

MECH 310 THERMODYNAMICS I
FINAL (OPEN BOOK 180 MINUTES) SOLUTIONS

2nd June 2010

MULTIPLE CHOICE QUESTIONS

1. D
2. D
3. A
4. C
5. C
6. D
7. A
8. A
9. D
10. B
11. B
12. D
13. C
14. D
15. D
16. B
17. A
18. D
19. D
20. A

PROBLEM 1

Insider diameter of the cylinder = 8 cm

Stiffness of the spring, $S = 150 \text{ N/cm}$

Initial pressure of air, $p_1 = 3 \times 10^5 \text{ N/m}^2$ or 30 N/cm^2

Initial volume of air, $V_1 = 0.000045 \text{ m}^3 = 45 \text{ cm}^3$

Initial temperature of air, $T_1 = 20 + 273 = 293 \text{ K}$

Specific heat at constant volume, $c_v = 0.71 \text{ kJ/kg K}$

Universal Gas constant for air, $R = 0.287 \text{ kJ/kg K}$

See Figure Below,

Let, $00 =$ An arbitrary datum from which the position of the lower face of the piston is to be measured,

$y =$ Distance of the lower face of the piston,

$y = y_0$, when spring length is its free length, and

$p =$ Pressure of air within the cylinder when $y = y_0$.

Now, force balance for the piston is given by

$$Ap = S(y - y_0) \tag{i}$$

where, $A =$ The area of the piston, and $S =$ Stiffness of the spring.

With heat transfer to the air, let the pressure inside the cylinder increase by dp forcing the piston to move upward by distance dy . Now the force balance for the piston is

$$A(p + dp) = S(y + dy - y_0) \quad (ii)$$

From eqns. (i) and (ii), we have

$$Adp = Sdy \quad (iii)$$

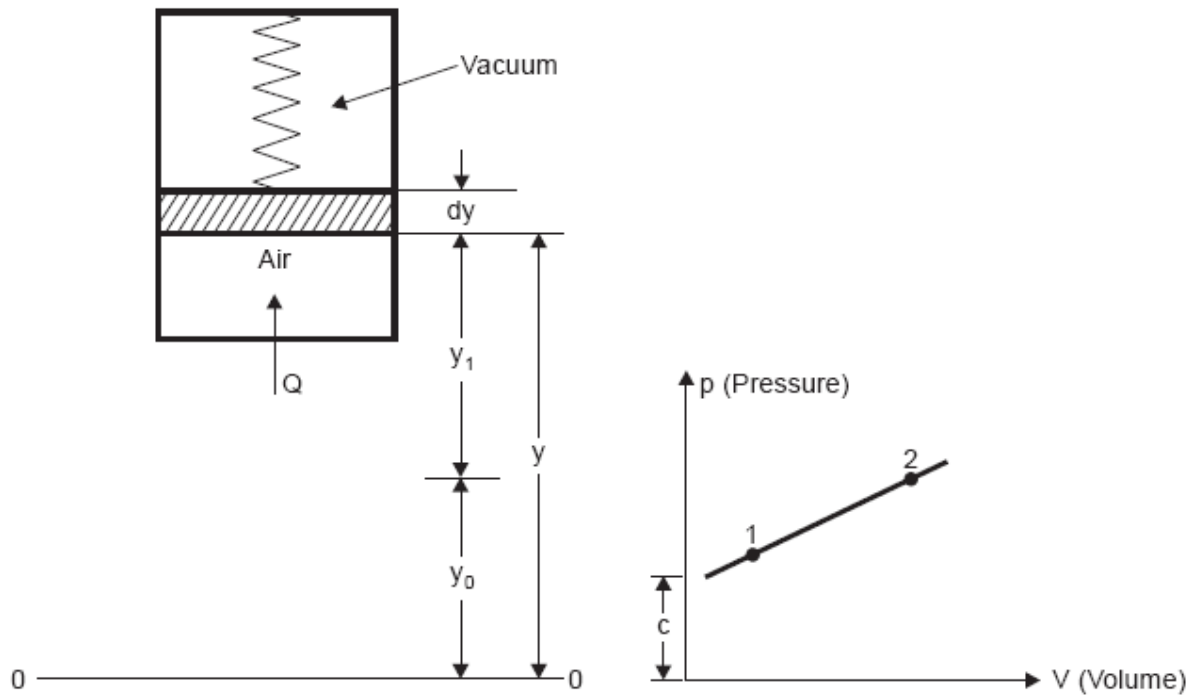
The increase in volume dV of the gas for the piston displacement is given by

$$dV = Ady \quad (iv)$$

$$\therefore dp = \frac{S}{A^2} dV \quad (v)$$

$$\therefore p = \frac{S}{A^2} V + C \quad (vi)$$

The p - V relationship for the process is a straight line (see below) having a slope of $\frac{S}{A^2}$ and pressure axis intercept of C . The value of C can be found out from the knowledge of pressure and volume at any state point.



