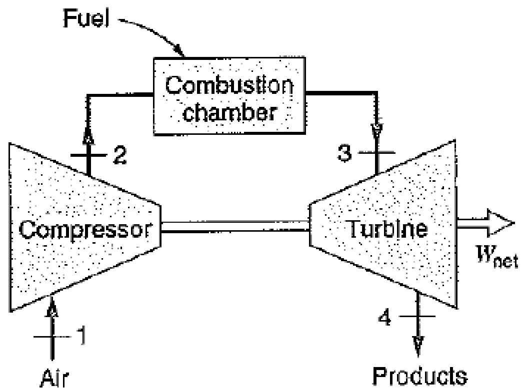
## Chapter 09 Extra Problems

### Extra problem 1

In a gas turbine operating at steady state, air enters the compressor with a mass flow rate of 5 kg/s at 0.1 MPa and 15°C and exits at 10 MPA. The air then passes through a heat exchanger where it experiences a pressure drop of 15 kPa before entering the turbine at 1100 °C. Air exits the turbine 0.1 MPa. The compressor and turbine operate adiabatically and kinetic and potential energy effects can be ignored.
Assuming ideal gas model with constant specific heats at 300K, determine the *net* power developed by the plant, in kJ/kg of air, if the compressor and turbine isentropic efficiencies are 80 and 85%, respectively.




*Control volume:* Compressor.
*Inlet state:* $P_1$, $T_1$ known; state fixed.
*Exit state:* $P_2$ known.

The first law for the real process is $w_c = h_2 - h_1$

The second law for the ideal process is $s_{2_s} = s_1$

$$\text{so that} \quad \frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

In addition, $\eta_c = \dfrac{h_{2_s} - h_1}{h_2 - h_1} = \dfrac{T_{2_s} - T_1}{T_2 - T_1}$

Solving for $T_{2_s}$ we get

$$\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = \frac{T_{2_s}}{T_1} = 10^{0.286} = 1.932, \qquad T_{2s} = 556.8\ \text{K}$$

The efficiency is $\eta_c = \dfrac{h_{2_s} - h_1}{h_2 - h_1} = \dfrac{T_{2_s} - T_1}{T_2 - T_1} = \dfrac{556.8 - 288.2}{T_2 - T_1} = 0.80$

Therefore, $T_2 - T_1 = \dfrac{556.8 - 288.2}{0.80} = 335.8, \qquad T_2 = 624.0\ \mathrm{K}$

$$\begin{aligned} w_c &= h_2 - h_1 = C_p(T_2 - T_1) \\ &= 1.004(624.0 - 288.2) = 337.0\ \mathrm{kJ/kg} \end{aligned}$$

For the turbine, we have:

*Control volume:* Turbine.
*Inlet state:* $P_3(P_2 - \text{drop})$ known, $T_3$ known; state fixed.
*Exit state:* $P_4$ known.

The first law for the real process is $w_t = h_3 - h_4$
The second law for the ideal process is $s_{4_s} = s_3$

so that $\dfrac{T_3}{T_{4_s}} = \left(\dfrac{P_3}{P_4}\right)^{(k-1)/k}$

In addition, $\eta_t = \dfrac{h_3 - h_4}{h_3 - h_{4_s}} = \dfrac{T_3 - T_4}{T_3 - T_{4_s}}$

Substituting numerical values, we obtain

$$P_3 = P_2 - \text{pressure drop} = 1.0 - 0.015 = 0.985\ \mathrm{MPa}$$

$$\left(\frac{P_3}{P_4}\right)^{(k-1)/k} = \frac{T_3}{T_{4_s}} = 9.85^{0.286} = 1.9236, \qquad T_{4_s} = 713.9\ \mathrm{K}$$

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4_s}} = \frac{T_3 - T_4}{T_3 - T_{4_s}} = 0.85$$

$$T_3 - T_4 = 0.85(1373.2 - 713.9) = 560.4\ \mathrm{K}$$

$$\begin{aligned} T_4 &= 812.8\ \mathrm{K} \\ w_t &= h_3 - h_4 = C_p(T_3 - T_4) \\ &= 1.004(1373.2 - 812.8) = 562.4\ \mathrm{kJ/kg} \\ w_{net} &= w_t - w_c = 562.4 - 337.0 = 225.4\ \mathrm{kJ/kg} \end{aligned}$$

Finally, for the heat exchanger:

*Control volume:* High-temperature heat exchanger.
*Inlet state:* State 2 fixed (as given).
*Exit state:* State 3 fixed (as given).
The first law is $q_H = h_3 - h_2$

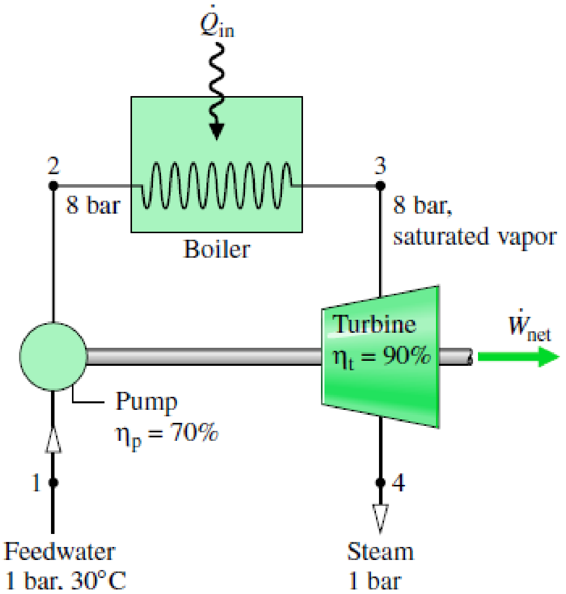
Substituting, $q_H = h_3 - h_2 = C_p(T_3 - T_2) = 1.004(1373.2 - 624.0) = 751.8\ \mathrm{kJ/kg}$

so that $\eta_{th} = \dfrac{w_{net}}{q_H} = \dfrac{225.4}{751.8} = 30.0\%$

**Extra problem 2**

3 devices operating at steady state: a pump, a boiler, and a turbine are shown on the figure below. The turbine provides the power required to drive the pump and also supplies power to other devices. For adiabatic operation of the pump and turbine, and ignoring kinetic and potential energy effects, determine, in kJ per kg of steam flowing:

1. the work required by the pump.
2. the net work developed by the turbine.
3. the heat transfer to the boiler




The work input to the pump if it were isentropic:

$$w_{Ps} = h_{2s} - h_1 = v_1(P_2 - P_1)$$

Fixing state 1: Compressed liquid: $v_1 = v_f(T_1)$ from table B1.1

Then, using the isentropic efficiency of the pump, the actual pump work becomes:

CV Pump $w_{Pac} = w_{Ps} / \eta_P = h_{2a} - h_1$

$$h_{2a} = w_{Pac} + h_1$$

CV Boiler: $q_H = h_3 - h_{2a}$

C.V. Turbine : $w_{Ts} = h_3 - h_{4s}$ ; $s_{4s} = s_3$ => $x_{4s}$ => $h_{4s}$

The actual work developed by the turbine can be found using the isentropic efficiency of the turbine:

$$w_{Tac} = w_{Ts} \times \eta_T$$

$$\eta_{cycle} = \frac{w_{Tac} - w_{Pac}}{q_H}$$