces inside and out

$$W = -\int S\, dA = -S\,(A_2 - A_1)$$

$$= -\,S(A_2) = -S(\,2\times \pi\, \mathbf{D}_2^2\,)$$

$$= -2\ \text{N/m} \times 2 \times \pi \times 1\ \text{m}^2 = -12.57\ \text{J}$$

$$W_{in} = -W = \mathbf{12.57\ J}$$

4.82

A sheet of rubber is stretched out over a ring of radius 0.25 m. I pour liquid water at 20^{o}C on it, as in Fig. P4.82, so the rubber forms a half sphere (cup). Neglect the rubber mass and find the surface tension near the ring?

Solution:

$F\uparrow = F\downarrow$; $F\uparrow =$ SL

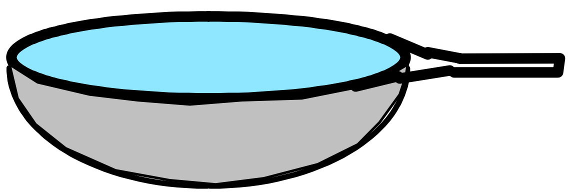
The length is the perimeter, $2\pi r$, and there is two surfaces

$$S \times 2 \times 2\pi r = m_{H2o}\, g = \rho_{H2o}\, Vg = \rho_{H2o} \times \frac{1}{12}\, \pi\, (2r)^{3} g = \rho_{H2o} \times \pi \frac{2}{3} r^{3}$$

$$S = \rho_{H2o} \frac{1}{6} r^2\, g$$

$$= 997\ \text{kg/m}^3 \times \frac{1}{6} \times 0.25^2\ \text{m}^2 \times 9.81\ \text{m/s}^2$$

$$= \mathbf{101.9\ N/m}$$

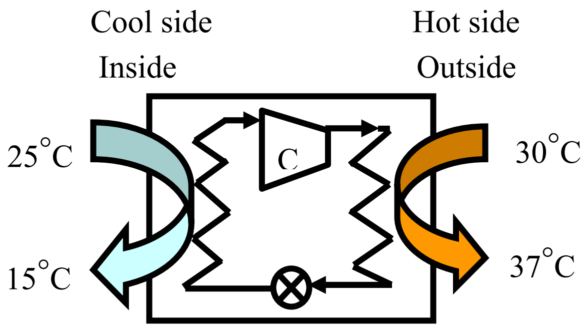


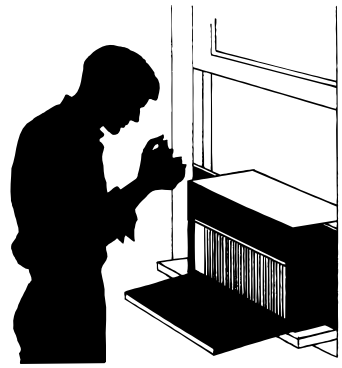
4.83

Consider a window-mounted air conditioning unit used in the summer to cool incoming air. Examine the system boundaries for rates of work and heat transfer, including signs.

Solution :

Air-conditioner unit, steady operation with no change of temperature of AC unit.





- electrical work (power) input operates unit,

+Q rate of heat transfer from the room which actually is room air being cooled,

a larger -Q rate of heat transfer (sum of the other two energy rates) out to the outside air.

4.84

Consider a light bulb that is on. Explain where we have rates of work and heat transfer (include modes) that moves energy.

Solution:

Electrical power comes in, that is rate of work.

In the wire filament this electrical work is converted to internal energy so the wire becomes hot and radiates energy out. There is also some heat transfer by conduction-convection to the gas inside the bulb so that become warm.

The gas in turn heats the glass by conduction/convection and some of the radiation may be absorbed by the glass (often the glass is coated white)

All the energy leaves the bulb as a combination of radiation over a range of wavelengths some of which you cannot see and conduction convection heat transfer to the air around the bulb.

4.85

Consider a household refrigerator that has just been filled up with room-temperature food. Define a control volume (mass) and examine its boundaries for rates of work and heat transfer, including sign.

a. Immediately after the food is placed in the refrigerator

b. After a long period of time has elapsed and the food is cold

Solution:

I. C.V. Food.

a) short term.: -Q from warm food to cold refrigerator air. Food cools.

b) Long term: -Q goes to zero after food has reached refrigerator T.

II. C.V. refrigerator space, not food, not refrigerator system

a) short term: +Q from the warm food, +Q from heat leak from room into cold space. -Q (sum of both) to refrigeration system. If not equal the refrigerator space initially warms slightly and then cools down to preset T.

b) long term: small -Q heat leak balanced by -Q to refrigeration system.

Note: For refrigeration system CV any Q in from refrigerator space plus electrical W input to operate system, sum of which is Q rejected to the room.

4.86

A room is heated with an electric space heater on a winter day. Examine the following control volumes, regarding heat transfer and work , including sign.

a) The space heater.

b) Room

c) The space heater and the room together

Solution:

a) The space heater.

Electrical work (power) input, and equal (after system warm up) Q out to the room.

b) Room

Q input from the heater balances Q loss to the outside, for steady (no temperature change) operation.

c) The space heater and the room together

Electrical work input balances Q loss to the outside, for steady operation.

Rates of work

4.87

A 100 hp car engine has a drive shaft rotating at 2000 RPM. How much torque is on the shaft for 25% of full power?

Solution:

Power = 0.25 × 100 hp = 0.25 × 73.5 kW (if SI hp) = 18.375 kW = Tω

ω = angular velocity (rad/s) = RPM 2 π /60

$$T = \text{Power}/\omega = \frac{\text{power} \times 60}{\text{RPM} \times 2\pi} = \frac{18.375 \text{ kW} \times 60 \text{ s/min}}{2000 \times 2\pi \text{ rad/min}} = \mathbf{87.73\ Nm}$$

We could also have used UK hp to get 0.25 × 74.6 kW. then T = 89 Nm.

4.88

A car uses 25 hp to drive at a horizontal level at constant 100 km/h. What is the traction force between the tires and the road?

Solution:

We need to relate the rate of work to the force and velocity

$$dW = F\,dx \quad => \quad \frac{dW}{dt} = \dot{W} = F\,\frac{dx}{dt} = F\mathbf{V}$$

$$F = \dot{W} / \mathbf{V}$$

$$\dot{W} = 25 \text{ hp} = 25 \times 0.7355 \text{ kW} = 18.39 \text{ kW}$$

$$\mathbf{V} = 100 \times \frac{1000}{3600} = 27.78 \text{ m/s}$$

$$F = \dot{W} / \mathbf{V} = \frac{18.39 \text{ kW}}{27.78 \text{ m/s}} = \mathbf{0.66\ kN}$$

Units: $\text{kW} / (\text{ms}^{-1}) = \text{kW s m}^{-1} = \text{kJ s}^{-1}\text{s m}^{-1} = \text{kN m m}^{-1} = \text{kN}$

4.89

An escalator raises a 100 kg bucket of sand 10 m in 1 minute. Determine the rate of work done during the process.