Now, substituting the values of p_1 , V_1 , A in eqn. (vi), we get:

$$p = \frac{150}{\left(\frac{\pi}{4} \times 8^2\right)^2} V + C$$

$$\text{Or } p = 0.0594 V + C \quad (vii)$$

where p is in N/cm^2 and V is in cm^3 .

$$\therefore p_1 = 0.0594 V_1 + C$$

$$30 = 0.0594 \times 45 + C$$

$$\therefore C = 27.33$$

Hence, p - V relationship for the process is,

$$p = 0.0594 V + 27.33 \quad (viii)$$

During the process the piston is moved by a distance of 3.5 cm. This increases the volume of gas by

$$3.5 A^2 = 3.5 \times \left(\frac{\pi}{4} \times 8^2\right) = 175.9 \text{ cm}^3$$

Hence, the final volume of air, $V_2 = 45 + 175.9 = 220.9 \text{ cm}^3$

Substituting this value in equation (viii), we get

$$p(=p_2) = 0.0594 \times 220.9 + 27.33 = 40.45 \text{ N/cm}^2$$

The work done W during the process is given by

$$\begin{aligned} W &= \frac{1}{2}(p_2 + p_1)(V_2 - V_1) & (ix) \\ &= \frac{1}{2}(40.45 + 35)(220.9 - 45) \\ &= 6196 \text{ N-cm or } 61.96 \text{ N-m} \end{aligned}$$

It may be noted that work done does not cross the system boundary when spring and cylinder are considered system.

Now, to find T_2 , use the ideal gas relation:

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\therefore T_2 = \frac{p_2 V_2 T_1}{p_1 V_1} = \frac{40.45 \times 220.9 \times 293}{30 \times 45} = 1,939.3 \text{ K}$$

The mass can be calculated from

$$m = \frac{p_1 V_1}{RT_1} = \frac{30 \times 45}{0.287 \times 10^3 \times 293 \times 100} = 0.0001605 \text{ kg}$$

Now the change in the internal energy is given by:

$$\Delta U = m \times c_v (T_2 - T_1) = 0.0001605 \times 0.71 \times (1939.3 - 293) = 0.1876 \text{ kJ}$$

According to first law,

$$Q_{1-2} = \Delta U + W = 0.1876 + 61.69 \times 10^{-3} = 0.2495 \text{ kJ}$$

\therefore **Amount of heat added to the system = 0.2495 kJ.**

PROBLEM 2

Assumptions

1. The heat pump operates steadily.
2. The kinetic and potential energy changes are zero.
3. Geothermal water is assumed to be in the liquid form at the inlet temperature given.
4. Steam table properties are used for geothermal water.
5. The pressure drops for the water stream and the R-134a stream in the evaporator are negligible (i.e. constant pressure).

Two intensive properties for R-134a at the inlet and two intensive properties for R-134a at the outlet of the evaporator are specified:

Inlet: $T = 20 \text{ }^\circ\text{C}$ and $x = 0.15$

Outlet: $T = 20 \text{ }^\circ\text{C}$ and $x = 1$ (saturated vapour)

From Table B.5.1 for R-134a, we can determine the enthalpies of R-134a at the inlet and outlet. These are given by:

Inlet ($T = 20\text{ }^{\circ}\text{C}$ and $x = 0.15$),

$$h_{R,in} = h_{R,f} + x h_{R,fg} = 227.49 + 0.15 \times 182.35 = 254.84 \text{ kJ/kg}$$

Outlet ($T = 20\text{ }^{\circ}\text{C}$ and $x = 1$ (saturated vapour)),

$$\text{From Table B.5.1 } h_{R,out} = 409.84 \text{ kJ/kg}$$

Two properties for geothermal water at the inlet and two properties for the geothermal water at the outlet of the evaporator are specified:

Inlet: $T = 50\text{ }^{\circ}\text{C}$ and $x = 0.0$ (Liquid)

Outlet: $T = 40\text{ }^{\circ}\text{C}$ and $x = 0$ (Liquid)

