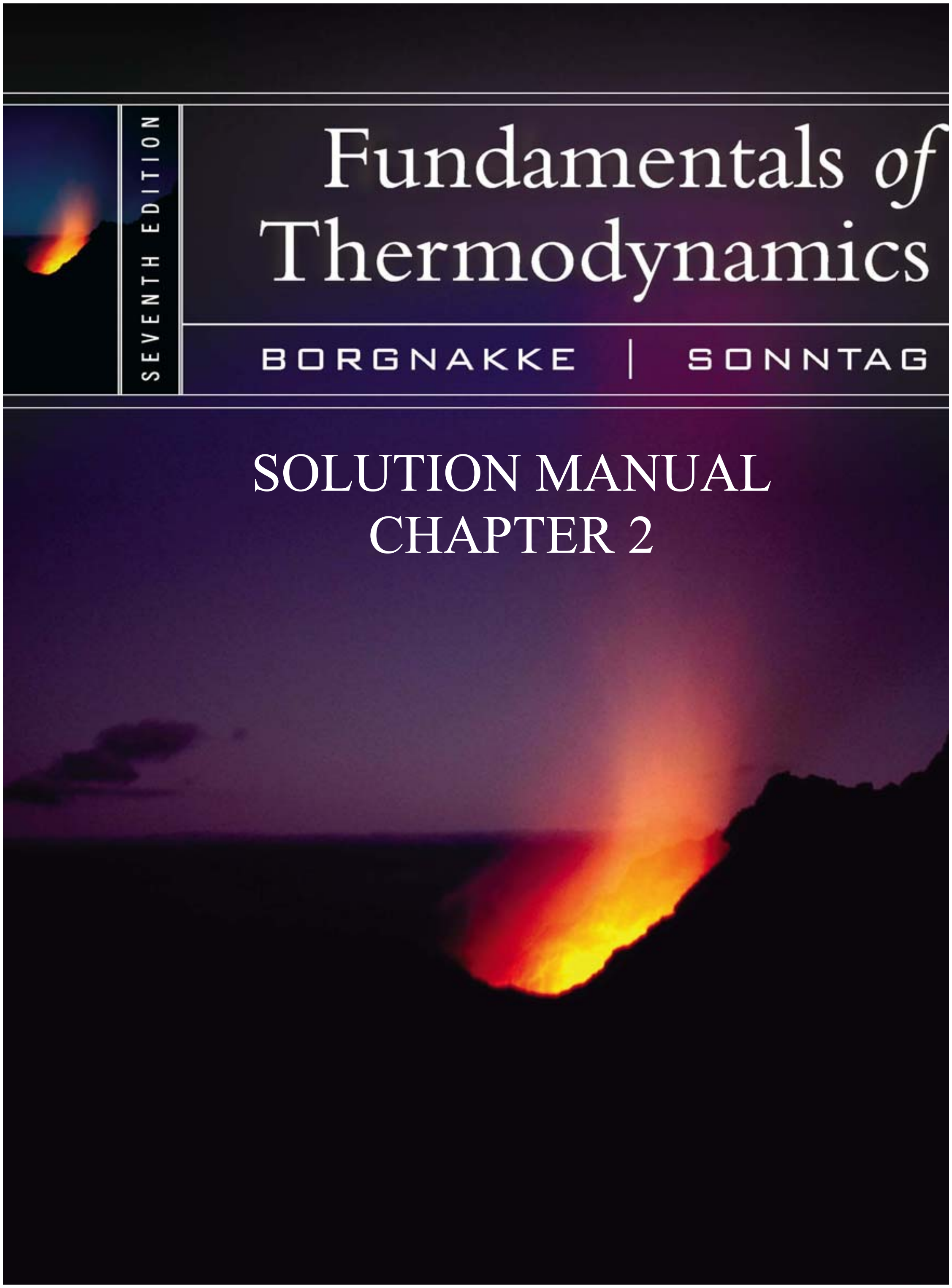

SEVENTH EDITION
Fundamentals of Thermodynamics
BORGNAKKE | SONNTAG
SOLUTION MANUAL
CHAPTER 2

CONTENT

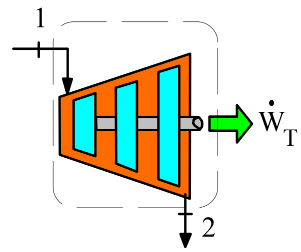
In-Text Concept Questions

2.a

Make a control volume around the turbine in the steam power plant in Fig. 1.1 and list the flows of mass and energy that are there.

Solution:

We see hot high pressure steam flowing in at state 1 from the steam drum through a flow control (not shown). The steam leaves at a lower pressure to the condenser (heat exchanger) at state 2. A rotating shaft gives a rate of energy (power) to the electric generator set.

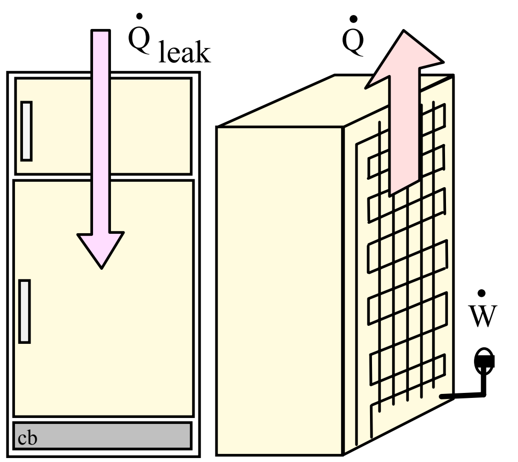


2.b

Take a control volume around your kitchen refrigerator and indicate where the components shown in Figure 1.6 are located and show all flows of energy transfer.

Solution:

The valve and the cold line, the evaporator, is inside close to the inside wall and usually a small blower distributes cold air from the freezer box to the refrigerator room.



The black grille in the back or at the bottom is the condenser that gives heat to the room air.

The compressor sits at the bottom.

2.c

Why do people float high in the water when swimming in the Dead Sea as compared with swimming in a fresh water lake?

As the dead sea is very salty its density is higher than fresh water density. The buoyancy effect gives a force up that equals the weight of the displaced water. Since density is higher the displaced volume is smaller for the same force.

2.d

Density of liquid water is $\rho = 1008 - T/2$ [kg/m^3] with T in ^{o}C. If the temperature increases, what happens to the density and specific volume?

Solution:

The density is seen to decrease as the temperature increases.

$$\Delta\rho = -\Delta T/2$$

Since the specific volume is the inverse of the density $v = 1/\rho$ it will increase.

2.e

A car tire gauge indicates 195 kPa; what is the air pressure inside?

The pressure you read on the gauge is a gauge pressure, ΔP, so the absolute pressure is found as

`

$$P = P_o + \Delta P = 101 + 195 = 296 \text{ kPa}$$

2.f

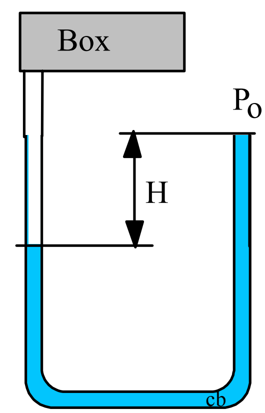
Can I always neglect ΔP in the fluid above location A in figure 2.12? What does that depend on?

If the fluid density above A is low relative to the manometer fluid then you neglect the pressure variation above position A, say the fluid is a gas like air and the manometer fluid is like liquid water. However, if the fluid above A has a density of the same order of magnitude as the manometer fluid then the pressure variation with elevation is as large as in the manometer fluid and it must be accounted for.

2.g

A U tube manometer has the left branch connected to a box with a pressure of 110 kPa and the right branch open. Which side has a higher column of fluid?

Solution:



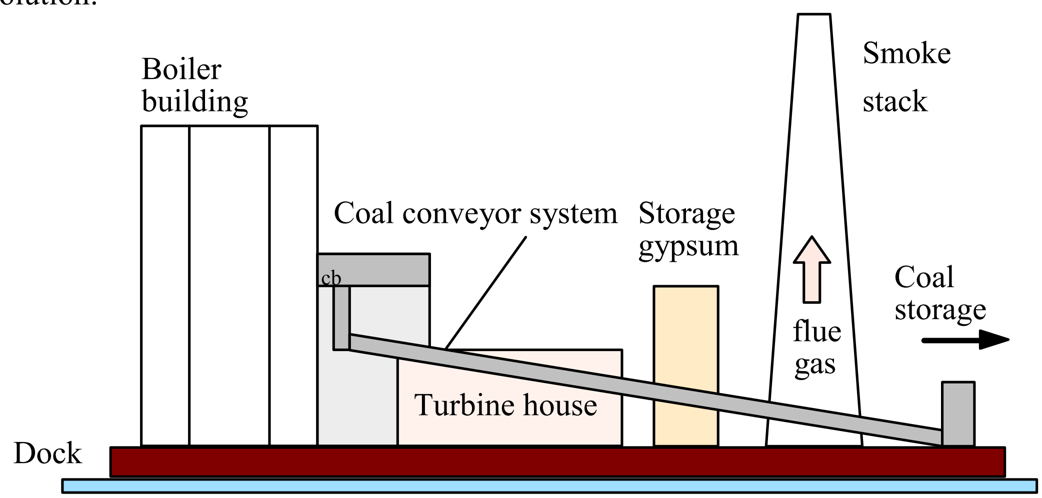
Since the left branch fluid surface feels 110 kPa and the right branch surface is at 100 kPa you must go further down to match the 110 kPa. The right branch has a higher column of fluid.

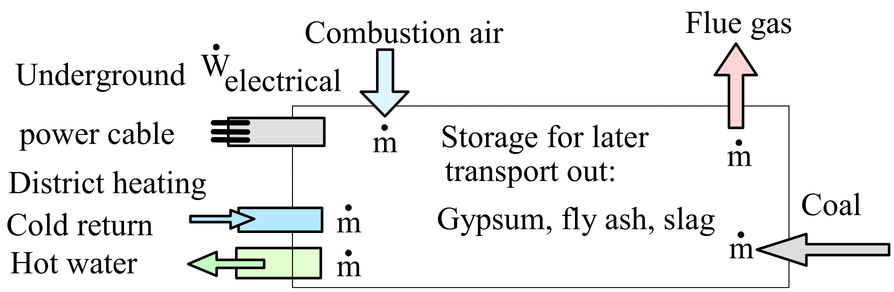
Concept Problems

2.1

Make a control volume around the whole power plant in Fig. 1.2 and with the help of Fig. 1.1 list what flows of mass and energy are in or out and any storage of energy. Make sure you know what is inside and what is outside your chosen C.V.

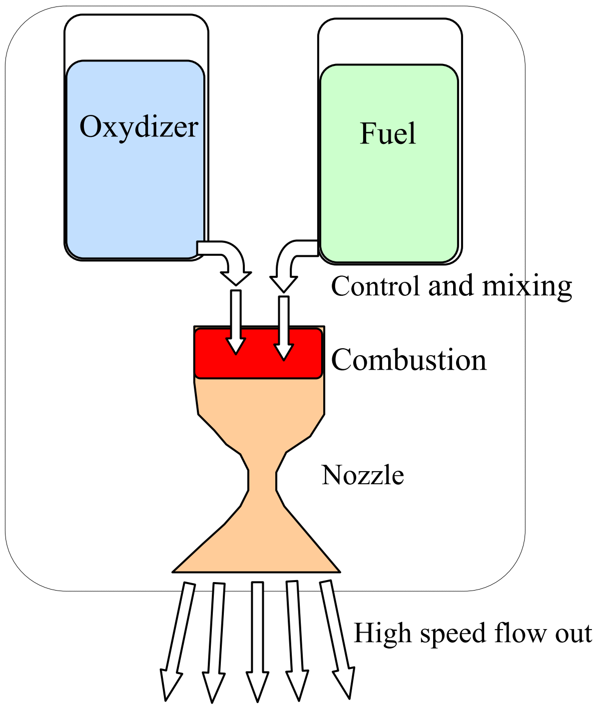
Solution:





2.2

Take a control volume around the rocket engine in Fig. 1.12. Identify the mass flows and where you have significant kinetic energy and where storage changes.