Solution:

The work is a force with a displacement and force is constant: F = mg

$$W = \int F\,dx = F \int dx = F\,\Delta x = 100\text{ kg} \times 9.80665\text{ m/s}^2 \times 10\text{ m} = 9807\text{ J}$$

The rate of work is work per unit time

$$\dot{W} = \frac{W}{\Delta t} = \frac{9807\text{ J}}{60\text{ s}} = \mathbf{163\ W}$$

4.90

A crane lifts a bucket of cement with a total mass of 450 kg vertically up with a constant velocity of 2 m/s. Find the rate of work needed to do that.

Solution:

Rate of work is force times rate of displacement. The force is due to gravity (a = 0) alone.

$\dot{W} = F\mathbf{V} = mg \times \mathbf{V} = 450 \text{ kg} \times 9.807 \text{ ms}^{-2} \times 2 \text{ ms}^{-1} = 8826 \text{ J/s}$

$\dot{W} =$ **8.83 kW**

4.91

A force of 1.2 kN moves a truck with 60 km/h up a hill. What is the power?

Solution:

$$\dot{W} = F\,\mathbf{V} = 1.2\text{ kN} \times 60\text{ (km/h)}$$

$$= 1.2 \times 10^3 \times 60 \times \frac{10^3}{3600}\ \frac{\text{Nm}}{\text{s}}$$

$$= 20\,000\text{ W} = \mathbf{20\text{ kW}}$$

4.92

A piston/cylinder of cross sectional area 0.01 m^2 maintains constant pressure. It contains 1 kg water with a quality of 5% at 150°C. If we heat so 1 g/s liquid turns into vapor what is the rate of work out?

$V_{vapor} = m_{vapor}\, v_g \,, \quad V_{liq} = m_{liq}\, v_f$

$m_{tot} = \text{constant} = m_{vapor}\, m_{liq}$

$V_{tot} = V_{vapor} + V_{liq}$

$\dot{m}_{tot} = 0 = \dot{m}_{vapor} + \dot{m}_{liq} \quad \Rightarrow \quad \dot{m}_{liq} = -\dot{m}_{vapor}$

$\dot{V}_{tot} = \dot{V}_{vapor} + \dot{V}_{liq} = \dot{m}_{vapor} v_g + \dot{m}_{liq} v_f$

$= \dot{m}_{vapor}\, (v_g - v_f) = \dot{m}_{vapor}\, v_{fg}$

$\dot{W} = P\dot{V} = P\, \dot{m}_{vapor}\, v_{fg}$

$= 475.9$ kPa × 0.001 kg/s × 0.39169 m^3/kg **= 0.1864 kW**

= 186 W

4.93

Consider the car with the rolling resistance as in problem 4.26. How fast can it drive using 30 hp?

$F = 0.006\ mg$

$\text{Power} = F \times \mathbf{V} = 30\ \text{hp} = \dot{W}$

$$\mathbf{V} = \dot{W} / F = \frac{\dot{W}}{0.006\ mg} = \frac{30 \times 0.7457 \times 1000}{0.006 \times 1200 \times 9.81} = \mathbf{271.5\ m/s}$$

Comment: This is a very high velocity, the rolling resistance is low relative to the air resistance.

4.94

Consider the car with the air drag force as in problem 4.27. How fast can it drive using 30 hp?

$$\rho = \frac{1}{v} = \frac{P}{RT} = \frac{100}{0.287 \times 290} = 1.2015 \frac{kg}{m^3} \quad \text{and} \quad A = 4 \; m^2$$

Drag force: $\quad F_{drag} = 0.225 \; A \; \rho \; \mathbf{V}^2$

Power for drag force: $\quad \dot{W}_{drag} = 30 \text{ hp} \times 0.7457 = 22.371 \text{ kW}$

$$\dot{W}_{drag} = F_{drag} \mathbf{V} = 0.225 \times 4 \times 1.2015 \times \mathbf{V}^3$$

$$\mathbf{V}^3 = \dot{W}_{drag} / (0.225 \times 4 \times 1.2015) = 20\,688$$

$$\mathbf{V} = 27.452 \text{ m/s} = 27.452 \times \frac{3600}{1000} = \mathbf{98.8 \; km/h}$$

4.95

Consider a 1400 kg car having the rolling resistance as in problem 4.26 and air resistance as in problem 4.27. How fast can it drive using 30 hp?

$F_{tot} = F_{rolling} + F_{air} = 0.006\ mg + 0.225\ A\rho\mathbf{V}^2$

$m = 1400\ kg\ ,\ A = 4\ m^2$

$\rho = P/RT = 1.2015\ kg/m^3$

$\dot{W} = F\mathbf{V} = 0.006\ mg\mathbf{V} + 0.225\ \rho A\mathbf{V}^3$

Nonlinear in V so solve by trial and error.

$\dot{W} = 30\ hp = 30 \times 0.7355\ kW = 22.06\ kW$

$= 0.0006 \times 1400 \times 9.807\ \mathbf{V} + 0.225 \times 1.2015 \times 4\ \mathbf{V}^3$

$= 82.379V + 1.08135\ \mathbf{V}^3$

$\mathbf{V} = 25\ m/s \Rightarrow \dot{W} = 18\ 956\ W$

$\mathbf{V} = 26\ m/s \quad \dot{W} = 21\ 148\ W$

$\mathbf{V} = 27\ m/s \quad \dot{W} = 23\ 508\ W$

Linear interpolation

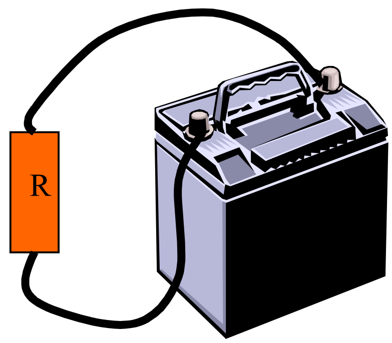
$\mathbf{V} = 26.4\ m/s$ **= 95 km/h**

4.96

A current of 10 amp runs through a resistor with a resistance of 15 ohms. Find the rate of work that heats the resistor up.

Solution:

$$\dot{W} = \text{power} = E\ i = R\ i^2 = 15 \times 10 \times 10 = \mathbf{1500\ W}$$



4.97

A battery is well insulated while being charged by 12.3 V at a current of 6 A. Take the battery as a control mass and find the instantaneous rate of work and the total work done over 4 hours.

Solution :

Battery thermally insulated ⇒ Q = 0

For constant voltage *E* and current i,

Power = *E* i = 12.3 × 6 = **73.8 W** [Units V × A = W]

$$W = \int \text{power } dt = \text{power } \Delta t$$

$$= 73.8 \times 4 \times 60 \times 60 = 1\ 062\ 720 \text{ J} = \mathbf{1062.7\ kJ}$$

4.98

A torque of 650 Nm rotates a shaft of diameter 0.25 m with $\omega = 50$ rad/s. What are the shaft surface speed and the transmitted power?

Solution:

$\mathbf{V} = \omega r = \omega\, D/2 = 50 \times 0.25 / 2 = \mathbf{6.25\ m/s}$

Power $= T\omega = 650 \times 50$ Nm/s $= 32\,500$ W $=$ **32.5 kW**

Recall also Power $= FV = (T/r)\, V = T\, V/r = T\omega$

4.99

Air at a constant pressure in a piston cylinder is at 300 kPa, 300 K and a volume of 0.1 m^3. It is heated to 600 K over 30 seconds in a process with constant piston velocity. Find the power delivered to the piston.