Therefore from Table B.1.1, the enthalpies of water at the inlet and outlet are given by:

$$h_{W,in} = 209.31 \text{ kJ/kg and } h_{W,out} = 167.54 \text{ kJ/kg}$$

The rate of heat transferred from the water is the energy change of the water from inlet to exit (i.e. energy decrease of the water) which is equal to the energy increase of the refrigerant; i.e.:

$$\dot{Q}_L = \dot{m}_w(h_{W,in} - h_{W,out}) = \dot{m}_R(h_{R,out} - h_{R,in})$$

$$\text{From the given information } \dot{Q}_L = 0.065(209.31 - 167.54) = 2,715.1 \text{ W} = \mathbf{2.715 \text{ kW}}$$

$$\text{And } \dot{m}_R = \frac{2.715}{409.84 - 254.84} = \mathbf{0.0175 \text{ kg/s (Answer to part (a))}}$$

(b) The heating Load,

$$\dot{Q}_H = \dot{Q}_L + \dot{W}_{in} = 2.715 + 1.2 = \mathbf{3.915 \text{ kW}}$$

(c) The COP of the Heat Pump is determined from its definition:

$$COP = \frac{\dot{Q}_H}{\dot{W}_{in}} = \frac{3.915}{1.2} = \mathbf{3.27}$$

The COP of a reversible heat pump operating between the same temperature limits is:

$$COP_{max} = \frac{1}{1 - T_L/T_H} = \frac{1}{1 - (25 + 273)/(50 + 273)} = 12.92$$

Therefore the minimum power input to the compressor for the same load would be:

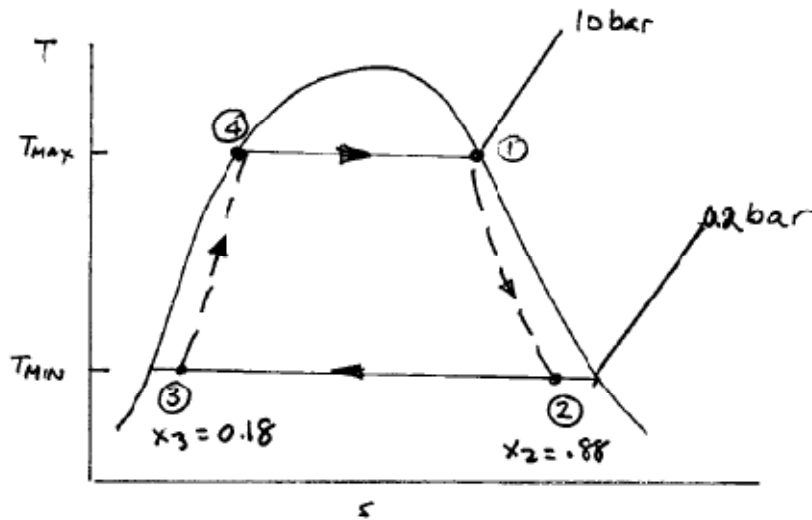
$$\text{(d) } \dot{W}_{in,min} = \frac{\dot{Q}_H}{COP_{max}} = \frac{3.915}{12.92} = \mathbf{0.303 \text{ kW}}$$

PROBLEM 3

Assumptions

1. This is a steady-flow process.
2. Kinetic and potential energy changes are negligible.
3. The only significant heat transfer occurs with the reservoirs.
4. The maximum (highest) and minimum (lowest) temperatures correspond to the saturation temperatures at 10 bar (1,000 kPa) and 0.2 bar (20 kPa) respectively. In other words, and from Table B.1.2, $T_{max} = T_H = 179.91\text{ }^{\circ}\text{C}$ and $T_{min} = T_L = 60.06\text{ }^{\circ}\text{C}$.

(a)



(b) The thermal efficiency for a heat engine is given by (Carnot Efficiency):

$$\text{Carnot Thermal Efficiency} = \frac{\dot{W}_{net}}{\dot{Q}_H} = 1 - \frac{\dot{Q}_L}{\dot{Q}_H}$$

This is usually the maximum possible! For the case in hand the efficiency of our heat engine will be given by:

$$\text{Heat Engine Thermal Efficiency} = 1 - \frac{\dot{Q}_C}{\dot{Q}_B}$$

Where \dot{Q}_C and \dot{Q}_B are the heats transferred in the condenser and boiler respectively. Note the thermal efficiency of the real heat engine is always smaller than the ideal Carnot efficiency!