We have storage in both tanks are reduced, mass flows out with modest velocities.

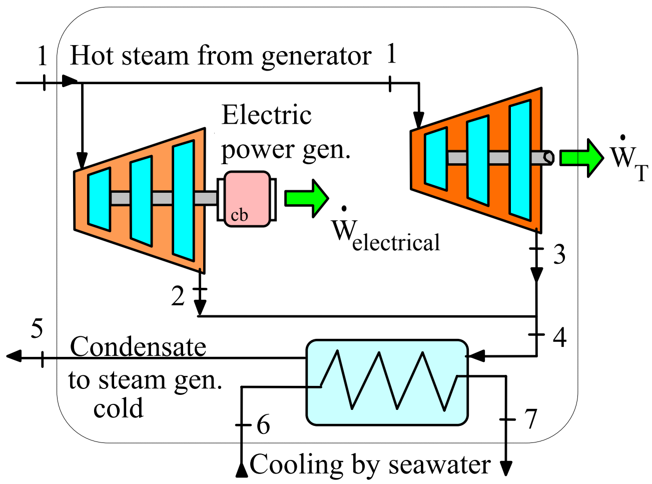
Energy conversion in the combustion process.

gas at high pressure expands towards lower pressure outside and thus accelerates to high velocity with significant kinetic energy flowing out.

2.3

Make a control volume that includes the steam flow around in the main turbine loop in the nuclear propulsion system in Fig.1.3. Identify mass flows (hot or cold) and energy transfers that enter or leave the C.V.

Solution:



The electrical power also leaves the C.V. to be used for lights, instruments and to charge the batteries.

2.4

Separate the list P, F, V, v, ρ, T, a, m, L, t, and **V** into intensive, extensive, and non-properties.

Solution:

Intensive properties are independent upon mass: P, v, ρ, T
Extensive properties scales with mass: V, m
Non-properties: F, a, L, t, **V**

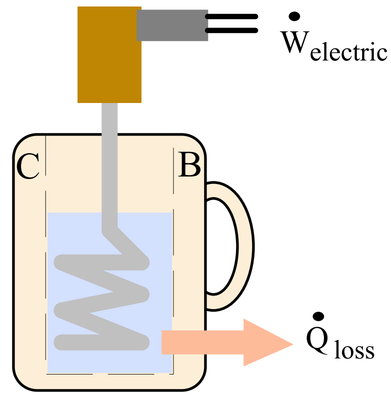
Comment: You could claim that acceleration a and velocity **V** are physical properties for the dynamic motion of the mass, but not thermal properties.

2.5

An electric dip heater is put into a cup of water and heats it from 20°C to 80°C. Show the energy flow(s) and storage and explain what changes.

Solution:

Electric power is converted in the heater element (an electric resistor) so it becomes hot and gives energy by heat transfer to the water. The water heats up and thus stores energy and as it is warmer than the cup material it heats the cup which also stores some energy. The cup being warmer than the air gives a smaller amount of energy (a rate) to the air as a heat loss.



2.6

Water in nature exists in different phases such as solid, liquid and vapor (gas). Indicate the relative magnitude of density and specific volume for the three phases.

Solution:

Values are indicated in Figure 2.7 as density for common substances. More accurate values are found in Tables A.3, A.4 and A.5

Water as solid (ice) has density of around 900 kg/m^3

Water as liquid has density of around 1000 kg/m^3

Water as vapor has density of around 1 kg/m^3 (sensitive to P and T)

Ice cube

Liquid drop

Cloud*

* Steam (water vapor) can not be seen what you see are tiny drops suspended in air from which we infer that there was some water vapor before it condensed.

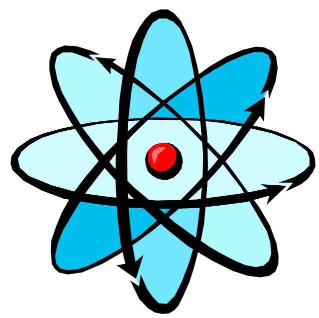
2.7

Is density a unique measure of mass distribution in a volume? Does it vary? If so, on what kind of scale (distance)?

Solution:

Density is an average of mass per unit volume and we sense if it is not evenly distributed by holding a mass that is more heavy in one side than the other. Through the volume of the same substance (say air in a room) density varies only little from one location to another on scales of meter, cm or mm. If the volume you look at has different substances (air and the furniture in the room) then it can change abruptly as you look at a small volume of air next to a volume of hardwood.

Finally if we look at very small scales on the order of the size of atoms the density can vary infinitely, since the mass (electrons, neutrons and positrons) occupy very little volume relative to all the empty space between them.



2.8

Density of fibers, rock wool insulation, foams and cotton is fairly low. Why is that?

Solution:

All these materials consist of some solid substance and mainly air or other gas. The volume of fibers (clothes) and rockwool that is solid substance is low relative to the total volume that includes air. The overall density is

$$\rho = \frac{m}{V} = \frac{m_{solid} + m_{air}}{V_{solid} + V_{air}}$$

where most of the mass is the solid and most of the volume is air. If you talk about the density of the solid only, it is high.

2.9

How much mass is there approximately in 1 L of engine oil? Atmospheric air?

Solution:

A volume of 1 L equals 0.001 m^3, see Table A.1. From Table A.4 the density is 885 kg/m^3 so we get

$$m = \rho V = 885 \text{ kg/m}^3 \times 0.001 \text{ m}^3 = \mathbf{0.885 \text{ kg}}$$

For the air we see in Figure 2.7 that density is about 1 kg/m^3 so we get

$$m = \rho V = 1 \text{ kg/m}^3 \times 0.001 \text{ m}^3 = \mathbf{0.001 \text{ kg}}$$

A more accurate value from Table A.5 is $\rho = 1.17$ kg/m^3 at 100 kPa, 25°C.

2.10

Can you carry 1 m^3 of liquid water?

Solution:

The density of liquid water is about 1000 kg/m^3 from Figure 2.7, see also Table A.3. Therefore the mass in one cubic meter is

$$m = \rho V = 1000 \text{ kg/m}^3 \times 1 \text{ m}^3 = 1000 \text{ kg}$$

and we can not carry that in the standard gravitational field.

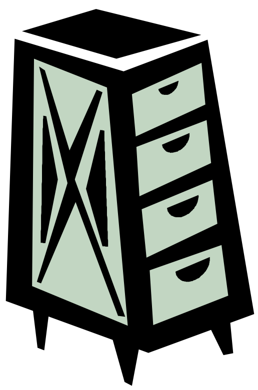
2.11

A heavy cabinet has four adjustable feet on it. What feature of the feet will ensure that they do not make dents in the floor?

Answer:
The area that is in contact with the floor supports the total mass in the gravitational field.

$$F = PA = mg$$

so for a given mass the smaller the area is the larger the pressure becomes.



2.12

The pressure at the bottom of a swimming pool is evenly distributed. Suppose we look at a cast iron plate of 7272 kg lying on the ground with an area of 100 m^2. What is the average pressure below that? Is it just as evenly distributed as the pressure at the bottom of the pool?

Solution:

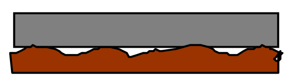
The pressure is force per unit area from page 25:

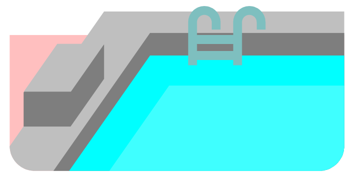
$$P = F/A = mg/A = 7272 \text{ kg} \times (9.81 \text{ m/s}^2) / 100 \text{ m}^2 = \mathbf{713.4 \text{ Pa}}$$

The iron plate being cast can be reasonable plane and flat, but it is stiff and rigid. However, the ground is usually uneven so the contact between the plate and the ground is made over an area much smaller than the 100 m^2. Thus the local pressure at the contact locations is much larger than the quoted value above.

The pressure at the bottom of the swimming pool is very even due to the ability of the fluid (water) to have full contact with the bottom by deforming itself. This is the main difference between a fluid behavior and a solid behavior.

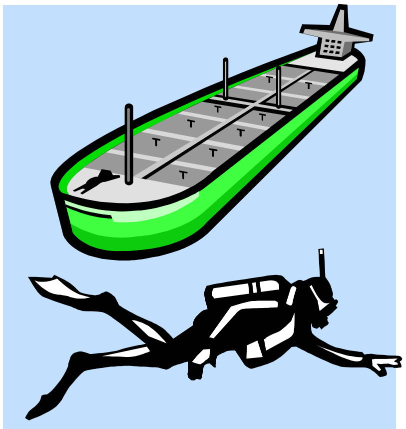
Iron plate
Ground





2.13

Two divers swim at 20 m depth. One of them swims right in under a supertanker; the other stays away from the tanker. Who feels a greater pressure?



Solution:

Each one feels the local pressure which is the static pressure only a function of depth.

$$P_{ocean} = P_0 + \Delta P = P_0 + \rho g H$$

So they feel exactly the same pressure.

2.14

A manometer with water shows a ΔP of $P_o/10$; what is the column height difference?

Solution:

$$\Delta P = P_o/10 = \rho H g$$

$$H = P_o/(10\ \rho\ g) = \frac{101.3 \times 1000 \text{ Pa}}{10 \times 997 \text{ kg/m}^3 \times 9.80665 \text{ m/s}^2}$$

$= \mathbf{1.036\ m}$

2.15

A water skier does not sink too far down in the water if the speed is high enough. What makes that situation different from our static pressure calculations?

The water pressure right under the ski is not a static pressure but a static plus dynamic pressure that pushes the water away from the ski. The faster you go, the smaller amount of water is displaced but at a higher velocity.

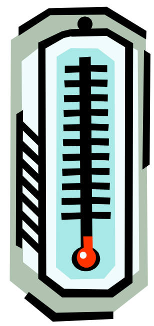
2.16

What is the smallest temperature in degrees Celsuis you can have? Kelvin?

Solution:

The lowest temperature is absolute zero which is at zero degrees Kelvin at which point the temperature in Celsius is negative

$$T_K = 0 \text{ K} = -273.15 \,^{o}\text{C}$$



2.17

Convert the formula for water density in In-text Concept Question “e” to be for T in degrees Kelvin.