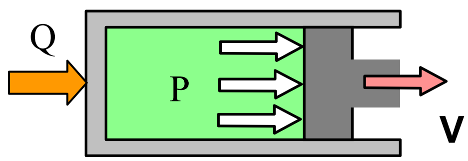
Solution:

Process: P = constant :

Boundary work: $dW = P\,dV \quad => \quad \dot{W} = P\dot{V}$

$V_2 = V_1 \times (T_2/T_1) = 0.1 \times (600/300) = 0.2\ m^3$

$$\dot{W} = P \frac{\Delta V}{\Delta t} = 300 \times \frac{0.2 - 0.1}{30}\ \text{kPa}\ \frac{m^3}{s} = \mathbf{1\ kW}$$



Remark: Since we do not know the area we do not know the velocity

4.100

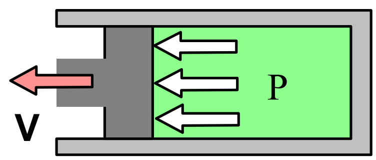
A pressure of 650 kPa pushes a piston of diameter 0.25 m with **V** = 5 m/s. What is the volume displacement rate, the force and the transmitted power?

$$A = \frac{\pi}{4} D^2 = 0.049087 \text{ m}^2$$

$$\dot{V} = A\mathbf{V} = 0049087 \text{ m}^2 \times 5 \text{ m/s} = \mathbf{0.2454\ m^3/s}$$

$$F = P\,A = 650 \text{ kPa} \times 0.049087 \text{ m}^2 = \mathbf{31.9\ kN}$$

$$\dot{W} = \text{power} = F\,\mathbf{V} = P\,\dot{V} = 650 \text{ kPa} \times 0.2454 \text{ m}^3\text{/s} = \mathbf{159.5\ kW}$$



4.101

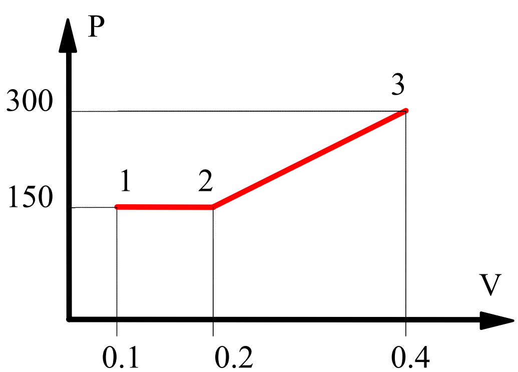
Assume the process in Problem 4.61 takes place with a constant rate of change in volume over 2 minutes. Show the power (rate of work) as a function of time.

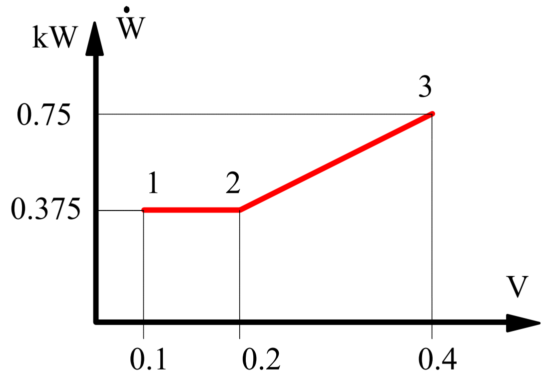
Solution:

$W = \int P\, dV$ since 2 min = 120 sec

$\dot{W} = P\ (\Delta V / \Delta t)$

$(\Delta V / \Delta t) = 0.3\ m^3 / 120\ s = 0.0025\ m^3/s$





Heat Transfer rates

4.102

Find the rate of conduction heat transfer through a 1.5 cm thick hardwood board, k = 0.16 W/m K, with a temperature difference between the two sides of 20^{o}C.

One dimensional heat transfer by conduction, we do not know the area so we can find the flux (heat transfer per unit area W/m^2).

$$\dot{q} = \dot{Q}/A = k \frac{\Delta T}{\Delta x} = 0.16 \frac{W}{m\ K} \times \frac{20}{0.015} \frac{K}{m} = \mathbf{213\ W/m^2}$$

4.103

A pot of steel, conductivity 50 W/m K, with a 5 mm thick bottom is filled with 15°C liquid water. The pot has a diameter of 20 cm and is now placed on an electric stove that delivers 250 W as heat transfer. Find the temperature on the outer pot bottom surface assuming the inner surface is at 15°C.

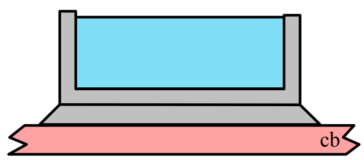
Solution :

Steady conduction through the bottom of the steel pot. Assume the inside surface is at the liquid water temperature.

$$\dot{Q} = k\ A\ \frac{\Delta T}{\Delta x} \quad \Rightarrow \quad \Delta T = \dot{Q}\ \Delta x / kA$$

$$\Delta T = 250\ \ W \times 0.005\ m /(50\ W/m\text{-}K \times \frac{\pi}{4} \times 0.2^2\ m^2) = 0.796\ K$$

$$T = 15 + 0.796 \cong \mathbf{15.8°C}$$



4104

The sun shines on a 150 m^2 road surface so it is at 45°C. Below the 5 cm thick asphalt, average conductivity of 0.06 W/m K, is a layer of compacted rubbles at a temperature of 15°C. Find the rate of heat transfer to the rubbles.

Solution :

This is steady one dimensional conduction through the asphalt layer.

$$\dot{Q} = k\,A\,\frac{\Delta T}{\Delta x}$$

$$= 0.06\,\frac{W}{m\text{-}K} \times 150\ m^2 \times \frac{45\text{-}15}{0.05}\,\frac{K}{m}$$

$$= \mathbf{5400\ W}$$

4.105

A water-heater is covered up with insulation boards over a total surface area of 3 m^2. The inside board surface is at 75°C and the outside surface is at 18°C and the board material has a conductivity of 0.08 W/m K. How thick a board should it be to limit the heat transfer loss to 200 W ?

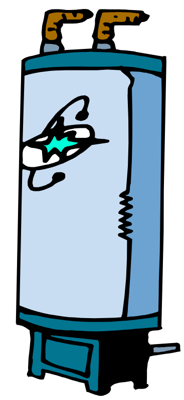
Solution :

Steady state conduction through a single layer board.

$$\dot{Q}_{cond} = k\ A\ \frac{\Delta T}{\Delta x} \quad \Rightarrow \quad \Delta x = k\ A\ \Delta T/\dot{Q}$$

$$\Delta x = 0.08\ \frac{W}{m\ K} \times 3 m^2 \times \frac{75 - 18}{200}\ \frac{K}{W}$$

$$= \mathbf{0.068\ m}$$



4.106

A large condenser (heat exchanger) in a power plant must transfer a total of 100 MW from steam running in a pipe to sea water being pumped through the heat exchanger. Assume the wall separating the steam and seawater is 4 mm of steel, conductivity 15 W/m K and that a maximum of 5°C difference between the two fluids is allowed in the design. Find the required minimum area for the heat transfer neglecting any convective heat transfer in the flows.