The heat transfer rates can be determined by carrying out a heat and mass balance over the condenser and boiler to give the following:

$$\begin{aligned}\dot{Q}_C &= \dot{m}_w(h_2 - h_3) \\ \dot{Q}_B &= \dot{m}_w(h_1 - h_4)\end{aligned}$$

Thus the heat engine efficiency can be written as

$$\text{Heat Engine Thermal Efficiency} = 1 - \frac{(h_2 - h_3)}{(h_1 - h_4)}$$

Note that the change in enthalpy in the boiler (i.e. $(h_1 - h_4)$) is equal to h_{fg} , which is the latent heat of evaporation of the water at the specified pressure of 10 bar (1,000 kPa). Therefore from Table B.1.2 we can determine $(h_1 - h_4) = 2015.29$ kJ/kg.

On the other hand, and from the information given in the diagram, we have two independent properties for each of streams 2 and 3 and therefore we can determine the enthalpies from the following equation:

$$\begin{aligned}h_2 &= h_{f,2} + x_2 h_{fg,2} \\ h_3 &= h_{f,3} + x_3 h_{fg,3}\end{aligned}$$

Using Table B.1.2 and at $P=20$ kPa (0.2 bar) we obtain the following:

$$\begin{aligned}h_2 &= 251.38 + 0.18 \times 2358.33 = 675.88 \text{ kJ} \\ h_3 &= 251.38 + 0.88 \times 2358.33 = 2326.71 \text{ kJ}\end{aligned}$$

Substituting in the efficiency equation we get:

$$\text{Heat Engine Thermal Efficiency} = 1 - \frac{(2326.7 - 675.88)}{2015.29} = 0.181$$

Therefore the efficiency of the heat engine is **18.1%**

On the other hand and in view of assumption 4, the Carnot ideal efficiency will be given by:

$$\text{Carnot Thermal Efficiency} = 1 - \frac{(60.06 + 273.15)}{(179.91 + 273.15)} = 0.265$$

Therefore the Carnot efficiency of a heat engine operating between the maximum and minimum temperature is equal to **26.5%**

PROBLEM 4

Assumptions:

- 1 This is a steady-flow process since there is no change with time.
- 2 Kinetic and potential energy changes are negligible.
- 3 The devices are adiabatic and thus heat transfer is negligible.
- 4 Air is an ideal gas with variable specific heats.

Properties:

Two intensive properties for the air at the inlet and two intensive properties for air at the outlet of the compressor are specified:

- Inlet: $T = 295\text{K}$ and $p = 98\text{ kPa}$
Outlet: $T = 620\text{K}$ and $p = 1,000\text{ kPa}$ (1MPa)

Therefore the enthalpies and entropies at the inlet and outlet of the compressor can be determined from Table A.71 (by interpolation for the inlet conditions and directly for the outlet conditions) to give:

$$h_{AIR,in} = 295.45\text{ kJ/kg and } s_{AIR,in}^0 = 6.85224\text{ kJ/kg-K}$$
$$h_{AIR,out} = 628.38\text{ kJ/kg and } s_{AIR,out}^0 = 7.6109\text{ kJ/kg-K}$$

Similarly two properties for steam at the inlet and two properties for the steam at the outlet of the Steam generator are specified:

- Inlet: $T = 500\text{ }^\circ\text{C}$ and $p = 12.5\text{ MPa}$ (12,500 kPa)
Outlet: $p = 10\text{ kPa}$ and $x = 0.92$

Therefore from Tables B.1.3 and B.1.2, the enthalpies and entropies of the steam at the inlet of the generator can be determined:

For inlet conditions and by interpolation from B.1.3, we obtain:

$$h_{STEAM,in} = 3341.08\text{ kJ/kg and } s_{STEAM,in} = 6.47035\text{ kJ/kg-K}$$

For outlet conditions, from Table B.1.2 and using equations, $h_{STEAM,out} = h_f + x_{out}h_{fg}$ and $s_{STEAM,out} = s_f + xs_{fg}$, we obtain:

$$h_{STEAM,out} = 191.8 + (0.92) \times 2392.8282 = 2393.20\text{ kJ/kg and}$$
$$s_{STEAM,out} = 0.6492 + (0.92) \times 7.5010 = 7.55012\text{ kJ/kg-K}$$

There is only one inlet and one exit for either device, and thus $\dot{m}_{in} = \dot{m}_{out} = \dot{m}$. We take either the steam turbine or the compressor as the system, which is a control volume since mass crosses the boundary. The energy balance for either steady-flow system can be expressed in the rate form:

$$\dot{E}_{in} - \dot{E}_{out} = \Delta\dot{E} \quad \text{where for steady state conditions } \Delta\dot{E} \text{ will be equal to zero; i.e. } \dot{E}_{in} = \dot{E}_{out}$$