Solution:

$$\rho = 1008 - T_C/2 \quad [kg/m^3]$$

We need to express degrees Celsius in degrees Kelvin

$$T_C = T_K - 273.15$$

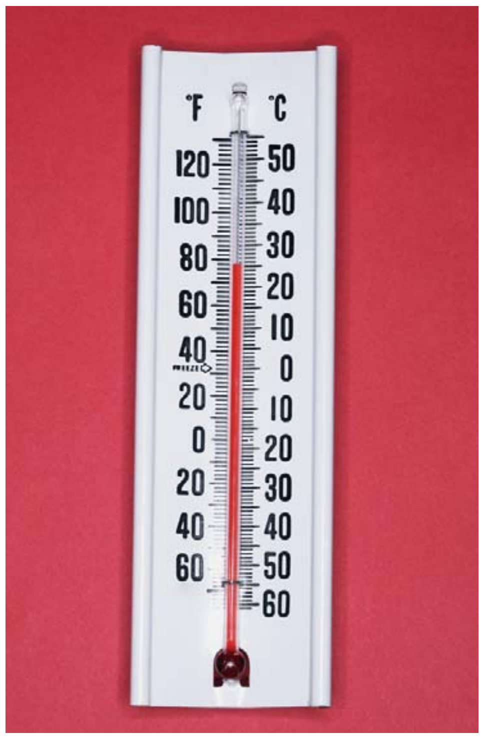
and substitute into formula

$$\rho = 1008 - T_C/2 = 1008 - (T_K - 273.15)/2 = 1144.6 - T_K/2$$

2.18

A thermometer that indicates the temperature with a liquid column has a bulb with a larger volume of liquid, why is that?

The expansion of the liquid volume with temperature is rather small so by having a larger volume expand with all the volume increase showing in the very small diameter column of fluid greatly increases the signal that can be read.



Properties and units

2.19

An apple "weighs" 60 g and has a volume of 75 cm^3 in a refrigerator at 8^oC. What is the apple density? List three intensive and two extensive properties of the apple.

Solution:

$$\rho = \frac{m}{V} = \frac{0.06}{0.000\ 075}\ \frac{kg}{m^3} = 800\ \frac{kg}{m^3}$$

Intensive

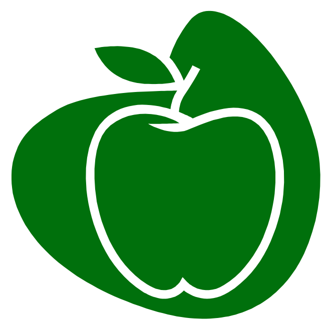
$\rho = 800\ \frac{kg}{m^3}$; $\quad v = \frac{1}{\rho} = 0.001\ 25\ \frac{m^3}{kg}$

T = 8°C; $\quad$ P = 101 kPa

Extensive

m = 60 g = 0.06 kg

V = 75 cm^3 = 0.075 L = 0.000 075 m^3



2.20

A steel cylinder of mass 2 kg contains 4 L of liquid water at 25°C at 200 kPa. Find the total mass and volume of the system. List two extensive and three intensive properties of the water

Solution:

Density of steel in Table A.3: $\rho = 7820 \text{ kg/m}^3$

Volume of steel: $V = m/\rho = \dfrac{2 \text{ kg}}{7820 \text{ kg/m}^3} = 0.000\,256 \text{ m}^3$

Density of water in Table A.4: $\rho = 997 \text{ kg/m}^3$

Mass of water: $m = \rho V = 997 \text{ kg/m}^3 \times 0.004 \text{ m}^3 = 3.988 \text{ kg}$

Total mass: $m = m_{steel} + m_{water} = 2 + 3.988 = \mathbf{5.988\ kg}$

Total volume: $V = V_{steel} + V_{water} = 0.000\,256 + 0.004$

$= \mathbf{0.004\,256\ m^3 = 4.26\ L}$

Extensive properties: m, V
Intensive properties: ρ (or v = 1/ρ), T, P

2.21

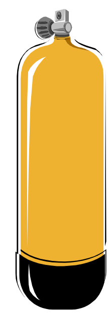
A storage tank of stainless steel contains 7 kg of oxygen gas and 5 kg of nitrogen gas. How many kmoles are in the tank?

Table A.2: $M_{O2} = 31.999$; $M_{N2} = 28.013$

$$n_{O2} = m_{O2} / M_{O2} = \frac{7}{31.999} = 0.21876 \text{ kmol}$$

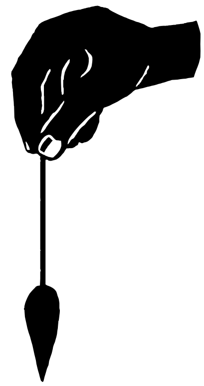
$$n_{O2} = m_{N2} / M_{N2} = \frac{5}{28.013} = 0.17848 \text{ kmol}$$

$$n_{tot} = n_{O2} + n_{N2} = 0.21876 + 0.17848 = \mathbf{0.3972\ kmol}$$



2.22

One kilopond (1 kp) is the weight of 1 kg in the standard gravitational field. How many Newtons (N) is that?



$$F = ma = mg$$

$$1 \text{ kp} = 1 \text{ kg} \times 9.807 \text{ m/s}^2 = \mathbf{9.807\ N}$$

Force and Energy

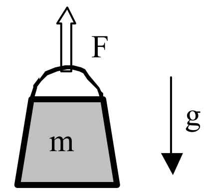
2.23

The “standard” acceleration (at sea level and 45° latitude) due to gravity is 9.80665 m/s^2. What is the force needed to hold a mass of 2 kg at rest in this gravitational field? How much mass can a force of 1 N support?

Solution:

$$ma = 0 = \sum F = F - mg$$

$$F = mg = 2 \text{ kg} \times 9.80665 \text{ m/s}^2 = \mathbf{19.613\ N}$$



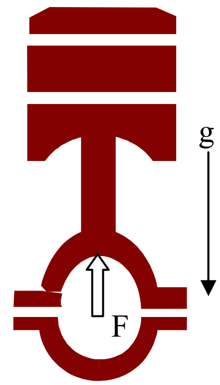
$F = mg$ =>

$$m = \frac{F}{g} = \frac{1 \text{ N}}{9.80665 \text{ m/s}^2} = \mathbf{0.102\ kg}$$

2.24

A steel piston of 2.5 kg is in the standard gravitational field where a force of 25 N is applied vertically up. Find the acceleration of the piston.

Solution:



$$F_{up} = ma = F - mg$$

$$a = \frac{F - mg}{m} = \frac{F}{m} - g$$

$$= \frac{25\ \text{N}}{2.5\ \text{kg}} - 9.807\ \text{m/s}^2$$

$$= \mathbf{0.193\ ms^{-2}}$$

2.25

When you move up from the surface of the earth the gravitation is reduced as g = 9.807 – 3.32 × 10^{-6} z, with z as the elevation in meters. How many percent is the weight of an airplane reduced when it cruises at 11 000 m?

Solution:

$$g_o = 9.807\ \text{ms}^{-2}$$

$$g_H = 9.807 - 3.32 \times 10^{-6} \times 11\,000 = 9.7705\ \text{ms}^{-2}$$

$$W_o = m\, g_o \;\; ; \;\; W_H = m\, g_H$$

$$W_H/W_o = g_H/g_o = \frac{9.7705}{9.807} = 0.9963$$

Reduction = 1 – 0.9963 = 0.0037 or **0.37%**

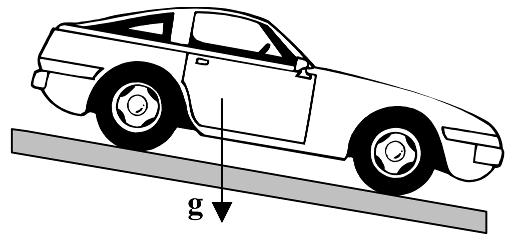
i.e. we can neglect that for most applications.

2.26

A model car rolls down an incline with a slope so the gravitational "pull" in the direction of motion is one third of the standard gravitational force (see Problem 2.23). If the car has a mass of 0.06 kg find the acceleration.

Solution:

$$ma = \sum F = mg / 3$$
$$a = mg / 3m = g/3$$
$$= 9.80665 \text{ (m/s}^2\text{)} / 3$$
$$= \mathbf{3.27\ m/s^2}$$



This acceleration does not depend on the mass of the model car.

2.27

A van drives at 60 km/h and is brought to a full stop with constant deceleration in 5 seconds. If the total car and driver mass is 2075 kg find the necessary force.

Solution:

Acceleration is the time rate of change of velocity.

$$a = \frac{d\mathbf{V}}{dt} = \frac{60 \times 1000}{3600 \times 5} = 3.333 \text{ m/s}^2$$

$ma = \sum F$;

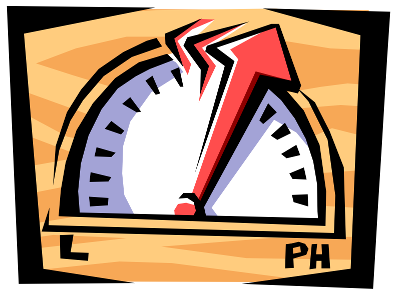
$F_{net} = ma = 2075 \text{ kg} \times 3.333 \text{ m/s}^2 =$ **6916 N**

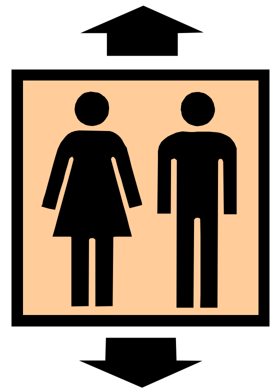
2.28

An escalator brings four people of total 300 kg, 25 m up in a building. Explain what happens with respect to energy transfer and stored energy.

Solution:

The four people (300 kg) have their potential energy raised, which is how the energy is stored. The energy is supplied as electrical power to the motor that pulls the escalator with a cable.





2.29

A car of mass 1775 kg travels with a velocity of 100 km/h. Find the kinetic energy. How high should it be lifted in the standard gravitational field to have a potential energy that equals the kinetic energy?

Solution:

Standard kinetic energy of the mass is

$$KIN = ½ \, m \, \mathbf{V}^2 = ½ \times 1775 \text{ kg} \times \left(\frac{100 \times 1000}{3600}\right)^2 \text{ m}^2/\text{s}^2$$

$$= ½ \times 1775 \times 27.778 \text{ Nm} = 684\,800 \text{ J}$$

$$= \mathbf{684.8 \text{ kJ}}$$

Standard potential energy is

$$POT = mgh$$

$$h = ½ \, m \, \mathbf{V}^2 / mg = \frac{684\,800 \text{ Nm}}{1775 \text{ kg} \times 9.807 \text{ m/s}^2} = \mathbf{39.3 \text{ m}}$$

2.30

A 1500-kg car moving at 20 km/h is accelerated at a constant rate of $4\ m/s^2$ up to a speed of 75 km/h. What are the force and total time required?

Solution:

$$a = \frac{d\mathbf{V}}{dt} = \frac{\Delta\mathbf{V}}{\Delta t} \quad => \quad \Delta t = \frac{\Delta\mathbf{V}}{a} = \frac{(75 - 20)\ 1000}{3600 \times 5} = \mathbf{3.82\ sec}$$

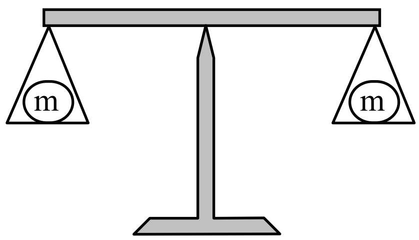
$$F = ma = 1500\ kg \times 4\ m/s^2 = \mathbf{6000\ N}$$

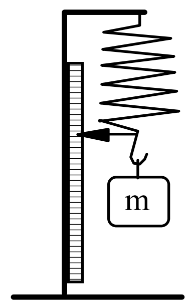
2.31

On the moon the gravitational acceleration is approximately one-sixth that on the surface of the earth. A 5-kg mass is "weighed" with a beam balance on the surface on the moon. What is the expected reading? If this mass is weighed with a spring scale that reads correctly for standard gravity on earth (see Problem 2.23), what is the reading?