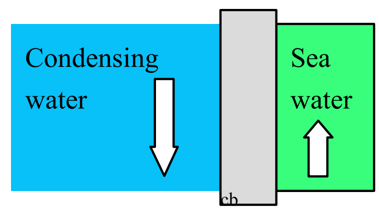
Solution :

Steady conduction through the 4 mm steel wall.

$$\dot{Q} = k\ A\ \frac{\Delta T}{\Delta x} \quad \Rightarrow \quad A = \dot{Q}\ \Delta x / k\Delta T$$

$$A = 100 \times 10^6\ \text{W} \times 0.004\ \text{m} / (15\ \text{W/mK} \times 5\ \text{K})$$

$$= \mathbf{480\ m^2}$$



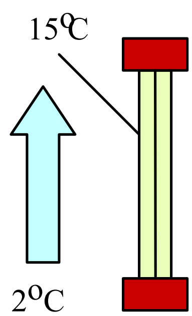
4.107

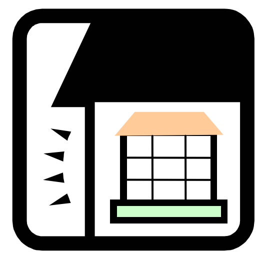
A 2 m^2 window has a surface temperature of 15^{o}C and the outside wind is blowing air at 2^{o}C across it with a convection heat transfer coefficient of h = 125 W/m^2K. What is the total heat transfer loss?

Solution:

$$\dot{Q} = h\ A\ \Delta T = 125\ W/m^2K \times 2\ m^2 \times (15 - 2)\ K = \mathbf{3250\ W}$$

as a rate of heat transfer out.





4.108

You drive a car on a winter day with the atmospheric air at –15°C and you keep the outside front windshield surface temperature at +2°C by blowing hot air on the inside surface. If the windshield is 0.5 m^2 and the outside convection coefficient is 250 W/m^2K find the rate of energy loos through the front windshield. For that heat transfer rate and a 5 mm thick glass with k = 1.25 W/m K what is then the inside windshield surface temperature?

Solution :

The heat transfer from the inside must match the loss on the outer surface to give a steady state (frost free) outside surface temperature.

$$\dot{Q}_{conv} = h\ A\ \Delta T = 250 \times 0.5 \times [2 - (-15)]$$

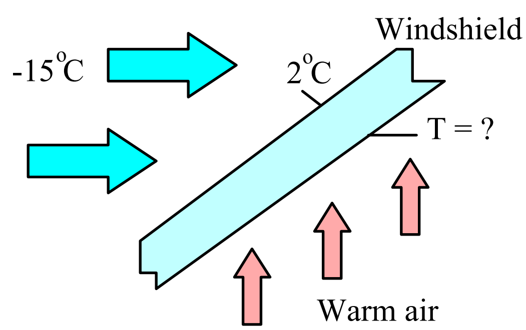
$$= 250 \times 0.5 \times 17 = \mathbf{2125\ W}$$

This is a substantial amount of power.

$$\dot{Q}_{cond} = k\ A\ \frac{\Delta T}{\Delta x} \qquad \Rightarrow \qquad \Delta T = \frac{\dot{Q}}{kA}\ \Delta x$$

$$\Delta T = \frac{2125\ \text{W}}{1.25\ \text{W/mK} \times 0.5\ \text{m}^2} \times 0.005\ \text{m} = 17\ \text{K}$$

$$T_{in} = T_{out} + \Delta T = 2 + 17 = \mathbf{19°C}$$



4.109

The brake shoe and steel drum on a car continuously absorbs 25 W as the car slows down. Assume a total outside surface area of 0.1 m^2 with a convective heat transfer coefficient of 10 W/m^2 K to the air at 20°C. How hot does the outside brake and drum surface become when steady conditions are reached?

Solution :

$$\dot{Q} = hA\Delta T \quad \Rightarrow \quad \Delta T = \frac{\dot{Q}}{hA}$$

$$\Delta T = (T_{BRAKE} - 20) = \frac{25}{10 \times 0.1} = \mathbf{25\ °C}$$

$$T_{BRAKE} = 20 + 25 = \mathbf{45°C}$$

4.110

Due to a faulty door contact the small light bulb (25 W) inside a refrigerator is kept on and limited insulation lets 50 W of energy from the outside seep into the refrigerated space. How much of a temperature difference to the ambient at 20°C must the refrigerator have in its heat exchanger with an area of 1 m^2 and an average heat transfer coefficient of 15 W/m^2 K to reject the leaks of energy.

Solution :

$$\dot{Q}_{tot} = 25 + 50 = 75 \text{ W to go out}$$

$$\dot{Q} = hA\,\Delta T = 15 \times 1 \times \Delta T = 75 \text{ W}$$

$$\Delta T = \frac{\dot{Q}}{hA} = \frac{75}{15 \times 1} = 5\ °C$$

OR T must be at least **25 °C**

4.111

The black grille on the back of a refrigerator has a surface temperature of 35°C with a total surface area of 1 m^2. Heat transfer to the room air at 20°C takes place with an average convective heat transfer coefficient of 15 W/m^2 K. How much energy can be removed during 15 minutes of operation?

Solution :

$$\dot{Q} = hA\ \Delta T; \quad Q = \dot{Q}\ \Delta t = hA\ \Delta T\ \Delta t$$

$$Q = 15\ W/m^2\ K \times 1\ m^2 \times (35\text{-}20)\ K \times 15\ min \times 60\ s/min$$

$$= 202\ 500\ J = \mathbf{202.5\ kJ}$$

4.112

A wall surface on a house is at 30°C with an emissivity of $\varepsilon = 0.7$. The surrounding ambient to the house is at 15°C, average emissivity of 0.9. Find the rate of radiation energy from each of those surfaces per unit area.

Solution :

$$\dot{Q}/A = \varepsilon \sigma A T^4, \quad \sigma = 5.67 \times 10^{-8} \ \mathrm{W/m^2K^4}$$

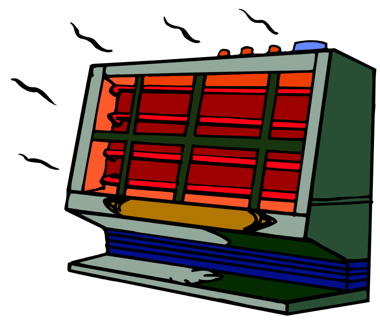
a) $\dot{Q}/A = 0.7 \times 5.67 \times 10^{-8} \times (273.15 + 30)^4 = \mathbf{335\ W/m^2}$

b) $\dot{Q}/A = 0.9 \times 5.67 \times 10^{-8} \times 288.15^4 = \mathbf{352\ W/m^2}$

4.113

A radiant heating lamp has a surface temperature of 1000 K with $\varepsilon = 0.8$. How large a surface area is needed to provide 250 W of radiation heat transfer?