Therefore for the turbine and compressor the energy balance equations will become:

$$\begin{aligned} \text{Compressor:} \quad & \dot{W}_{compressor,in} = \dot{m}_{AIR}(h_{AIR,out} - h_{AIR,in}) \\ \text{Turbine} \quad & \dot{W}_{turbine,out} = \dot{m}_{STEAM}(h_{STEAM,in} - h_{STEAM,out}) \end{aligned}$$

Substituting the values from the given information we obtain:

$$\begin{aligned} \dot{W}_{compressor,in} &= 10 \times (628.38 - 295.45) = 3,329.3 \text{ kW} \\ \dot{W}_{turbine,out} &= 25 \times (3341.08 - 2393.20) = 23,697 \text{ kW} \end{aligned}$$

Therefore,

$$(a) \quad \dot{W}_{net,out} = \dot{W}_{turbine,out} - \dot{W}_{compressor,in} = 23,697 - 3,329.3 = \mathbf{20,368 \text{ kW}}$$

Noting that the system is adiabatic, the total rate of entropy change (or generation) during this process is the sum of the entropy changes of both fluids (or sum of entropy generated by each process, which are equivalent):

$$\dot{S}_{gen} = \dot{S}_{gen,compressor} + \dot{S}_{gen,turbine} = \dot{m}_{AIR}(s_{AIR,out} - s_{AIR,in}) + \dot{m}_{STEAM}(s_{STEAM,in} - s_{STEAM,out})$$

Where

$$\begin{aligned} \dot{m}_{AIR}(s_{AIR,out} - s_{AIR,in}) &= \dot{m}_{AIR} \left\{ (s_{AIR,out}^0 - s_{AIR,in}^0) - R \ln \frac{P_2}{P_1} \right\} \\ &= 10 \left\{ (7.61090 - 6.85224) - 0.287 \ln \frac{1000}{98} \right\} = 0.92 \text{ kW/K} \end{aligned}$$

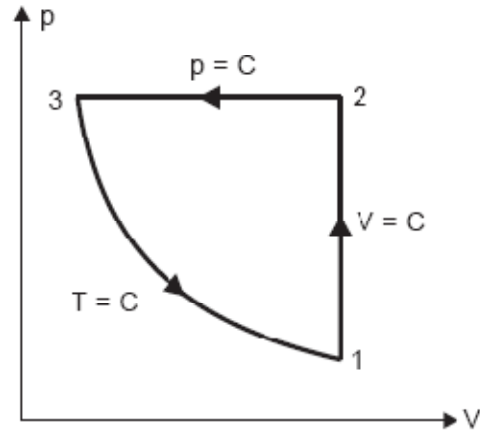
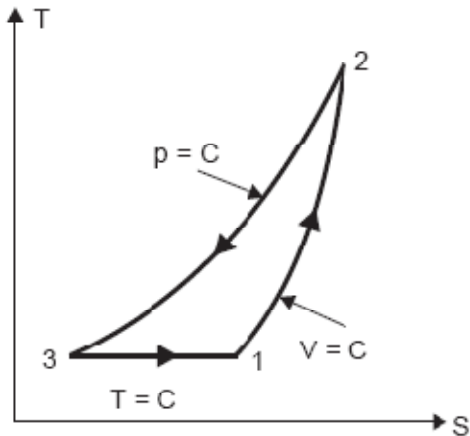
And

$$\dot{m}_{STEAM}(s_{STEAM,in} - s_{STEAM,out}) = 25 \times (7.55012 - 6.47035) = 26.99 \text{ kW/K}$$

Substituting the above two values in the entropy generation equation we get:

$$\dot{S}_{gen} = \dot{S}_{gen,compressor} + \dot{S}_{gen,turbine} = 0.92 + 26.99 = \mathbf{27.91 \text{ kW/K}}$$

PROBLEM 5



Given : $p_1 = 1 \text{ bar}$; $T_1 = 300 \text{ K}$; $V_1 = 0.018 \text{ m}^3$; $p_2 = 5 \text{ bar}$; $c_v = 0.718 \text{ kJ/kg K}$; $R = 0.287 \text{ kJ/kg K}$.