Solution:

Moon gravitation is: $g = g_{earth}/6$





Beam Balance Reading is **5 kg**
This is mass comparison

Spring Balance Reading is in kg units
Force comparison length $\propto$ F $\propto$ g

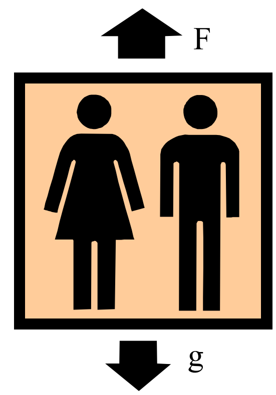
Reading will be $\mathbf{\frac{5}{6}}$ **kg**

2.32

The escalator cage in Problem 2.28 has a mass of 500 kg in addition to the people. How much force should the cable pull up with to have an acceleration of 1 m/s^2 in the upwards direction?

Solution:

The total mass moves upwards with an acceleration plus the gravitations acts with a force pointing down.



$$ma = \sum F = F - mg$$

$$F = ma + mg = m(a + g)$$
$$= (500 + 300)\ \text{kg} \times (1 + 9.81)\ \text{m/s}^2$$
$$= \mathbf{8648\ N}$$

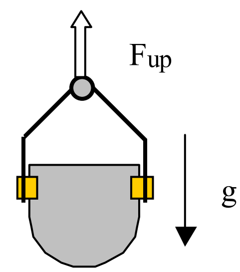
2.33

A bucket of concrete of total mass 200 kg is raised by a crane with an acceleration of 2 m/s^2 relative to the ground at a location where the local gravitational acceleration is 9.5 m/s^2. Find the required force.

Solution:

$$F = ma = F_{up} - mg$$

$$F_{up} = ma + mg = 200\ (\ 2 + 9.5\ ) = \mathbf{2300\ N}$$



2.34

A bottle of 12 kg steel has 1.75 kmole of liquid propane. It accelerates horizontal with 3 m/s^2, what is the needed force?

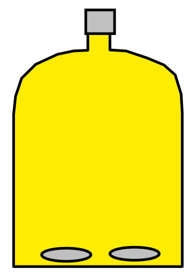
Solution:

The molecular weight for propane is $M = 44.094$ from Table A.2. The force must accelerate both the container mass and the propane mass.

$$m = m_{steel} + m_{propane} = 12 + (1.75 \times 44.094) = 90.645 \text{ kg}$$

$$ma = \sum F \quad \Rightarrow$$

$$F = ma = 90.645 \text{ kg} \times 3 \text{ m/s}^2 = \mathbf{271.9\ N}$$

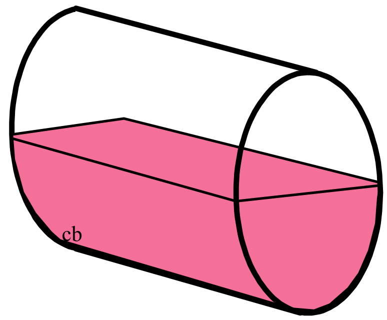


Specific Volume

2.35

A 15-kg steel gas tank holds 300 L of liquid gasoline, having a density of 800 kg/m^3. If the system is decelerated with 2g what is the needed force?

Solution:



$$m = m_{tank} + m_{gasoline}$$
$$= 15 \text{ kg} + 0.3 \text{ m}^3 \times 800 \text{ kg/m}^3$$
$$= 255 \text{ kg}$$
$$F = ma = 255 \text{ kg} \times 2 \times 9.81 \text{ m/s}^2$$
$$= \mathbf{5003 \ N}$$

2.36

A power plant that separates carbon-dioxide from the exhaust gases compresses it to a density of 110 kg/m^3 and stores it in an un-minable coal seam with a porous volume of 100 000 m^3. Find the mass they can store.

Solution:

$$m = \rho V = 110 \text{ kg/m}^3 \times 100\,000 \text{ m}^3 = 11 \times 10^{6} \text{ kg}$$

Just to put this in perspective a power plant that generates 2000 MW by burning coal would make about 20 million tons of carbon-dioxide a year. That is 2000 times the above mass so it is nearly impossible to store all the carbon-dioxide being produced.

2.37

A 1 m^3 container is filled with 400 kg of granite stone, 200 kg dry sand and 0.2 m^3 of liquid 25°C water. Use properties from tables A.3 and A.4. Find the average specific volume and density of the masses when you exclude air mass and volume.

Solution:

Specific volume and density are ratios of total mass and total volume.

$$m_{liq} = V_{liq}/v_{liq} = V_{liq}\,\rho_{liq} = 0.2\ m^3 \times 997\ kg/m^3 = 199.4\ kg$$

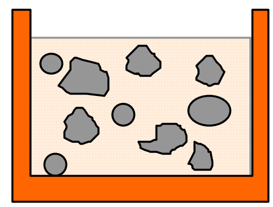
$$m_{TOT} = m_{stone} + m_{sand} + m_{liq} = 400 + 200 + 199.4 = 799.4\ kg$$

$$V_{stone} = mv = m/\rho = 400\ kg/\ 2750\ kg/m^3 = 0.1455\ m^3$$

$$V_{sand} = mv = m/\rho = 200/\ 1500 = 0.1333\ m^3$$

$$V_{TOT} = V_{stone} + V_{sand} + V_{liq}$$

$$= 0.1455 + 0.1333 + 0.2 = 0.4788\ m^3$$



$$v = V_{TOT} / m_{TOT} = 0.4788/799.4 = \mathbf{0.000599\ m^3/kg}$$

$$\rho = 1/v = m_{TOT}/V_{TOT} = 799.4/0.4788 = \mathbf{1669.6\ kg/m^3}$$

2.38

One kilogram of diatomic oxygen (O_2 molecular weight 32) is contained in a 500-L tank. Find the specific volume on both a mass and mole basis (v and $\bar{v}$).

Solution:

From the definition of the specific volume

$$v = \frac{V}{m} = \frac{0.5}{1} = \mathbf{0.5\ m^3/kg}$$

$$\bar{v} = \frac{V}{n} = \frac{V}{m/M} = M\,v = 32 \times 0.5 = \mathbf{16\ m^3/kmol}$$

2.39

A tank has two rooms separated by a membrane. Room A has 1 kg air and volume 0.5 m^3, room B has 0.75 m^3 air with density 0.8 kg/m^3. The membrane is broken and the air comes to a uniform state. Find the final density of the air.

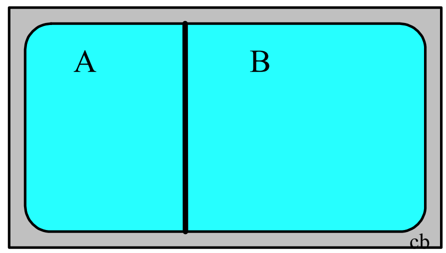
Solution:

Density is mass per unit volume

$$m = m_A + m_B = m_A + \rho_B V_B = 1 + 0.8 \times 0.75 = 1.6 \text{ kg}$$

$$V = V_A + V_B = 0.5 + 0.75 = 1.25 \text{ m}^3$$

$$\rho = \frac{m}{V} = \frac{1.6}{1.25} = \mathbf{1.28\ kg/m^3}$$



2.40

A 5 m^3 container is filled with 900 kg of granite (density 2400 kg/m^3) and the rest of the volume is air with density 1.15 kg/m^3. Find the mass of air and the overall (average) specific volume.

Solution:

$$m_{air} = \rho V = \rho_{air} \left(V_{tot} - \frac{m_{granite}}{\rho} \right)$$

$$= 1.15 \left[5 - \frac{900}{2400} \right] = 1.15 \times 4.625 = \mathbf{5.32\ kg}$$

$$v = \frac{V}{m} = \frac{5}{900 + 5.32} = \mathbf{0.005\ 52\ m^3/kg}$$

Comment: Because the air and the granite are not mixed or evenly distributed in the container the overall specific volume or density does not have much meaning.

Pressure

2.41

The hydraulic lift in an auto-repair shop has a cylinder diameter of 0.2 m. To what pressure should the hydraulic fluid be pumped to lift 40 kg of piston/arms and 700 kg of a car?

Solution:

Force acting on the mass by the gravitational field

$$F\downarrow = ma = mg = 740 \times 9.80665 = 7256.9 \text{ N} = 7.257 \text{ kN}$$

Force balance: $F\uparrow = (P - P_0) A = F\downarrow \quad => \quad P = P_0 + F\downarrow / A$

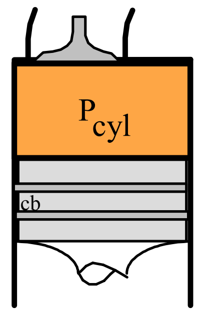
$$A = \pi D^2 (1 / 4) = 0.031416 \text{ m}^2$$

$$P = 101 \text{ kPa} + \frac{7.257 \text{ kN}}{0.031416 \text{ m}^2} = \mathbf{332 \text{ kPa}}$$

2.42

A valve in a cylinder has a cross sectional area of 11 cm^2 with a pressure of 735 kPa inside the cylinder and 99 kPa outside. How large a force is needed to open the valve?

$$F_{net} = P_{in}A - P_{out}A$$
$$= (735 - 99)\ \text{kPa} \times 11\ \text{cm}^2$$
$$= 6996\ \text{kPa cm}^2$$
$$= 6996 \times \frac{\text{kN}}{\text{m}^2} \times 10^{-4}\ \text{m}^2$$
$$= \mathbf{700\ N}$$



2.43

A hydraulic lift has a maximum fluid pressure of 500 kPa. What should the piston-cylinder diameter be so it can lift a mass of 850 kg?

Solution:

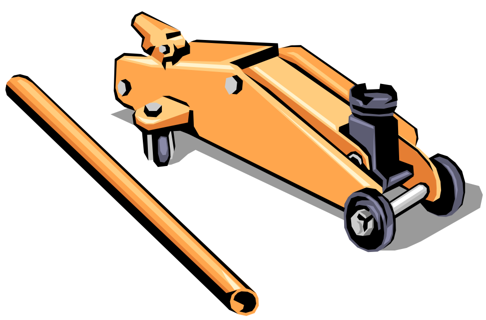
With the piston at rest the static force balance is

$$F\uparrow = P\ A = F\downarrow = mg$$

$$A = \pi\ r^2 = \pi\ D^2/4$$

$$PA = P\ \pi\ D^2/4 = mg \quad \Rightarrow \quad D^2 = \frac{4mg}{P\ \pi}$$

$$D = 2\sqrt{\frac{mg}{P\pi}} = 2\sqrt{\frac{850\text{ kg} \times 9.807\text{ m/s}^2}{500\text{ kPa} \times \pi \times 1000\text{ (Pa/kPa)}}} = \mathbf{0.146\ m}$$



2.44

A laboratory room keeps a vacuum of 0.1 kPa. What net force does that put on the door of size 2 m by 1 m?

Solution:

The net force on the door is the difference between the forces on the two sides as the pressure times the area

$$F = P_{outside} A - P_{inside} A = \Delta P\ A = 0.1\ kPa \times 2\ m \times 1\ m = \mathbf{200\ N}$$

Remember that kPa is kN/m^2.

2.45

A vertical hydraulic cylinder has a 125-mm diameter piston with hydraulic fluid inside the cylinder and an ambient pressure of 1 bar. Assuming standard gravity, find the piston mass that will create a pressure inside of 1500 kPa.