Radiation heat transfer. We do not know the ambient so let us find the area for an emitted radiation of 250 W from the surface



$$\dot{Q} = \varepsilon\sigma A T^4$$

$$A = \frac{\dot{Q}}{\varepsilon\sigma T^4} = \frac{250}{0.8 \times 5.67 \times 10^{-8} \times 1000^4} \frac{\text{W}}{\text{W/m}^2}$$

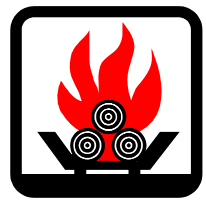
$$= \mathbf{0.0055\ m^2}$$

4.114

A log of burning wood in the fireplace has a surface temperature of 450°C. Assume the emissivity is 1 (perfect black body) and find the radiant emission of energy per unit surface area.

Solution :

$$\dot{Q}/A = 1 \times \sigma\ T^4$$
$$= 5.67 \times 10^{-8} \times (273.15 + 450)^4$$
$$= 15\ 505\ \text{W/m}^2$$
$$= \mathbf{15.5\ kW/m^2}$$



4.115

A radiant heat lamp is a rod, 0.5 m long and 0.5 cm in diameter, through which 400 W of electric energy is deposited. Assume the surface has an emissivity of 0.9 and neglect incoming radiation. What will the rod surface temperature be ?

Solution :

For constant surface temperature outgoing power equals electric power.

$\dot{Q}_{rad} = \varepsilon\sigma A T^4 = \dot{Q}_{el} \Rightarrow$

$T^4 = \dot{Q}_{el} / \varepsilon\sigma A = 400 / (0.9 \times 5.67 \times 10^{-8} \times 0.5 \times \pi \times 0.005)$

$= 9.9803 \times 10^{11}\ K^4 \quad \Rightarrow \quad T \cong$ **1000 K OR 725 °C**

Review Problems

4.116

A nonlinear spring has the force versus displacement relation of $F = k_{ns}(x - x_0)^n$. If the spring end is moved to x_1 from the relaxed state, determine the formula for the required work.

Solution:

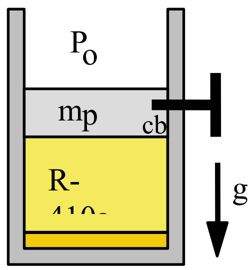
In this case we know F as a function of x and can integrate

$$W = \int F dx = \int k_{ns}(x - x_o)^n \, d(x - x_o) = \frac{k_{ns}}{n + 1} (x_1 - x_o)^{n+1}$$

4.117

A vertical cylinder (Fig. P4.117) has a 61.18-kg piston locked with a pin trapping 10 L of R-410a at 10°C, 90% quality inside. Atmospheric pressure is 100 kPa, and the cylinder cross-sectional area is 0.006 m^2. The pin is removed, allowing the piston to move and come to rest with a final temperature of 10°C for the R-410a. Find the final pressure, final volume and the work done by the R-410a.

Solution:



State 1: (T, x) from table B.4.1

$$v_1 = 0.0008 + 0.9 \times 0.03391 = 0.03132 \text{ m}^3/\text{kg}$$

$$m = V_1/v_1 = 0.010/0.03132 = 0.319 \text{ kg}$$

Force balance on piston gives the equilibrium pressure

$$P_2 = P_0 + m_P g/ A_P = 100 + \frac{61.18 \times 9.807}{0.006 \times 1000} = \mathbf{200\ kPa}$$

State 2: (T,P) in Table B.4.2 $v_2 = 0.13129 \text{ m}^3/\text{kg}$

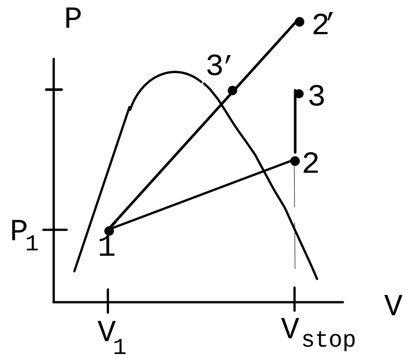
$$V_2 = mv_2 = 0.319 \text{ kg} \times 0.13129 \text{ m}^3/\text{kg} = 0.04188 \text{ m}^3 = \mathbf{41.88\ L}$$

$$_1W_2 = \int P_{equil}\, dV = P_2(V_2\text{-}V_1) = 200 \text{ kPa } (0.04188\text{- } 0.010) \text{ m}^3 = \mathbf{6.38\ kJ}$$

4.118

Two kilograms of water is contained in a piston/cylinder (Fig. P4.110) with a massless piston loaded with a linear spring and the outside atmosphere. Initially the spring force is zero and $P_1 = P_o = 100$ kPa with a volume of 0.2 m^3. If the piston just hits the upper stops the volume is 0.8 m^3 and $T = 600°C$. Heat is now added until the pressure reaches 1.2 MPa. Find the final temperature, show the *P–V* diagram and find the work done during the process.

Solution:



State 1: $v_1 = V/m = 0.2 / 2 = 0.1$ m^3/kg

Process: $1 \rightarrow 2 \rightarrow 3$ or $1 \rightarrow 3'$

State at stops: 2 or 2'

$v_2 = V_{stop}/m = 0.4$ m^3/kg & $T_2 = 600°C$

Table B.1.3 $\Rightarrow$ $P_{stop} = 1$ MPa $< P_3$

since $P_{stop} < P_3$ the process is as $1 \rightarrow 2 \rightarrow 3$

State 3: $P_3 = 1.2$ MPa, $v_3 = v_2 = 0.4$ m^3/kg $\Rightarrow$ $T_3 \cong$ **770°C**

$$W_{13} = W_{12} + W_{23} = \frac{1}{2}(P_1 + P_2)(V_2 - V_1) + 0$$

$$= \frac{1}{2}(100 + 1000) \text{ kPa} \times (0.8 - 0.2) \text{ m}^3$$

= **330 kJ**

4.119

A piston/cylinder contains butane, C_4H_{10}, at 300°C, 100 kPa with a volume of 0.02 m^3. The gas is now compressed slowly in an isothermal process to 300 kPa.

a. Show that it is reasonable to assume that butane behaves as an ideal gas during this process.

b. Determine the work done by the butane during the process.

Solution:

a) $T_{r1} = \frac{T}{T_c} = \frac{573.15}{425.2} = 1.35;$ $\quad P_{r1} = \frac{P}{P_c} = \frac{100}{3800} = 0.026$

From the generalized chart in figure D.1 $\quad Z_1 = 0.99$

$T_{r2} = \frac{T}{T_c} = \frac{573.15}{425.2} = 1.35;$ $\quad P_{r2} = \frac{P}{P_c} = \frac{300}{3800} = 0.079$

From the generalized chart in figure D.1 $\quad Z_2 = 0.98$

Ideal gas model is adequate for both states.

b) Ideal gas $\quad T = \text{constant} \;\Rightarrow\; PV = mRT = \text{constant}$

$$W = \int P\,dV = P_1V_1 \ln\frac{P_1}{P_2} = 100\,\text{kPa} \times 0.02\ m^3 \times \ln\frac{100}{300}$$

$= \mathbf{-2.2\ kJ}$

4.120

Consider the process described in Problem 3.116. With 1 kg water as a control mass, determine the boundary work during the process.