The mass of air can be calculated from: $m = \frac{p_1 V_1}{RT_1} = \frac{10^5 \times 0.018}{0.287 \times 10^3 \times 300} = 0.0209 \text{ kg}$

- Constant volume process 1-2:

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \text{ or } T_2 = T_1 \times \frac{p_2}{p_1} = 300 \times \left(\frac{5}{1}\right) = 1500 \text{ K}$$

$$\begin{aligned} \therefore \text{Change in entropy, } S_2 - S_1 &= mc_v \ln\left(\frac{T_2}{T_1}\right) \\ S_2 - S_1 &= 0.0209 \times 0.718 \times \ln\left(\frac{1500}{300}\right) = \mathbf{0.0241 \text{ kJ/K}} \end{aligned}$$

- Constant pressure process 2-3:

$$T_3 = T_1 = 300 \text{ K}$$

The change in entropy will be given by:

$$\begin{aligned} S_3 - S_2 &= mc_p \ln\left(\frac{T_3}{T_2}\right) = m(c_v + R) \ln\left(\frac{T_3}{T_2}\right) \\ S_3 - S_2 &= 0.0209 \times (0.718 + 0.287) \times \ln\left(\frac{300}{1500}\right) = \mathbf{-0.0338 \text{ kJ/K}} \end{aligned}$$

- Constant temperature (isothermal) process 3-1:

$$p_3 = p_2 = 5 \text{ bar}$$

The change in entropy will be given by:

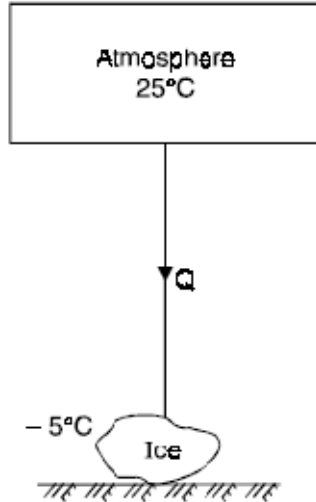
$$\begin{aligned} S_1 - S_3 &= mR \ln\left(\frac{p_3}{p_1}\right) \\ S_1 - S_3 &= 0.0209 \times 0.287 \times \ln\left(\frac{5}{1}\right) = \mathbf{0.00965 \text{ kJ/K}} \end{aligned}$$

\therefore The total net change in entropy will be given by:

$$(S_2 - S_1) + (S_3 - S_2) + (S_1 - S_3) = 0.0241 + (-0.0338) + 0.00965 = \mathbf{0}$$

BONUS PROBLEMS

BP1



Temperature of ice = -5°C ($= -5 + 273 = 268\text{ K}$)

Temperature of atmosphere = 25°C ($= 25 + 273 = 298\text{ K}$)

Heat absorbed by ice from the atmosphere (From Figure above)

$$\begin{aligned} &= \text{Heat absorbed in solid phase} + \text{latent heat} + \text{heat absorbed in liquid phase} \\ &= 1 \times 2.093 \times [0 - (-5)] + 1 \times 333.33 + 1 \times 4.187 \times (25 - 0) \\ &= 10.46 + 333.33 + 104.67 = 448.46\text{ kJ.} \end{aligned}$$

Entropy increase of the universe is given by:

$$(\Delta S)_{\text{universe}} = (\Delta S)_{\text{atmosphere}} + (\Delta S)_{\text{system}}$$

Entropy change of the atmosphere is given by:

$$(\Delta S)_{\text{atmosphere}} = -\frac{Q}{T} = -\frac{448}{298} = -1.5049\text{ kJ/K}$$

Entropy change of the system is given by:

$$(\Delta S)_{\text{system}} = (\Delta S_I)_{\text{system}} + (\Delta S_{II})_{\text{system}} + (\Delta S_{III})_{\text{system}}$$

Where $(\Delta S_I)_{\text{system}}$ is the change of system as it gets heated from -5°C to 0°C , $(\Delta S_{II})_{\text{system}}$ is the entropy change as ice melts at 0°C to become water at 0°C and $(\Delta S_{III})_{\text{system}}$ is the entropy change as water gets heated from 0°C to 25°C .

Entropy change of system as ice gets heated from -5°C to 0°C ,

$$(\Delta S_I)_{\text{system}} = \int_{268}^{273} mc_p \frac{dT}{T} = 1 \times 2.093 \ln \frac{273}{268} = 0.0386\text{ kJ/K}$$