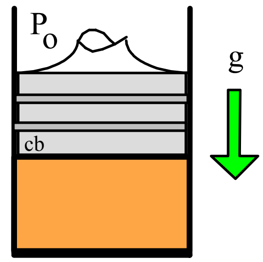
Solution:

Force balance:

$$F\uparrow = PA = F\downarrow = P_0A + m_pg;$$

$$P_0 = 1 \text{ bar} = 100 \text{ kPa}$$

$$A = (\pi/4)\, D^2 = (\pi/4) \times 0.125^2 = 0.01227 \text{ m}^2$$



$$m_p = (P - P_0)\frac{A}{g} = (\ 1500 - 100\ ) \times 1000 \times \frac{0.01227}{9.80665} = \mathbf{1752\ kg}$$

2.46

A piston/cylinder with cross sectional area of 0.01 m^2 has a piston mass of 100 kg resting on the stops, as shown in Fig. P2.46. With an outside atmospheric pressure of 100 kPa, what should the water pressure be to lift the piston?

Solution:

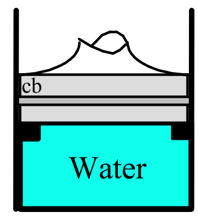
The force acting down on the piston comes from gravitation and the outside atmospheric pressure acting over the top surface.

Force balance: $F\uparrow = F\downarrow = PA = m_p g + P_0 A$

Now solve for P (divide by 1000 to convert to kPa for 2nd term)

$$P = P_0 + \frac{m_p g}{A} = 100 \text{ kPa} + \frac{100 \times 9.80665}{0.01 \times 1000} \text{ kPa}$$

$$= 100 \text{ kPa} + 98.07 \text{ kPa} = \mathbf{198\ kPa}$$



2.47

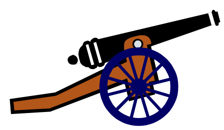
A cannon-ball of 5 kg acts as a piston in a cylinder of 0.15 m diameter. As the gun-powder is burned a pressure of 7 MPa is created in the gas behind the ball. What is the acceleration of the ball if the cylinder (cannon) is pointing horizontally?

Solution:

The cannon ball has 101 kPa on the side facing the atmosphere.

$$ma = F = P_1 \times A - P_0 \times A = (P_1 - P_0) \times A$$
$$= (7000 - 101)\text{ kPa} \times \pi\,(0.15^2/4)\text{ m}^2 = 121.9\text{ kN}$$

$$a = \frac{F}{m} = \frac{121.9\text{ kN}}{5\text{ kg}} = \mathbf{24\ 380\ m/s^2}$$



2.48

Repeat the previous problem for a cylinder (cannon) pointing 40 degrees up relative to the horizontal direction.

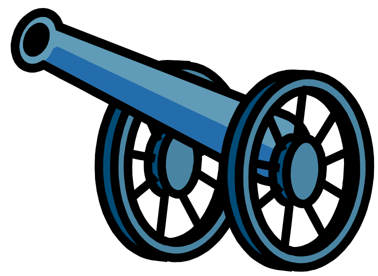
Solution:

$$ma = F = (P_1 - P_0) A - mg \sin 40^0$$

$$ma = (7000 - 101) \text{ kPa} \times \pi \times (0.15^2 / 4) \text{ m}^2 - 5 \times 9.807 \times 0.6428 \text{ N}$$

$$= 121.9 \text{ kN} - 31.52 \text{ N} = 121.87 \text{ kN}$$

$$a = \frac{F}{m} = \frac{121.87 \text{ kN}}{5 \text{ kg}} = \mathbf{24\ 374\ m/s^2}$$



2.49

A large exhaust fan in a laboratory room keeps the pressure inside at 10 cm water relative vacuum to the hallway. What is the net force on the door measuring 1.9 m by 1.1 m?

Solution:

The net force on the door is the difference between the forces on the two sides as the pressure times the area

$$F = P_{outside}\ A - P_{inside}\ A = \Delta P \times A$$
$$= 10 \text{ cm } H_2O \times 1.9 \text{ m} \times 1.1 \text{ m}$$
$$= 0.10 \times 9.80638 \text{ kPa} \times 2.09 \text{ m}^2$$
$$= \mathbf{2049\ N}$$

Table A.1: 1 m H_2O is 9.80638 kPa and kPa is kN/m^2.

2.50

A tornado rips off a 100 m^2 roof with a mass of 1000 kg. What is the minimum vacuum pressure needed to do that if we neglect the anchoring forces?

Solution:

The net force on the roof is the difference between the forces on the two sides as the pressure times the area

$$F = P_{inside} A - P_{outside} A = \Delta P \ A$$

That force must overcome the gravitation mg, so the balance is

$$\Delta P \ A = mg$$

$$\Delta P = mg/A = (1000 \text{ kg} \times 9.807 \text{ m/s}^2)/100 \text{ m}^2 = \mathbf{98 \ Pa = 0.098 \ kPa}$$

Remember that kPa is kN/m^2.

2.51

A 2.5 m tall steel cylinder has a cross sectional area of 1.5 m^2. At the bottom with a height of 0.5 m is liquid water on top of which is a 1 m high layer of gasoline. This is shown in Fig. P2.51. The gasoline surface is exposed to atmospheric air at 101 kPa. What is the highest pressure in the water?

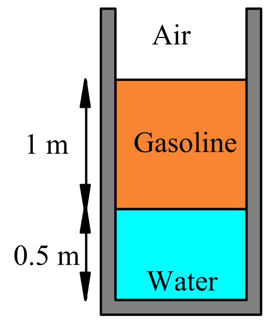
Solution:

The pressure in the fluid goes up with the depth as

$$P = P_{top} + \Delta P = P_{top} + \rho g h$$

and since we have two fluid layers we get

$$P = P_{top} + [(\rho h)_{gasoline} + (\rho h)_{water}]\ g$$



The densities from Table A.4 are:

$$\rho_{gasoline} = 750\ kg/m^3; \quad \rho_{water} = 997\ kg/m^3$$

$$P = 101 + [750 \times 1 + 997 \times 0.5]\ \frac{9.807}{1000} = \mathbf{113.2\ kPa}$$

2.52

What is the pressure at the bottom of a 5 m tall column of fluid with atmospheric pressure 101 kPa on the top surface if the fluid is

a) water at 20°C b) glycerine 25°C or c) gasoline 25°C

Solution:

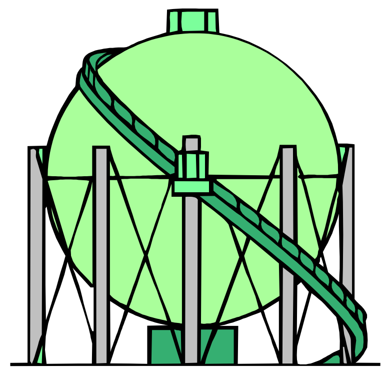
Table A.4: $\rho_{H2O} = 997\ kg/m^3$; $\rho_{Glyc} = 1260\ kg/m^3$; $\rho_{gasoline} = 750\ kg/m^3$

$\Delta P = \rho gh$ $P = P_{top} + \Delta P$

a) $\Delta P = \rho gh = 997 \times 9.807 \times 5 = 48\ 888$ Pa
$P = 101 + 48.99 =$ **149.9 kPa**

b) $\Delta P = \rho gh = 1260 \times 9.807 \times 5 = 61\ 784$ Pa
$P = 101 + 61.8 =$ **162.8 kPa**

c) $\Delta P = \rho gh = 750 \times 9.807 \times 5 = 36\ 776$ Pa
$P = 101 + 36.8 =$ **137.8 kPa**



2.53

At the beach, atmospheric pressure is 1025 mbar. You dive 15 m down in the ocean and you later climb a hill up to 250 m elevation. Assume the density of water is about 1000 kg/m^3 and the density of air is 1.18 kg/m^3. What pressure do you feel at each place?

Solution:

$\Delta P = \rho g h$,

Units from A.1: 1 mbar = 100 Pa (1 bar = 100 kPa).

$$P_{ocean} = P_0 + \Delta P = 1025 \times 100 + 1000 \times 9.81 \times 15$$

$$= 2.4965 \times 10^5 \text{ Pa} = \mathbf{250\ kPa}$$

$$P_{hill} = P_0 - \Delta P = 1025 \times 100 - 1.18 \times 9.81 \times 250$$

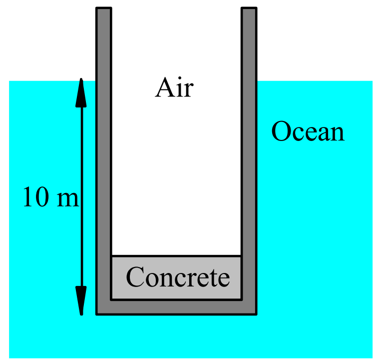
$$= 0.99606 \times 10^5 \text{ Pa} = \mathbf{99.61\ kPa}$$

2.54

A steel tank of cross sectional area 3 m^2 and 16 m tall weighs 10 000 kg and it is open at the top. We want to float it in the ocean so it sticks 10 m straight down by pouring concrete into the bottom of it. How much concrete should I put in?

Solution:

The force up on the tank is from the water pressure at the bottom times its area. The force down is the gravitation times mass and the atmospheric pressure.



$$F\uparrow = PA = (\rho_{ocean}gh + P_0)A$$

$$F\downarrow = (m_{tank} + m_{concrete})g + P_0A$$

The force balance becomes

$$F\uparrow = F\downarrow = (\rho_{ocean}gh + P_0)A = (m_{tank} + m_{concrete})g + P_0A$$

Solve for the mass of concrete

$$m_{concrete} = (\rho_{ocean}hA - m_{tank}) = 997 \times 10 \times 3 - 10\ 000 = \mathbf{19\ 910\ kg}$$

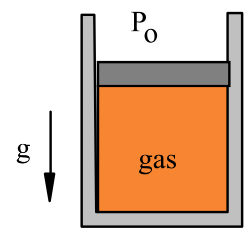
Notice: The first term is the mass of the displaced ocean water. The net force up is the weight (mg) of this mass called bouyancy, P_0 cancel.

2.55

A piston, m_p= 5 kg, is fitted in a cylinder, A = 15 cm^2, that contains a gas. The setup is in a centrifuge that creates an acceleration of 25 m/s^2 in the direction of piston motion towards the gas. Assuming standard atmospheric pressure outside the cylinder, find the gas pressure.

Solution:

Force balance: $F\uparrow = F\downarrow = P_0 A + m_p g = PA$



$$P = P_0 + \frac{m_p g}{A}$$

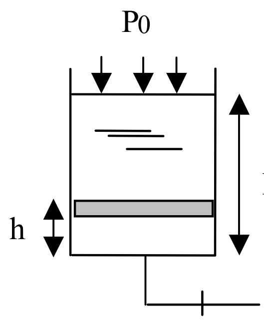
$$= 101.325 + \frac{5 \times 25}{1000 \times 0.0015} \ \frac{\text{kPa kg m/s}^2}{\text{Pa m}^2}$$

= **184.7 kPa**

2.56

Liquid water with density ρ is filled on top of a thin piston in a cylinder with cross-sectional area *A* and total height *H,* as shown in Fig. P2.56. Air is let in under the piston so it pushes up, spilling the water over the edge. Derive the formula for the air pressure as a function of piston elevation from the bottom, *h.*

Solution:



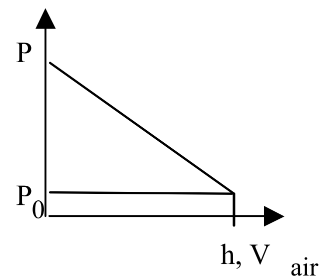
Force balance

Piston: F↑ = F↓

$$PA = P_0A + m_{H_2O}g$$

$$P = P_0 + m_{H_2O}g/A$$

$$\mathbf{P = P_0 + (H - h)\rho g}$$



Manometers and Barometers

2.57

You dive 5 m down in the ocean. What is the absolute pressure there?