A spring-loaded piston/cylinder contains water at 500°C, 3 MPa. The setup is such that pressure is proportional to volume, *P = CV*. It is now cooled until the water becomes saturated vapor. Sketch the *P-v* diagram and find the final pressure.

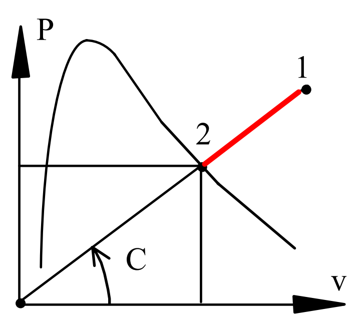
Solution :

State 1: Table B.1.3: $v_1 = 0.11619 \text{ m}^3/\text{kg}$

Process: m is constant and $P = C_0V = C_0 m\, v = C\, v$

$P = Cv \quad \Rightarrow \quad C = P_1/v_1 = 3000/0.11619 = 25820 \text{ kPa kg/m}^3$

State 2: $x_2 = 1$ & $P_2 = Cv_2$ (on process line)



Trial & error on T_{2sat} or P_{2sat}:

Here from B.1.2:

at 2 MPa $v_g = 0.09963 \Rightarrow C = P/v_g = 20074$ (low)

2.5 MPa $v_g = 0.07998 \Rightarrow C = P/v_g = 31258$ (high)

2.25 MPa $v_g = 0.08875 \Rightarrow C = P/v_g = 25352$ (low)

Now interpolate to match the right slope C:

$P_2 = 2270$ kPa, $v_2 = P_2/C = 2270/25820 = 0.0879 \text{ m}^3/\text{kg}$

P is linear in V so the work becomes (area in P-v diagram)

$${}_1w_2 = \int P\, dv = \frac{1}{2}(P_1 + P_2)(v_2 - v_1)$$

$$= \frac{1}{2}(3000 + 2270)(0.0879 - 0.11619) = \mathbf{-\ 74.5\ kJ/kg}$$

4.121

A cylinder having an initial volume of 3 m^3 contains 0.1 kg of water at 40°C. The water is then compressed in an isothermal quasi-equilibrium process until it has a quality of 50%. Calculate the work done in the process splitting it into two steps. Assume the water vapor is an ideal gas during the first step of the process.

Solution: C.V. Water

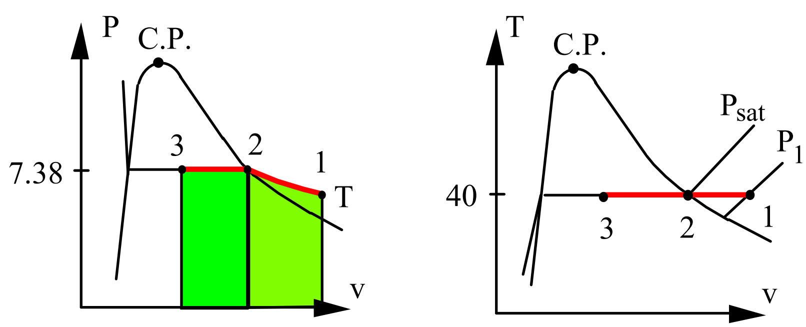
State 2: (40°C, x = 1) Tbl B.1.1 => P_G = 7.384 kPa, $v_G = 19.52$ m^3/kg

State 1: $v_1 = V_1/m = 3 / 0.1 = 30$ m^3/kg (> v_G)

so H_2O ~ ideal gas from 1-2 so since constant T

$$P_1 = P_G \frac{v_G}{v_1} = 7.384 \times \frac{19.52}{30} = 4.8 \text{ kPa}$$

$$V_2 = mv_2 = 0.1 \times 19.52 = 1.952 \text{ m}^3$$



Process T = C: and ideal gas gives work from Eq.4.5

$${}_1W_2 = \int_1^2 PdV = P_1V_1\ln\frac{V_2}{V_1} = 4.8 \times 3.0 \times \ln\frac{1.952}{3} = -6.19 \text{ kJ}$$

$$v_3 = 0.001008 + 0.5 \times 19.519 = 9.7605 \quad \Rightarrow \quad V_3 = mv_3 = 0.976 \text{ m}^3$$

$P = C = P_g$: This gives a work term as

$${}_2W_3 = \int_2^3 PdV = P_g\,(V_3 - V_2) = 7.384 \text{ kPa } (0.976 - 1.952) \text{ m}^3 = -7.21 \text{ kJ}$$

Total work:

$${}_1W_3 = {}_1W_2 + {}_2W_3 = -6.19 - 7.21 = \mathbf{-13.4\ kJ}$$

4.122

A piston/cylinder (Fig. P4.72) contains 1 kg of water at 20°C with a volume of 0.1 m^3. Initially the piston rests on some stops with the top surface open to the atmosphere, P_o and a mass so a water pressure of 400 kPa will lift it. To what temperature should the water be heated to lift the piston? If it is heated to saturated vapor find the final temperature, volume and the work, ${}_1W_2$.

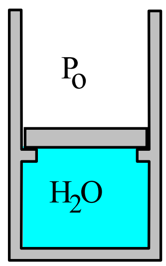
Solution:

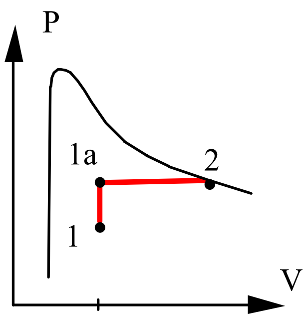
(a) State to reach lift pressure of P = 400 kPa, v = V/m = 0.1 m^3/kg

Table B.1.2: $v_f < v < v_g = 0.4625\ m^3/kg$

=> $T = T_{sat}$ = **143.63°C**

(b) State 2 is saturated vapor at 400 kPa since state 1a is two-phase.





$v_2 = v_g = 0.4625\ m^3/kg$, $V_2 = m\, v_2 = 0.4625\ m^3$,

Pressure is constant as volume increase beyond initial volume.

${}_1W_2 = \int P\, dV = P\,(V_2 - V_1) = P_{lift}\,(V_2 - V_1)$

$= 400\ kPa\ (0.4625 - 0.1)\ m^3 =$ **145 kJ**

4.123

Find the work for Problem 3.112.

Refrigerant-410a in a piston/cylinder arrangement is initially at 50°C, $x = 1$. It is then expanded in a process so that $P = Cv^{-1}$ to a pressure of 100 kPa. Find the final temperature and specific volume.

Solution:

Knowing the process (P versus V) and states 1 and 2 allows calculation of W.