Entropy change of system as ice melts at 0°C to become water at 0°C ,

$$(\Delta S_{II})_{\text{system}} = \frac{333.33}{273} = 1.2209\text{ kJ/K}$$

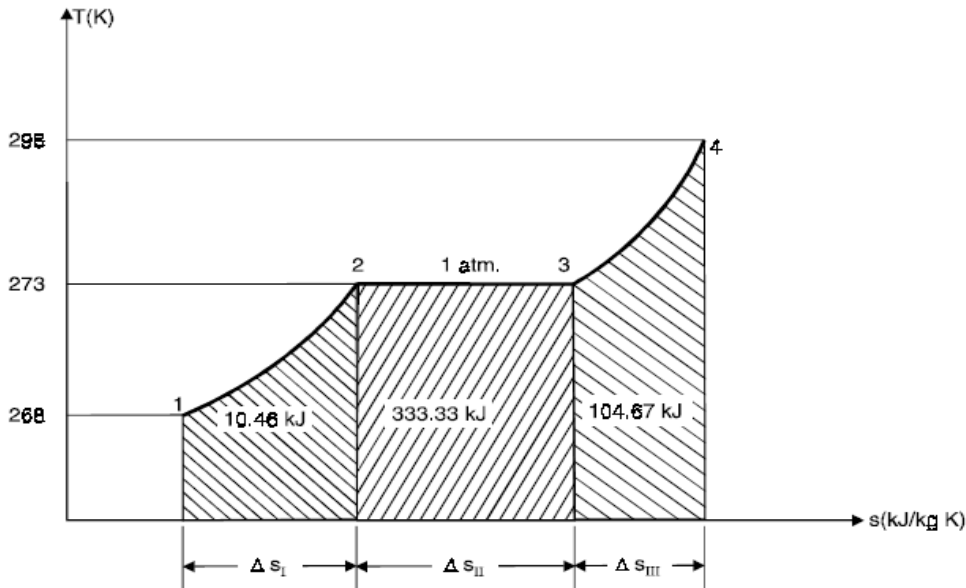
Entropy change of system as water gets heated from 0°C to 25°C ,

$$(\Delta S_{III})_{\text{system}} = \int_{273}^{298} mc_p \frac{dT}{T} = 1 \times 4.187 \ln \frac{298}{273} = 0.3668\text{ kJ/K}$$

Therefore, the Total entropy of the system as ice melts into water:

$$(\Delta S)_{\text{system}} = 0.0386 + 1.2209 + 0.3668 = 1.6263\text{ kJ/K}$$

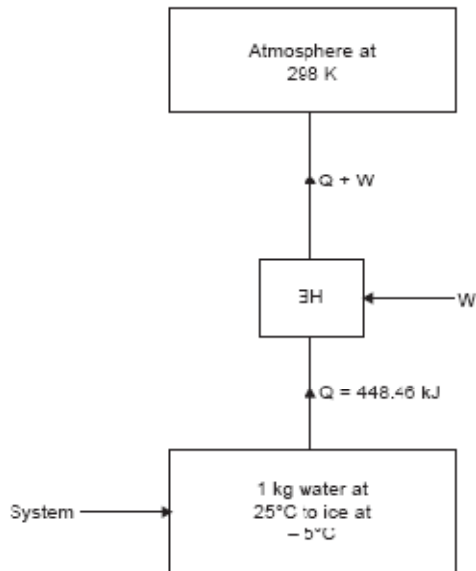
Then the temperature-entropy diagram for the system as ice at -5°C converts to water at 25°C is shown in the figure below:



$$\begin{aligned} \therefore (\Delta s)_{universe} &= (\Delta s)_{atmosphere} + (\Delta s)_{system} \\ &= 1.6263 + (-1.5049) = \mathbf{0.1214 \text{ kJ/K}} \quad (\text{Answer to (a)}) \end{aligned}$$

To convert 1 kg of water at 25°C to ice at -5°C, 448.46 kJ of heat have to be removed from it, and the system has to be brought from state 4 to state 1 (Figure above). A refrigerator cycle, as shown in the figure below, is assumed to accomplish this. The entropy change of the system would be the same, *i.e.*, $s_4 - s_1$, with the only difference that its sign will be negative, because heat is removed from the system (see below). *i.e.*:

$$(\Delta s)_{system} = s_1 - s_4 \text{ (negative)}$$



The entropy change of the working fluid in the refrigerator would be zero, since it is operating in a cycle; *i.e.*: $(\Delta s)_{refrigerator} = 0$.

The entropy change of the atmosphere will be positive and given by: $(\Delta s)_{atmosphere} = \frac{Q+W}{T}$.

