Chapter 07 Solved Problems

**Problem 7.21**

A room is heated with a 1500 W electric heater. How much power can be saved if a heat pump with a COP of 2.0 is used instead?

Assume the heat pump has to deliver 1500 W as the $\dot{Q}_H$.

Heat pump: $\beta' = \dot{Q}_H/\dot{W}_{IN}$

$$\dot{W}_{IN} = \dot{Q}_H/\beta' = \frac{1500}{2} = 750 \text{ W}$$

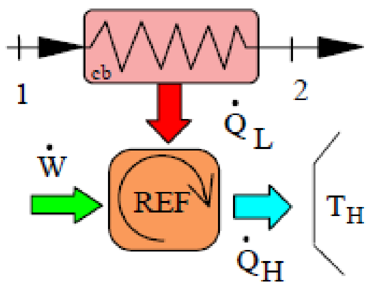
So the heat pump requires an input of 750 W thus saving the difference

$$\dot{W}_{saved} = 1500 \text{ W} - 750 \text{ W} = \mathbf{750 \text{ W}}$$

**Problem 7.25**

The mass flow rate is

$$\dot{m} = \rho\dot{V} = \frac{25 \times 10^{-3}}{0.001002}\frac{1}{3600} \text{ kg/s} = 6.93 \text{ g/s}$$




Energy equation for heat exchanger

$$\dot{Q}_L = \dot{m}(h_1 - h_2) = \dot{m}\, C_P\, (T_1 - T_2)$$
$$= 6.93 \times 10^{-3} \times 4.18 \times (18 - 10) = 0.2318 \text{ kW}$$

$$\beta = \text{COP} = \dot{Q}_L / \dot{W} \quad \Rightarrow \quad \dot{W} = \dot{Q}_L / \beta = 0.2318 / 2.5 = \mathbf{0.093 \text{ kW}}$$

**Problem 7.31**

a. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 4$ kW, $\dot{W} = 2$ kW

b. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 0$ kW, $\dot{W} = 6$ kW

c. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 2$ kW, $\dot{W} = 5$ kW

d. $\dot{Q}_H = 6$ kW, $\dot{Q}_L = 6$ kW, $\dot{W} = 0$ kW

Solution:

| | $1^{st}$. law | $2^{nd}$ law |
|---|---|---|
| a | Yes | Yes (possible) |
| b | Yes | No, impossible Kelvin - Planck |
| c | No | Yes, but energy not conserved |
| d | Yes | Yes (Irreversible $\dot{Q}$ over $\Delta T$) |

**Problem 7.59**

$T_H = T_{sat} = 138.88^oC = 412 \text{ K}, \quad h_{fg} = 2148.1 \text{ kJ/kg}$

$$\beta_{HP\ Carnot} = \frac{\dot{Q}_H}{\dot{W}_{in}} = \frac{T_H}{T_H - T_L} = \frac{412}{138.88 - 80} = 7$$

$\beta_{HP\ ac} = 0.6 \times 7 = 4.2 = \dot{Q}_H / \dot{W}_{in}$

$\dot{Q}_H = 4.2\ \dot{W}_{in} = 4.2 \times 2.5 \text{ MW} = 10.5 \text{ MW} = \dot{m}\, h_{fg}$

$\dot{m} = \dot{Q}_H / h_{fg} = 10\ 500 \text{ kW} / 2148.1 \text{ kJ/kg} = \mathbf{4.89\ kg/s}$

**Problem 7.65**

$\dot{W}_{NET} = 10^6 \text{ kW}, \quad T_H = 550°C = 823.3 \text{ K}$

$P_{COND} = 10 \text{ kPa} \rightarrow T_L = T_G\ (P = 10 \text{ kPa}) = 45.8°C = 319 \text{ K}$

$$\eta_{TH\ CARNOT} = \frac{T_H - T_L}{T_H} = \frac{823.2 - 319}{823.2} = 0.6125$$

$$\Rightarrow \dot{Q}_{L\ MIN} = 10^6\left(\frac{1 - 0.6125}{0.6125}\right) = 0.6327 \times 10^6 \text{ kW}$$

But $\dot{m}_{H_2O} = \dfrac{60 \times 8 \times 10/60}{0.001} = 80\ 000 \text{ kg/s}$ having an energy flow of

$$\dot{Q}_{L\ MIN} = \dot{m}_{H_2O}\ \Delta h = \dot{m}_{H_2O}\ C_{P\ LIQ\ H_2O}\ \Delta T_{H_2O\ MIN}$$

$$\Rightarrow \Delta T_{H_2O\ MIN} = \frac{\dot{Q}_{L\ MIN}}{\dot{m}_{H_2O} C_{P\ LIQ\ H_2O}} = \frac{0.6327 \times 10^6}{80000 \times 4.184} = \mathbf{1.9°C}$$

**Problem 7.90**

$q_H = 250 \text{ kJ/kg}, \quad T_H = 600 \text{ K}, \quad T_L = 300 \text{ K}, \quad P_3 = 75 \text{ kPa}$