State 1: 50°C, x = 1 Table B.4.1: $P_1 = 3065.2$ kPa, $v_1 = 0.00707 \text{ m}^3/\text{kg}$

Process: $Pv = C = P_1v_1$; => $P_2 = C/v_2 = P_1v_1/v_2$

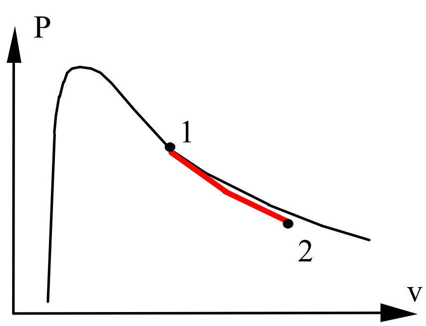
State 2: 100 kPa and $v_2 = v_1P_1/P_2 = 0.2167 \text{ m}^3/\text{kg}$

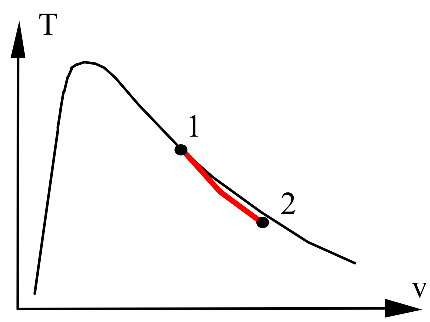
$v_2 < v_g$ at 100 kPa, $T_2 \cong$ **-51.65°C** from Table B.3.2

Notice T is **not** constant. It is not an ideal gas in this range

The constant C for the work term is P_1v_1 so per unit mass we get

$$_1w_2 = P_1v_1 \ln\frac{v_2}{v_1} = 3065.2 \times 0.0707 \times \ln\frac{0.2167}{0.0707} = \mathbf{242.7\ kJ/kg}$$





4.124

A cylinder fitted with a piston contains propane gas at 100 kPa, 300 K with a volume of 0.2 m^3. The gas is now slowly compressed according to the relation $PV^{1.1}$ = constant to a final temperature of 340 K. Justify the use of the ideal gas model. Find the final pressure and the work done during the process.

Solution:

The process equation and T determines state 2. Use ideal gas law to say

$$P_2 = P_1 \left(\frac{T_2}{T_1}\right)^{\frac{n}{n-1}} = 100 \left(\frac{340}{300}\right)^{\frac{1.1}{0.1}} = \mathbf{396\ kPa}$$

$$V_2 = V_1 \left(\frac{P_1}{P_2}\right)^{1/n} = 0.2 \left(\frac{100}{396}\right)^{1/1.1} = 0.0572\ m^3$$

For propane Table A.2: T_c = 370 K, P_c = 4260 kPa, Figure D.1 gives Z.

$T_{r1} = 0.81$, $P_{r1} = 0.023$ => $Z_1 = 0.98$

$T_{r2} = 0.92$, $P_{r2} = 0.093$ => $Z_2 = 0.95$

Ideal gas model **OK** for both states, minor corrections could be used. The work is integrated to give Eq.4.4

$$_1W_2 = \int P\,dV = \frac{P_2V_2 - P_1V_1}{1-n} = \frac{(396 \times 0.0572) - (100 \times 0.2)}{1 - 1.1}\ \text{kPa m}^3$$

$$= \mathbf{-26.7\ kJ}$$

4.125

Consider the nonequilibrium process described in Problem 3.122. Determine the work done by the carbon dioxide in the cylinder during the process.

A cylinder has a thick piston initially held by a pin as shown in Fig. P3.122. The cylinder contains carbon dioxide at 200 kPa and ambient temperature of 290 K. The metal piston has a density of 8000 kg/m^3 and the atmospheric pressure is 101 kPa. The pin is now removed, allowing the piston to move and after a while the gas returns to ambient temperature. Is the piston against the stops?

Solution:

Knowing the process (P vs. V) and the states 1 and 2 we can find W.

If piston floats or moves:

$$P = P_{lift} = P_o + \rho Hg = 101.3 + 8000 \times 0.1 \times 9.807 / 1000 = 108.8 \text{ kPa}$$

Assume the piston is at the stops (since $P_1 > P_{lift}$ piston would move)

$V_2 = V_1 \times 150 / 100 = (\pi/4)\ 0.1^2 \times 0.1 \times 1.5 = 0.000785 \times 1.5 = 0.001\ 1775\ m^3$

For max volume we must have $P > P_{lift}$ so check using ideal gas and constant T process: $P_2 = P_1 V_1 / V_2 = 200/1.5 = 133 \text{ kPa} > P_{lift}$ and piston is at stops.

$$_1W_2 = \int P_{lift}\, dV = P_{lift} (V_2 - V_1) = 108.8 \text{ kPa } (0.0011775 - 0.000785)\ m^3$$
$$= \mathbf{0.0427\ kJ}$$

Remark: The work is determined by the equilibrium pressure, P_{lift}, and not the instantaneous pressure that will accelerate the piston (give it kinetic energy). We need to consider the quasi-equilibrium process to get W.

4.126

The gas space above the water in a closed storage tank contains nitrogen at 25°C, 100 kPa. Total tank volume is 4 m^3, and there is 500 kg of water at 25°C. An additional 500 kg water is now forced into the tank. Assuming constant temperature throughout, find the final pressure of the nitrogen and the work done on the nitrogen in this process.

Solution:

The water is compressed liquid and in the process the pressure goes up so the water stays as liquid. Incompressible so the specific volume does not change. The nitrogen is an ideal gas and thus highly compressible.

State 1: $V_{H_2O\,1} = 500 \times 0.001003 = 0.5015 \text{ m}^3$

$V_{N_2\,1} = 4.0 - 0.5015 = 3.4985 \text{ m}^3$

State 2: $V_{N_2\,2} = 4.0 - 2 \times 0.5015 = 2.997 \text{ m}^3$

Process: T = C gives $P_1V_1 = mRT = P_2V_2$

$$P_{N_2\,2} = 100 \times \frac{3.4985}{2.997} = \mathbf{116.7\ kPa}$$

Constant temperature gives P = mRT/V i.e. pressure inverse in V for which the work term is integrated to give Eq.4.5

$$W_{by\ N_2} = \int_1^2 P_{N_2} dV_{N_2} = P_1V_1 \ln(V_2/V_1)$$

$$= 100 \times 3.4985 \times \ln\frac{2.997}{3.4985} = \mathbf{-54.1\ kJ}$$

4.127

Consider the problem of inflating the helium balloon, as described in problem 3.124. For a control volume that consists of the helium inside the balloon determine the work done during the filling process when the diameter changes from 1 m to 4 m.

Solution :

Inflation at constant $P = P_0 = 100$ kPa to $D_1 = 1$ m, then

$$P = P_0 + C\,(D^{*\,-1} - D^{*\,-2}), \qquad D^* = D / D_1,$$

to $D_2 = 4$ m, $P_2 = 400$ kPa, from which we find the constant C as:

$$400 = 100 + C[\ (1/4) - (1/4)^2\ ] \quad => \quad C = 1600 \text{ kPa}$$

The volumes are: $V = \frac{\pi}{6} D^3 \quad => \quad V_1 = 0.5236 \text{ m}^3; \quad V_2 = 33.51 \text{ m}^3$

$$W_{CV} = \int_1^2 P dV$$

$$= P_0(V_2 - V_1) + \int_1^2 C(D^{*\,-1} - D^{*\,-2})dV$$

$$V = \frac{\pi}{6} D^3, \qquad dV = \frac{\pi}{2} D^2\, dD = \frac{\pi}{2} D_1^{\,3}\, D^{*\,2}\, dD^*$$

$$\Rightarrow W_{CV} = P_0(V_2 - V_1) + 3CV_1 \int_{D_1^*=1}^{D_2^*=4} (D^*-1)dD^*$$

$$= P_0(V_2 - V_1) + 3CV_1\Big[\frac{D_2^{*\,2} - D_1^{*\,2}}{2} - (D_2^* - D_1^*)\Big]_1^4$$

$$= 100 \times (33.51 - 0.5236) + 3 \times 1600 \times 0.5236\ [\frac{16-1}{2} - (4-1)]$$

= **14 608 kJ**

4.128

Air at 200 kPa, 30°C is contained in a cylinder/piston arrangement with initial volume 0.1 m^3 . The inside pressure balances ambient pressure of 100 kPa plus an externally imposed force that is proportional to $V^{0.5}$. Now heat is transferred to the system to a final pressure of 225 kPa. Find the final temperature and the work done in the process.