Therefore the entropy change of the universe will be given by:

$$\begin{aligned} (\Delta s)_{universe} &= (\Delta s)_{refrigerator} + (\Delta s)_{atmosphere} + (\Delta s)_{system} \\ &= 0 + \frac{Q+W}{T} + (s_1 - s_4) \end{aligned}$$

By the principle of increase of entropy:

$$(\Delta s)_{universe} \geq 0$$

$$\therefore \frac{Q + W}{T} + (s_1 - s_4) \geq 0$$

$$\therefore \frac{Q + W}{T} \geq (s_4 - s_1)$$

$$W \geq T(s_4 - s_1) - Q$$

$$W_{min} = T(s_4 - s_1) - Q$$

Where $Q = 448.46$ kJ and $T = 298$ K and $(s_4 - s_1) = 1.6263$ kJ/K

Therefore:

$$Q = 298 \times 1.6263 - 448.46 = \mathbf{36.17 \text{ kJ}} \text{ (Answer to (b))}$$

BP2

Assumptions

- 1 The room is maintained at 20°C and 95 kPa at all times.
- 2 Air is an ideal gas with constant specific heats at room temperature.
- 3 The moisture is condensed at an average temperature of 4°C.
- 4 Half of the air volume in the refrigerator is replaced by the warmer kitchen air each time the door is opened.

Properties:

The gas constant of air is $R = 0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}$. The specific heat of air at room temperature is $c_p = 1.005 \text{ kJ/kg}\cdot\text{K}$ (Table A-5). The heat of vaporization of water at 4°C, i.e. h_{fg} can be obtained from Table B.1.1 by interpolation to be equal to 2492 kJ/kg.

The volume of the refrigerated air replaced each time the refrigerator is opened is 0.3 m^3 (half of the 0.6 m^3 air volume in the refrigerator). Then the total volume of refrigerated air replaced by room air per year is

$$\dot{V}_{\text{air, replaced}} = (0.3 \text{ m}^3)(8/\text{day})(365 \text{ days/year}) = 876 \text{ m}^3/\text{year}$$

The density of air at the refrigerated space conditions of 95 kPa and 4°C and the mass of air replaced per year are:

$$\rho_o = \frac{P_o}{RT_o} = \frac{95 \text{ kPa}}{(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(4 + 273 \text{ K})} = 1.195 \text{ kg/m}^3$$

$$m_{\text{air}} = \rho V_{\text{air}} = (1.195 \text{ kg/m}^3)(876 \text{ m}^3/\text{year}) = 1047 \text{ kg/year}$$

The amount of moisture condensed and removed by the refrigerator is

$$\begin{aligned} m_{\text{moisture}} &= m_{\text{air}} (\text{moisture removed per kg air}) = (1047 \text{ kg air/year})(0.006 \text{ kg/kg air}) \\ &= 6.28 \text{ kg/year} \end{aligned}$$

The sensible, latent, and total heat gains of the refrigerated space become

$$\begin{aligned} Q_{\text{gain, sensible}} &= m_{\text{air}} c_p (T_{\text{room}} - T_{\text{refrig}}) \\ &= (1047 \text{ kg/year})(1.005 \text{ kJ/kg}\cdot\text{K})(20 - 4)^\circ\text{C} = 16,836 \text{ kJ/year} \end{aligned}$$

$$\begin{aligned} Q_{\text{gain, latent}} &= m_{\text{moisture}} h_{fg} \\ &= (6.28 \text{ kg/year})(2492 \text{ kJ/kg}) = 15,650 \text{ kJ/year} \end{aligned}$$

$$Q_{\text{gain, total}} = Q_{\text{gain, sensible}} + Q_{\text{gain, latent}} = 16,836 + 15,650 = 32,486 \text{ kJ/year}$$

For a COP of 1.4, the amount of electrical energy the refrigerator will consume to remove this heat from the refrigerated space and its cost are

$$\text{Electrical energy used (total)} = \frac{Q_{\text{gain, total}}}{\text{COP}} = \frac{32,486 \text{ kJ/year}}{1.4} \left(\frac{1 \text{ kWh}}{3600 \text{ kJ}} \right) = 6.45 \text{ kWh/year}$$

$$\begin{aligned} \text{Cost of energy used (total)} &= (\text{Energy used})(\text{Unit cost of energy}) \\ &= (6.45 \text{ kWh/year})(\$0.075/\text{kWh}) = \mathbf{\$0.48/\text{year}} \end{aligned}$$

If the room air is very dry and thus latent heat gain is negligible, then the amount of electrical energy the refrigerator will consume to remove the sensible heat from the refrigerated space and its cost become

$$\text{Electrical energy used (sensible)} = \frac{Q_{\text{gain, sensible}}}{\text{COP}} = \frac{16,836 \text{ kJ/year}}{1.4} \left(\frac{1 \text{ kWh}}{3600 \text{ kJ}} \right) = 3.34 \text{ kWh/year}$$

$$\begin{aligned} \text{Cost of energy used (sensible)} &= (\text{Energy used})(\text{Unit cost of energy}) \\ &= (3.34 \text{ kWh/year})(\$0.075/\text{kWh}) = \mathbf{\$0.25/\text{year}} \end{aligned}$$