The states as shown in figure 7.21

1: 600 K , 2: 600 K, 3: 75 kPa, 300 K 4: 300 K

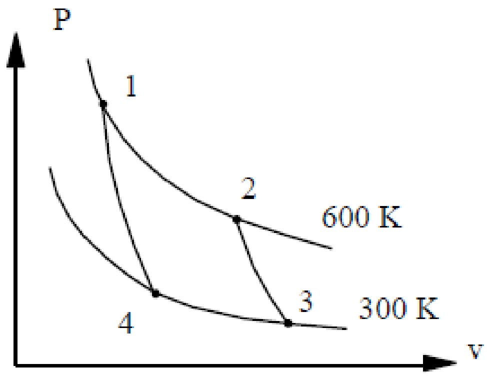
Since this is a Carnot cycle and we know the temperatures the efficiency is

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{600} = 0.5$$

and the net work becomes

$$w_{NET} = \eta q_H = 0.5 \times 250$$

$$= \mathbf{125\ kJ/kg}$$




The heat rejected is

$$q_L = q_H - w_{NET} = 125\ \text{kJ/kg}$$

After heat rejection is state 4. From equation 7.9

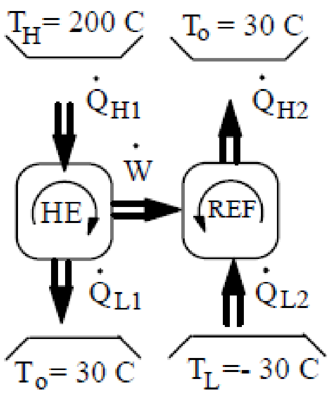
$3 \rightarrow 4$ Eq.7.9 : $\quad q_L = RT_L \ln (v_3/v_4)$

$v_3 = RT_3 / P_3 = 0.287 \times 300 / 75 = 1.148\ \text{m}^3\text{/kg}$

$v_4 = v_3 \exp(-q_L/RT_L) = 1.148 \exp(-125/0.287 \times 300) = \mathbf{0.2688\ m^3/kg}$

$P_4 = RT_4 / v_4 = 0.287 \times 300 / 0.2688 = \mathbf{320\ kPa}$

**Problem 7.96**




$$W = Q_{H1}\left(\frac{T_H - T_0}{T_H}\right)$$

also

$$W = Q_{L2}\left(\frac{T_0 - T_L}{T_L}\right)$$

$$\frac{Q_{H1}}{Q_{L2}} = \left(\frac{T_o - T_L}{T_L}\right)\left(\frac{T_H}{T_H - T_o}\right) = \left(\frac{60}{243.2}\right)\left(\frac{473.2}{170}\right) = \mathbf{0.687}$$

**Problem 7.101**

Waste supply: $\quad \dot{Q}_{w1} + \dot{Q}_{w2} = 5\ \text{MW}$

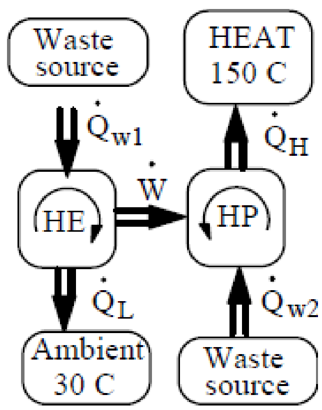
Heat Engine:

$$\dot{W} = \eta\, \dot{Q}_{w1} = (1 - T_{L1} / T_{H1})\, \dot{Q}_{w1}$$

Heat pump:

$$\dot{W} = \dot{Q}_H / \beta_{HP} = \dot{Q}_{w2} / \beta'$$

$$= \dot{Q}_{w2} / [T_{H1} / (T_H - T_{H1})]$$



Equate the two work terms:

$$( 1 - T_{L1} / T_{H1} ) \dot{Q}_{w1} = \dot{Q}_{w2} \times (T_H - T_{H1} ) / T_{H1}$$

Substitute $\dot{Q}_{w1} = 5 \text{ MW} - \dot{Q}_{w2}$

$$(1 - 303.15/323.15)(5 - \dot{Q}_{w2} ) = \dot{Q}_{w2} \times (150 - 50) / 323.15$$

$$20 ( 5 - \dot{Q}_{w2} ) = \dot{Q}_{w2} \times 100 \quad \Rightarrow \quad \dot{Q}_{w2} = 0.8333 \text{ MW}$$

$$\dot{Q}_{w1} = 5 - 0.8333 = 4.1667 \text{ MW}$$

$$\dot{W} = \eta\, \dot{Q}_{w1} = 0.06189 \times 4.1667 = 0.258 \text{ MW}$$

$\dot{Q}_H = \dot{Q}_{w2} + \dot{W} =$ **1.09 MW**

(For the heat pump $\beta' = 423.15 / 100 = 4.23$)