Solution:

C.V. Air. This is a control mass. Use initial state and process to find T_2

$P_1 = P_0 + CV^{1/2}$; $200 = 100 + C(0.1)^{1/2}$, $C = 316.23 \Rightarrow$

$225 = 100 + CV_2^{1/2} \Rightarrow V_2 = 0.156 \text{ m}^3$

$$P_2V_2 = mRT_2 = \frac{P_1V_1}{T_1} T_2 \Rightarrow$$

$T_2 = (P_2V_2 / P_1V_1)\, T_1 = 225 \times 0.156 \times 303.15 / (200 \times 0.1) = 532 \text{ K} = 258.9°\text{C}$

$$W_{12} = \int P\, dV = \int \left(P_0 + CV^{1/2}\right) dV$$

$$= P_0 (V_2 - V_1) + C \times \frac{2}{3} \times (V_2^{3/2} - V_1^{3/2})$$

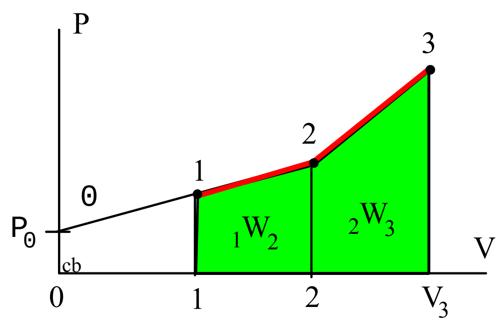
$$= 100\,(0.156 - 0.1) + 316.23 \times \frac{2}{3} \times (0.156^{3/2} - 0.1^{3/2})$$

$= 5.6 + 6.32 =$ **11.9 kJ**

4.129

Two springs with same spring constant are installed in a massless piston/cylinder with the outside air at 100 kPa. If the piston is at the bottom, both springs are relaxed and the second spring comes in contact with the piston at $V = 2\ m^3$. The cylinder (Fig. P4.129) contains ammonia initially at −2°C, $x = 0.13$, $V = 1\ m^3$, which is then heated until the pressure finally reaches 1200 kPa. At what pressure will the piston touch the second spring? Find the final temperature and the total work done by the ammonia.

Solution :



State 1: P = 399.7 kPa Table B.2.1

$v = 0.00156 + 0.13 \times 0.3106 = 0.0419$

At bottom state 0: $0\ m^3$, 100 kPa

State 2: $V = 2\ m^3$ and on line 0-1-2

Final state 3: 1200 kPa, on line segment 2.

Slope of line 0-1-2: $\Delta P/\Delta V = (P_1 - P_0)/\Delta V = (399.7\text{-}100)/1 = 299.7\ kPa/m^3$

$P_2 = P_1 + (V_2 - V_1)\Delta P/\Delta V = 399.7 + (2\text{-}1) \times 299.7 =$ **699.4 kPa**

State 3: Last line segment has twice the slope.

$P_3 = P_2 + (V_3 - V_2)2\Delta P/\Delta V \quad \Rightarrow \quad V_3 = V_2 + (P_3 - P_2)/(2\Delta P/\Delta V)$

$V_3 = 2 + (1200\text{-}699.4)/599.4 = 2.835\ m^3$

$v_3 = v_1 V_3/V_1 = 0.0419 \times 2.835/1 = 0.1188 \quad \Rightarrow \quad T =$ **51°C**

$${}_1W_3 = {}_1W_2 + {}_2W_3 = \frac{1}{2}(P_1 + P_2)(V_2 - V_1) + \frac{1}{2}(P_3 + P_2)(V_3 - V_2)$$

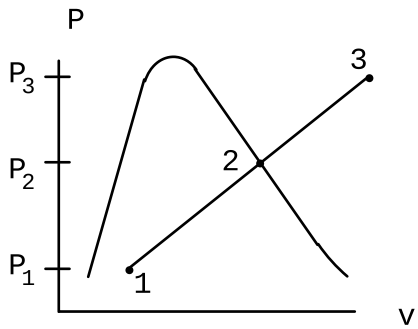
$= 549.6 + 793.0 =$ **1342.6 kJ**

4.130

A spring-loaded piston/cylinder arrangement contains R-134a at 20°C, 24% quality with a volume 50 L. The setup is heated and thus expands, moving the piston. It is noted that when the last drop of liquid disappears the temperature is 40°C. The heating is stopped when $T = 130°C$. Verify the final pressure is about 1200 kPa by iteration and find the work done in the process.

Solution:

C.V. R-134a. This is a control mass.



State 1: Table B.5.1 =>

$v_1 = 0.000817 + 0.24*0.03524 = 0.009274$

$P_1 = 572.8$ kPa,

$m = V/ v_1 = 0.050 / 0.009274 = 5.391$ kg

Process: Linear Spring

$P = A + Bv$

State 2: $x_2 = 1$, T_2 $\Rightarrow$ $P_2 = 1.017$ MPa, $v_2 = 0.02002$ m^3/kg

Now we have fixed two points on the process line so for final state 3:

$$P_3 = P_1 + \frac{P_2 - P_1}{v_2 - v_1}(v_3 - v_1) = \text{RHS} \qquad \text{Relation between } P_3 \text{ and } v_3$$

State 3: T_3 and on process line $\Rightarrow$ iterate on P_3 given T_3

at $P_3 = 1.2$ MPa => $v_3 = 0.02504$ => $P_3 - \text{RHS} = -0.0247$

at $P_3 = 1.4$ MPa => $v_3 = 0.02112$ => $P_3 - \text{RHS} = 0.3376$

Linear interpolation gives :

$$P_3 \cong 1200 + \frac{0.0247}{0.3376 + 0.0247}(1400\text{-}1200) = 1214 \text{ kPa}$$

$$v_3 = 0.02504 + \frac{0.0247}{0.3376 + 0.0247}(0.02112\text{-}0.02504) = 0.02478 \text{ m}^3/\text{kg}$$

$$W_{13} = \int P\, dV = \frac{1}{2}(P_1 + P_3)(V_3 - V_1) = \frac{1}{2}(P_1 + P_3)\, m\, (v_3 - v_1)$$

$$= \frac{1}{2}\ 5.391 \text{ kg } (572.8 + 1214) \text{ kPa } (0.02478 - 0.009274) \text{ m}^3/\text{kg}$$

$= \mathbf{74.7\ kJ}$