Solution:

The pressure difference for a column is from Eq.2.2 and the density of water is from Table A.4.

$$\Delta P = \rho g H$$
$$= 997 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2 \times 5 \text{ m}$$
$$= 48\,903 \text{ Pa} = 48.903 \text{ kPa}$$

$$P_{ocean} = P_0 + \Delta P$$
$$= 101.325 + 48.903$$
$$= \mathbf{150\ kPa}$$

2.58

A barometer to measure absolute pressure shows a mercury column height of 725 mm. The temperature is such that the density of the mercury is 13 550 kg/m^3. Find the ambient pressure.

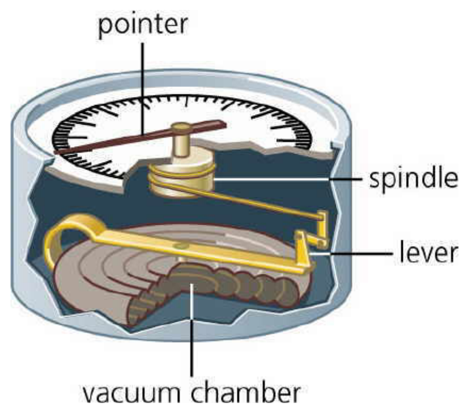
Solution:

Hg : $L = 725$ mm = 0.725 m; $\rho = 13\ 550\ kg/m^3$

The external pressure P balances the column of height L so from Fig. 2.10

$$P = \rho L g = 13\ 550\ kg/m^3 \times 9.80665\ m/s^2 \times 0.725\ m \times 10^{-3}\ kPa/Pa$$

$= \mathbf{96.34\ kPa}$



This is a more common type that does not involve mercury as the wall mounted unit to the left.

2.59

The density of atmospheric air is about 1.15 kg/m^3, which we assume is constant. How large an absolute pressure will a pilot see when flying 2000 m above ground level where the pressure is 101 kPa.

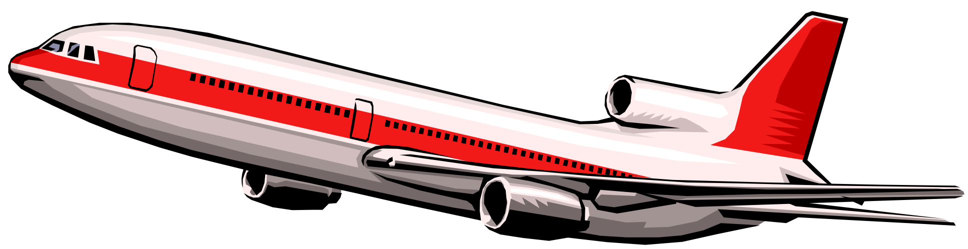
Solution:

Assume g and ρ are constant then the pressure difference to carry a column of height 1500 m is from Fig.2.10

$$\Delta P = \rho g h = 1.15 \text{ kg/m}^3 \times 9.807 \text{ ms}^{-2} \times 2000 \text{ m}$$
$$= 22\ 556 \text{ Pa} = 22.6 \text{ kPa}$$

The pressure on top of the column of air is then

$$P = P_0 - \Delta P = 101 - 22.6 = \mathbf{78.4\ kPa}$$



2.60

A differential pressure gauge mounted on a vessel shows 1.25 MPa and a local barometer gives atmospheric pressure as 0.96 bar. Find the absolute pressure inside the vessel.

Solution:

Convert all pressures to units of kPa.

$P_{gauge} = 1.25$ MPa = 1250 kPa;

$P_0 = 0.96$ bar = 96 kPa

$P = P_{gauge} + P_0 = 1250 + 96 =$ **1346 kPa**

2.61

A manometer shows a pressure difference of 1 m of liquid mercury. Find ΔP in kPa.

Solution:

Hg : L = 1 m; ρ = 13 580 kg/m^3 from Table A.4 (or read Fig 2.8)

The pressure difference ΔP balances the column of height L so from Eq.2.2

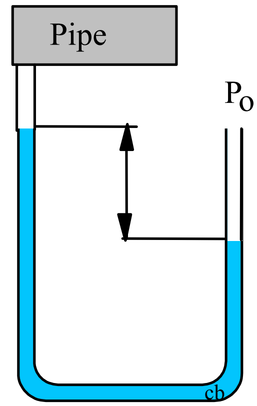
$$\Delta P = \rho g L = 13\,580 \text{ kg/m}^3 \times 9.80665 \text{ m/s}^2 \times 1.0 \text{ m} \times 10^{-3} \text{ kPa/Pa}$$

$$= \mathbf{133.2\ kPa}$$

2.62

Blue manometer fluid of density 925 kg/m^3 shows a column height difference of 3 cm vacuum with one end attached to a pipe and the other open to P_0 = 101 kPa. What is the absolute pressure in the pipe?

Solution:



Since the manometer shows a vacuum we have

$P_{PIPE} = P_0 - \Delta P$

$\Delta P = \rho g h = 925 \times 9.807 \times 0.03$

$= 272.1 \text{ Pa} = 0.272 \text{ kPa}$

$P_{PIPE} = 101 - 0.272 =$ **100.73 kPa**

2.63

What pressure difference does a 10 m column of atmospheric air show?

Solution:

The pressure difference for a column is from Eq.2.2

$$\Delta P = \rho g H$$

So we need density of air from Fig.2.7, $\rho = 1.2 \text{ kg/m}^3$

$$\Delta P = 1.2 \text{ kg/m}^3 \times 9.81 \text{ ms}^{-2} \times 10 \text{ m} = 117.7 \text{ Pa} = \mathbf{0.12 \text{ kPa}}$$

2.64

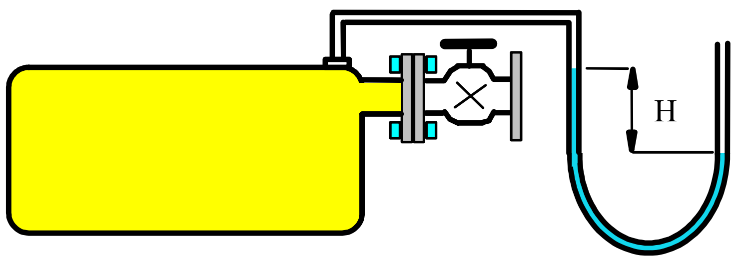
The absolute pressure in a tank is 85 kPa and the local ambient absolute pressure is 97 kPa. If a U-tube with mercury, density 13550 kg/m^3, is attached to the tank to measure the vacuum, what column height difference would it show?

Solution:

$$\Delta P = P_0 - P_{tank} = \rho g\ H$$

$$H = (P_0 - P_{tank}) / \rho g = [(97 - 85) \times 1000] / (13550 \times 9.80665)$$

$$= \mathbf{0.090\ m = 90\ mm}$$



2.65

The pressure gauge on an air tank shows 75 kPa when the diver is 10 m down in the ocean. At what depth will the gauge pressure be zero? What does that mean?

Ocean H_20 pressure at 10 m depth is

$$P_{water} = P_0 + \rho Lg = 101.3 + \frac{997 \times 10 \times 9.80665}{1000} = 199 \text{ kPa}$$

Air Pressure (absolute) in tank

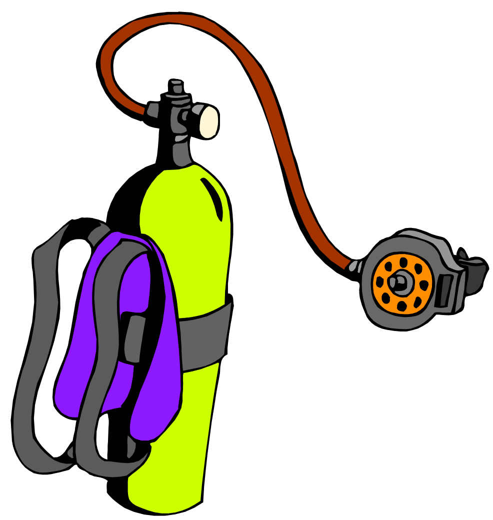
$$P_{tank} = 199 + 75 = 274 \text{ kPa}$$

Tank Pressure (gauge) reads zero at H_20 local pressure

$$274 = 101.3 + \frac{997 \times 9.80665}{1000} L$$

$$L = \mathbf{17.66\ m}$$

At this depth you will have to suck the air in, it can no longer push itself through a valve.



2.66

An exploration submarine should be able to go 4000 m down in the ocean. If the ocean density is 1020 kg/m^3 what is the maximum pressure on the submarine hull?

Solution:

Assume we have atmospheric pressure inside the submarine then the pressure difference to the outside water is

$$\Delta P = \rho L g = (1020 \text{ kg/m}^3 \times 4000 \text{ m} \times 9.807 \text{ m/s}^2) / 1000$$
$$= 40\ 012 \text{ kPa} \approx \mathbf{40\ MPa}$$

2.67

A submarine maintains 101 kPa inside it and it dives 240 m down in the ocean having an average density of 1030 kg/m^3. What is the pressure difference between the inside and the outside of the submarine hull?

Solution:

Assume the atmosphere over the ocean is at 101 kPa, then ΔP is from the 240 m column water.

$$\Delta P = \rho L g$$

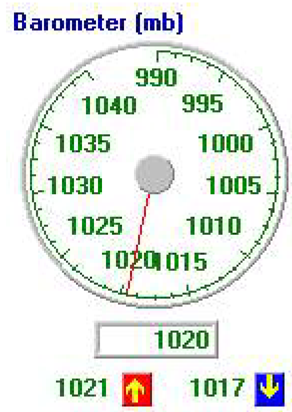
$$= (1030 \text{ kg/m}^3 \times 240 \text{ m} \times 9.807 \text{ m/s}^2) / 1000 = \mathbf{2424 \text{ kPa}}$$

2.68

Assume we use a pressure gauge to measure the air pressure at street level and at the roof of a tall building. If the pressure difference can be determined with an accuracy of 1 mbar (0.001 bar) what uncertainty in the height estimate does that corresponds to?

Solution:

$\rho_{air} = 1.169\ \text{kg/m}^3$ from Table A.5



$\Delta P = 0.001\ \text{bar} = 100\ \text{Pa}$

$$L = \frac{\Delta P}{\rho g} = \frac{100}{1.169 \times 9.807} = \mathbf{8.72\ m}$$

2.69

A barometer measures 760 mmHg at street level and 735 mmHg on top of a building. How tall is the building if we assume air density of 1.15 kg/m^3?

Solution:

$$\Delta P = \rho g H$$

$$H = \Delta P/\rho g = \frac{760 - 735}{1.15 \times 9.807} \frac{\text{mmHg}}{\text{kg/m}^2\text{s}^2} \frac{133.32 \text{ Pa}}{\text{mmHg}} = \mathbf{295\ m}$$

2.70

An absolute pressure gauge attached to a steel cylinder shows 135 kPa. We want to attach a manometer using liquid water a day that P_{atm} = 101 kPa. How high a fluid level difference must we plan for?

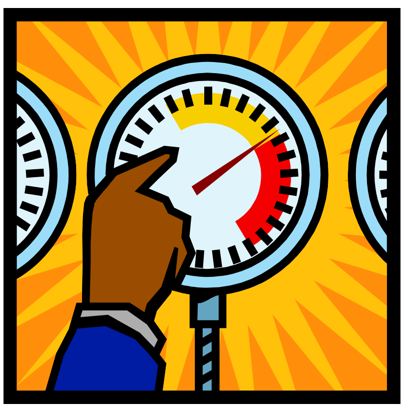
Solution:

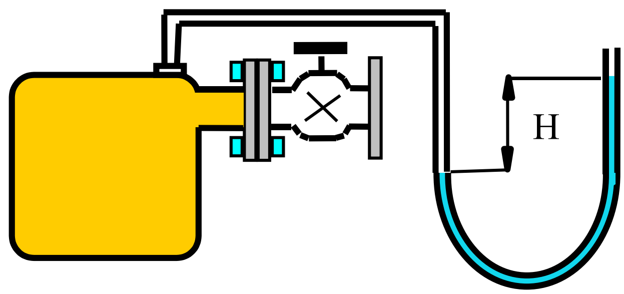
Since the manometer shows a pressure difference we have

$$\Delta P = P_{CYL} - P_{atm} = \rho L g$$

$$L = \Delta P / \rho g = \frac{(135 - 101)\text{ kPa}}{997\text{ kg m}^{-3} \times 10 \times 9.807\text{ m/s}^2} \frac{1000\text{ Pa}}{\text{kPa}}$$

$= \mathbf{3.467\ m}$



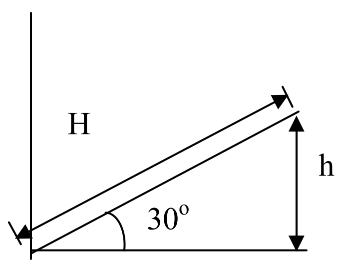


2.71

A U-tube manometer filled with water, density 1000 kg/m^3, shows a height difference of 25 cm. What is the gauge pressure? If the right branch is tilted to make an angle of 30° with the horizontal, as shown in Fig. P2.71, what should the length of the column in the tilted tube be relative to the U-tube?

Solution:

Same height in the two sides in the direction of g.



$$\Delta P = F/A = mg/A = V\rho g/A = h\rho g$$
$$= 0.25 \text{ m} \times 1000 \text{ kg/m}^3 \times 9.807 \text{m/s}^2$$
$$= 2452.5 \text{ Pa}$$
$$= \mathbf{2.45 \text{ kPa}}$$

$$h = H \times \sin 30°$$
$$\Rightarrow H = h/\sin 30° = 2h = \mathbf{50 \text{ cm}}$$

2.72

A pipe flowing light oil has a manometer attached as shown in Fig. P2.72. What is the absolute pressure in the pipe flow?

Solution:

Table A.3: $\rho_{oil} = 910 \text{ kg/m}^3$; $\rho_{water} = 997 \text{ kg/m}^3$

$$P_{BOT} = P_0 + \rho_{water}\, g\, H_{tot} = P_0 + 997 \text{ kg/m}^3 \times 9.807 \text{ m/s}^2 \times 0.8 \text{ m}$$
$$= P_o + 7822 \text{ Pa}$$

$$P_{PIPE} = P_{BOT} - \rho_{water}\, g\, H_1 - \rho_{oil}\, g\, H_2$$
$$= P_{BOT} - 997 \times 9.807 \times 0.1 - 910 \times 9.807 \times 0.2$$
$$= P_{BOT} - 977.7 \text{ Pa} - 1784.9 \text{ Pa}$$

$$P_{PIPE} = P_o + (7822 - 977.7 - 1784.9) \text{ Pa}$$
$$= P_o + 5059.4 \text{ Pa} = 101.325 + 5.06 = \mathbf{106.4\ kPa}$$

2.73

The difference in height between the columns of a manometer is 200 mm with a fluid of density 900 kg/m^3. What is the pressure difference? What is the height difference if the same pressure difference is measured using mercury, density 13600 kg/ m^3, as manometer fluid?

Solution:

$$\Delta P = \rho_1 g h_1 = 900 \text{ kg/m}^3 \times 9.807 \text{ m/s}^2 \times 0.2 \text{ m} = 1765.26 \text{ Pa} = \mathbf{1.77\ kPa}$$

$$h_{Hg} = \Delta P / (\rho_{hg}\ g) = (\rho_1\ g h_1) / (\rho_{hg}\ g) = \frac{900}{13600} \times 0.2$$

$$= \mathbf{0.0132\ m = 13.2\ mm}$$

2.74

Two cylinders are filled with liquid water, $\rho = 1000 \text{ kg/m}^3$, and connected by a line with a closed valve. A has 100 kg and B has 500 kg of water, their cross-sectional areas are $A_A = 0.1 \text{ m}^2$ and $A_B = 0.25 \text{ m}^2$ and the height h is 1 m. Find the pressure on each side of the valve. The valve is opened and water flows to an equilibrium. Find the final pressure at the valve location.

Solution:

$$V_A = v_{H_2O} m_A = m_A/\rho = 0.1 = A_A h_A \quad => \quad h_A = 1 \text{ m}$$

$$V_B = v_{H_2O} m_B = m_B/\rho = 0.5 = A_B h_B \quad => \quad h_B = 2 \text{ m}$$

$P_{VB} = P_0 + \rho g(h_B + H) = 101325 + 1000 \times 9.81 \times 3 = 130\ 755 \text{ Pa}$

$P_{VA} = P_0 + \rho g h_A = 101325 + 1000 \times 9.81 \times 1 = 111\ 135 \text{ Pa}$

Equilibrium: same height over valve in both

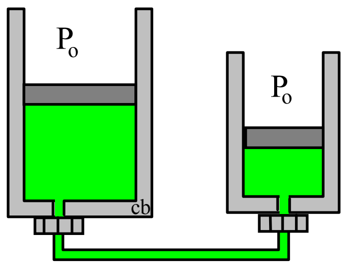
$$V_{tot} = V_A + V_B = h_2 A_A + (h_2 - H)A_B \Rightarrow h_2 = \frac{h_A A_A + (h_B + H)A_B}{A_A + A_B} = 2.43 \text{ m}$$

$P_{V2} = P_0 + \rho g h_2 = 101.325 + (1000 \times 9.81 \times 2.43)/1000 = \mathbf{125.2\ kPa}$

2.75

Two piston/cylinder arrangements, A and B, have their gas chambers connected by a pipe. Cross-sectional areas are A_A = 75 cm^2 and A_B = 25 cm^2 with the piston mass in A being m_A = 25 kg. Outside pressure is 100 kPa and standard gravitation. Find the mass m_B so that none of the pistons have to rest on the bottom.

Solution:



Force balance for both pistons: $F\uparrow = F\downarrow$

A: $m_{PA}g + P_0A_A = PA_A$

B: $m_{PB}g + P_0A_B = PA_B$

Same P in A and B gives no flow between them.

$$\frac{m_{PA}g}{A_A} + P_0 = \frac{m_{PB}g}{A_B} + P_0$$

=> $m_{PB} = m_{PA}\ A_A/\ A_B\ = 25 \times 25/75 =$ **8.33 kg**

2.76

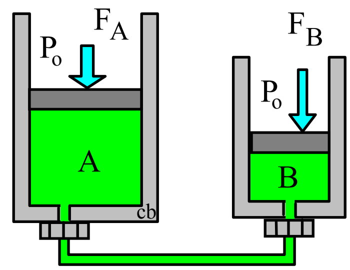
Two hydraulic piston/cylinders are of same size and setup as in Problem 2.75, but with negligible piston masses. A single point force of 250 N presses down on piston A. Find the needed extra force on piston B so that none of the pistons have to move.

Solution:

$A_A = 75 \text{ cm}^2$;

$A_B = 25 \text{ cm}^2$

No motion in connecting pipe: $P_A = P_B$



Forces on pistons balance

$$P_A = P_0 + F_A / A_A = P_B = P_0 + F_B / A_B$$

$$F_B = F_A \times \frac{A_B}{A_A} = 250 \times \frac{25}{75} = \mathbf{83.33\ N}$$

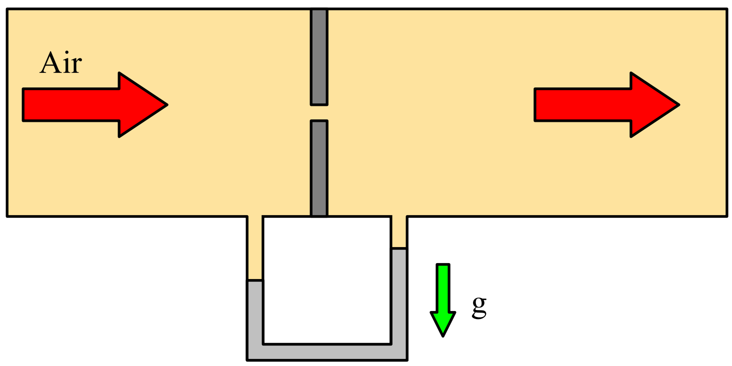
2.77

A piece of experimental apparatus is located where $g = 9.5$ m/s^2 and the temperature is 5°C. An air flow inside the apparatus is determined by measuring the pressure drop across an orifice with a mercury manometer (see Problem 2.79 for density) showing a height difference of 200 mm. What is the pressure drop in kPa?

Solution:

$\Delta P = \rho g h$; $\rho_{Hg} = 13600$ kg/m^3

$\Delta P = 13\,600$ kg/m^3 $\times$ 9.5 m/s^2 $\times$ 0.2 m = 25840 Pa = **25.84 kPa**

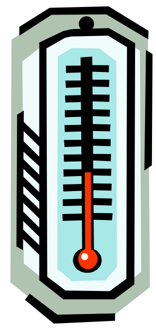


Temperature

2.78

What is a temperature of $-5^{o}C$ in degrees Kelvin?

Solution:



The offset from Celsius to Kelvin is 273.15 K, so we get

$$T_K = T_C + 273.15 = -5 + 273.15$$
$$= \mathbf{268.15\ K}$$

2.79

The density of mercury changes approximately linearly with temperature as

$$\rho_{Hg} = 13595 - 2.5\,T \text{ kg/ m}^3 \qquad T \text{ in Celsius}$$

so the same pressure difference will result in a manometer reading that is influenced by temperature. If a pressure difference of 100 kPa is measured in the summer at 35°C and in the winter at −15°C, what is the difference in column height between the two measurements?

Solution:

The manometer reading h relates to the pressure difference as

$$\Delta P = \rho\, L\, g \quad \Rightarrow \quad L = \frac{\Delta P}{\rho g}$$

The manometer fluid density from the given formula gives

$$\rho_{su} = 13595 - 2.5 \times 35 = 13507.5 \text{ kg/m}^3$$

$$\rho_{w} = 13595 - 2.5 \times (-15) = 13632.5 \text{ kg/m}^3$$

The two different heights that we will measure become

$$L_{su} = \frac{100 \times 10^3}{13507.5 \times 9.807} \frac{\text{kPa (Pa/kPa)}}{(\text{kg/m}^3)\text{ m/s}^2} = 0.7549 \text{ m}$$

$$L_{w} = \frac{100 \times 10^3}{13632.5 \times 9.807} \frac{\text{kPa (Pa/kPa)}}{(\text{kg/m}^3)\text{ m/s}^2} = 0.7480 \text{ m}$$

$\Delta L = L_{su} - L_{w} =$ **0.0069 m = 6.9 mm**

2.80

A mercury thermometer measures temperature by measuring the volume expansion of a fixed mass of liquid Hg due to a change in the density, see problem 2.35. Find the relative change (%) in volume for a change in temperature from 10°C to 20°C.

Solution:

From 10°C to 20°C

At 10°C : $\rho_{Hg} = 13595 - 2.5 \times 10 = 13570 \text{ kg/m}^3$

At 20°C : $\rho_{Hg} = 13595 - 2.5 \times 20 = 13545 \text{ kg/m}^3$

The volume from the mass and density is: $V = m/\rho$

$$\text{Relative Change} = \frac{V_{20} - V_{10}}{V_{10}} = \frac{(m/\rho_{20}) - (m/\rho_{10})}{m/\rho_{10}}$$

$$= \frac{\rho_{10}}{\rho_{20}} - 1 = \frac{13570}{13545} - 1 = \mathbf{0.0018\ (0.18\%)}$$

2.81

Density of liquid water is $\rho = 1008 - T/2$ [kg/m^3] with T in ^{o}C. If the temperature increases 10^{o}C how much deeper does a 1 m layer of water become?

Solution:

The density change for a change in temperature of 10^{o}C becomes

$$\Delta\rho = -\Delta T/2 = -5 \text{ kg/m}^3$$

from an ambient density of

$$\rho = 1008 - T/2 = 1008 - 25/2 = 995.5 \text{ kg/m}^3$$

Assume the area is the same and the mass is the same $m = \rho V = \rho A H$, then we have

$$\Delta m = 0 = V\Delta\rho + \rho\Delta V \quad \Rightarrow \quad \Delta V = - V\Delta\rho/\rho$$

and the change in the height is

$$\Delta H = \frac{\Delta V}{A} = \frac{H\Delta V}{V} = \frac{-H\Delta\rho}{\rho} = \frac{-1 \times (-5)}{995.5} = \mathbf{0.005\ m}$$

barely measurable.

2.82

Using the freezing and boiling point temperatures for water in both Celsius and Fahrenheit scales, develop a conversion formula between the scales. Find the conversion formula between Kelvin and Rankine temperature scales.

Solution:

$$T_{Freezing} = 0\ ^{o}C = 32\ F; \qquad T_{Boiling} = 100\ ^{o}C = 212\ F$$

$$\Delta T = 100\ ^{o}C = 180\ F \ \Rightarrow \ T_{oC} = (T_F - 32)/1.8 \quad \text{or} \quad T_F = 1.8\ T_{oC} + 32$$

For the absolute K & R scales both are zero at absolute zero.

$$T_R = 1.8 \times T_K$$

2.83

The atmosphere becomes colder at higher elevation. As an average the standard atmospheric absolute temperature can be expressed as $T_{atm} = 288 - 6.5 \times 10^{-3}\,z$, where z is the elevation in meters. How cold is it outside an airplane cruising at 12 000 m expressed in Kelvin and in Celsius?

Solution:

For an elevation of $z = 12\,000$ m we get

$$T_{atm} = 288 - 6.5 \times 10^{-3}\,z = \mathbf{210\ K}$$

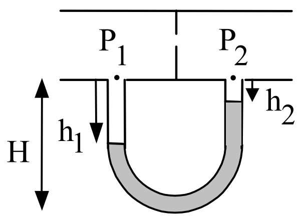
To express that in degrees Celsius we get

$$T_C = T - 273.15 = \mathbf{-63.15^{o}C}$$

2.84

Repeat problem 2.77 if the flow inside the apparatus is liquid water, $\rho \cong 1000$ kg/m^3, instead of air. Find the pressure difference between the two holes flush with the bottom of the channel. You cannot neglect the two unequal water columns.

Solution:



Balance forces in the manometer:

$$(H - h_2) - (H - h_1) = \Delta h_{Hg} = h_1 - h_2$$

$$P_1A + \rho_{H_2O}h_1gA + \rho_{Hg}(H - h_1)gA$$
$$= P_2A + \rho_{H_2O}h_2gA + \rho_{Hg}(H - h_2)gA$$

$$\Rightarrow P_1 - P_2 = \rho_{H_2O}(h_2 - h_1)g + \rho_{Hg}(h_1 - h_2)g$$

$$P_1 - P_2 = \rho_{Hg}\Delta h_{Hg}g - \rho_{H_2O}\Delta h_{Hg}g = 13\,600 \times 0.2 \times 9.5 - 1000 \times 0.2 \times 9.5$$

$= 25\,840 - 1900 = 23940$ Pa = **23.94 kPa**

2.85

A dam retains a lake 6 m deep. To construct a gate in the dam we need to know the net horizontal force on a 5 m wide and 6 m tall port section that then replaces a 5 m section of the dam. Find the net horizontal force from the water on one side and air on the other side of the port.

Solution:

$P_{bot} = P_0 + \Delta P$

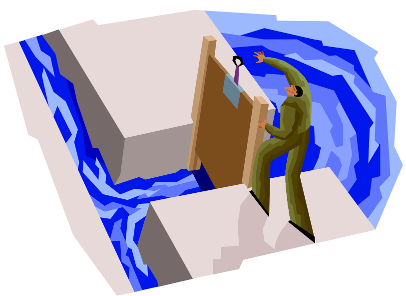
$\Delta P = \rho g h = 997 \times 9.807 \times 6 = 58\ 665 \text{ Pa} = 58.66 \text{ kPa}$

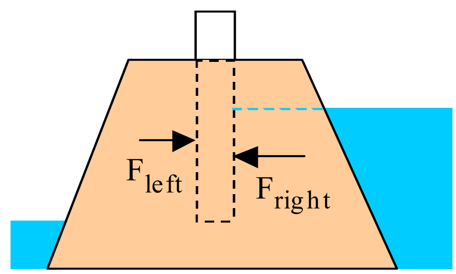
Neglect ΔP in air

$F_{net} = F_{right} - F_{left} = P_{avg} A - P_0 A$

$P_{avg} = P_0 + 0.5\ \Delta P$ Since a linear pressure variation with depth.

$F_{net} = (P_0 + 0.5\ \Delta P)A - P_0 A = 0.5\ \Delta P\ A = 0.5 \times 58.66 \times 5 \times 6 =$ **880 kN**



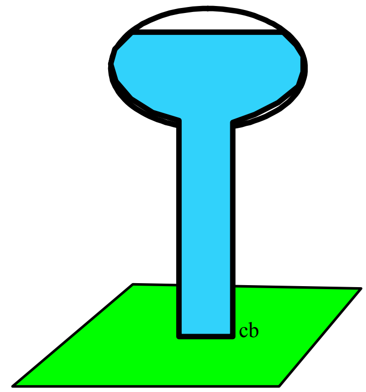


2.86

In the city water tower, water is pumped up to a level 25 m above ground in a pressurized tank with air at 125 kPa over the water surface. This is illustrated in Fig. P2.86. Assuming the water density is 1000 kg/m^3 and standard gravity, find the pressure required to pump more water in at ground level.

Solution:

$$\Delta P = \rho\ L\ g$$
$$= 1000\ \text{kg/m}^3 \times 25\ \text{m} \times 9.807\ \text{m/s}^2$$
$$= 245\ 175\ \text{Pa} = 245.2\ \text{kPa}$$
$$P_{bottom} = P_{top} + \Delta P$$
$$= 125 + 245.2$$
$$= \mathbf{370\ kPa}$$



2.87

The main waterline into a tall building has a pressure of 600 kPa at 5 m elevation below ground level. How much extra pressure does a pump need to add to ensure a water line pressure of 200 kPa at the top floor 150 m above ground?

Solution:

The pump exit pressure must balance the top pressure plus the column ΔP. The pump inlet pressure provides part of the absolute pressure.

$P_{after\ pump} = P_{top} + \Delta P$

$\Delta P = \rho g h = 997\ kg/m^3 \times 9.807\ m/s^2 \times (150 + 5)\ m$

$= 1\ 515\ 525\ Pa = 1516\ kPa$

$P_{after\ pump} = 200 + 1516 = 1716\ kPa$

$\Delta P_{pump} = 1716 - 600 =$ **1116 kPa**

2.88

Two cylinders are connected by a piston as shown in Fig. P2.88. Cylinder A is used as a hydraulic lift and pumped up to 500 kPa. The piston mass is 25 kg and there is standard gravity. What is the gas pressure in cylinder B?

Solution:

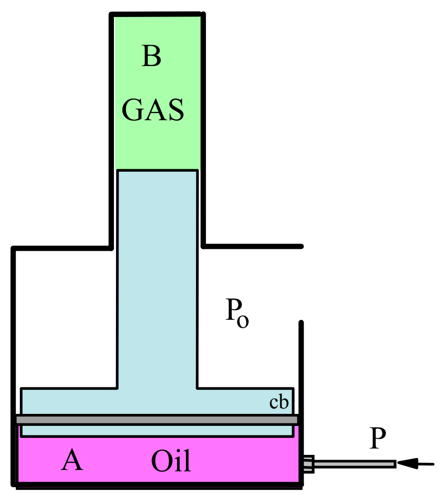
Force balance for the piston: $P_B A_B + m_p g + P_0(A_A - A_B) = P_A A_A$

$A_A = (\pi/4)0.1^2 = 0.00785 \text{ m}^2$; $A_B = (\pi/4)0.025^2 = 0.000\,491 \text{ m}^2$

$P_B A_B = P_A A_A - m_p g - P_0(A_A - A_B) = 500\times 0.00785 - (25 \times 9.807/1000)$

$- 100\,(0.00785 - 0.000\,491) = 2.944 \text{ kN}$

$P_B = 2.944/0.000\,491 = 5996 \text{ kPa} = \mathbf{6.0\ MPa}$



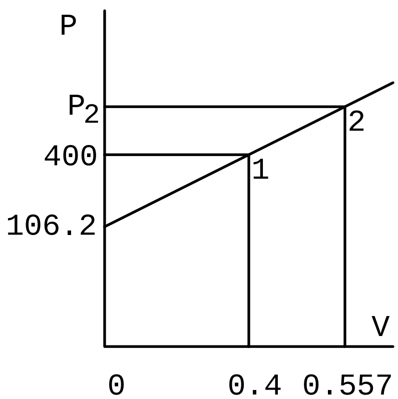
2.89

A 5-kg piston in a cylinder with diameter of 100 mm is loaded with a linear spring and the outside atmospheric pressure of 100 kPa as shown in Fig. P2.89. The spring exerts no force on the piston when it is at the bottom of the cylinder and for the state shown, the pressure is 400 kPa with volume 0.4 L. The valve is opened to let some air in, causing the piston to rise 2 cm. Find the new pressure.

Solution:

A linear spring has a force linear proportional to displacement. F = k x, so the equilibrium pressure then varies linearly with volume: P = a + bV, with an intersect a and a slope b = dP/dV. Look at the balancing pressure at zero volume (V -> 0) when there is no spring force $F = PA = P_oA + m_pg$ and the initial state. These two points determine the straight line shown in the P-V diagram.

Piston area = $A_P = (\pi/4) \times 0.1^2 = 0.00785 \text{ m}^2$



$$a = P_0 + \frac{m_p g}{A_p} = 100 \text{ kPa} + \frac{5 \times 9.80665}{0.00785} \text{ Pa}$$

$= 106.2$ kPa intersect for zero volume.

$$V_2 = 0.4 + 0.00785 \times 20 = 0.557 \text{ L}$$

$$P_2 = P_1 + \frac{dP}{dV} \Delta V$$

$$= 400 + \frac{(400\text{-}106.2)}{0.4 - 0} (0.557 - 0.4)$$

= **515.3 kPa**