MECH 310 Quiz 2 **American University of Beirut** **January 11, 2010**

This is an open book 90 minutes exam.
• It is recommended that you read the whole exam before you start solving.
• Make sure that the units are consistent.
• Write your name and section number on both the question and answer sheets.
• Clearly identify your control mass / control volume.
• State any assumptions you need and provide a convincing justification.

Problem 1 (35%)
A spherical balloon initially contains 25 m^3 of helium gas at 25 °C and 150 kPa. A valve is now opened, and the helium is allowed to escape slowly. The valve is closed when the pressure inside the balloon drops to atmospheric pressure of 100 kPa. The elasticity of the balloon material is such that the pressure inside the balloon during the process varies with the volume according to the relation:

$p = a + b\,V$ where a = - 100 kPa and b is constant

Disregarding any heat transfer and assuming constant specific heats, determine:
(a) (25 %) The final temperature inside the balloon.
(b) (10 %)The mass of the helium that has escaped

Problem 1 Solution
Given: V_1, T_1, p_1, p_2 and the p-V relationship. Q=0.

(a) Helium is modeled as an ideal gas so that

$$\frac{p_1 V_1}{m_1 T_1} = \frac{p_2 V_2}{m_2 T_2}$$

From p_1 and V_1, we get b. Using $p_2 = a + b\,V_2$, we get V_2. Now the only two unknowns are T_2 and m_2. Applying the first law for unsteady process for a control volume, and noting that there is a single inlet, and that the enthalpy of flow leaving the ballon is at the balloon conditions, i.e. $h_o = c_p\,T$, then for an infinitesimal process and neglecting changes in kinetic energy and potential energy,

$$\begin{aligned}
& d(mu) = -pdV - h_o dm \\
\Rightarrow\quad & mdu + udm = -pdV - c_p T dm \\
\Rightarrow\quad & mdu + c_v T dm = -pdV - c_p T dm \\
\Rightarrow\quad & mc_v dT + (c_v + c_p) T dm = -pdV \\
\Rightarrow\quad & c_v \frac{dT}{T} + R\frac{dm}{m} = -\frac{p}{mT} dV \\
\Rightarrow\quad & c_v \frac{dT}{T} + R\frac{dm}{m} = -R\frac{dV}{V}
\end{aligned}$$

Integrating between initial and final states, we get

$$c_v \ln \frac{T_2}{T_1} + R \ln \frac{m_2}{m_1} = -R \ln \frac{V_2}{V_1}$$

Now, we have two equations with two unknowns, one of which is nonlinear. So use iterative technique or a plotting technique to get T_2 and m_2.

Problem 2 (30%)
You are asked to assess energy production of 100 kg/s of (geothermal) saturated steam at 1 atm.

(a) (10%) What is the maximum efficiency of a power generation cyclic engine operating between the steam as a high temperature reservoir and atmosphere (at 27 °C) as a low temperature reservoir.

(b) (10%) What is the maximum work produced per unit time assuming that steam enters a heat exchanger and leaves it as saturated liquid at the same pressure.

(c) (10%) What are the conditions for satisfying Carnot cycle requirements.

Problem 1 Solution
(a) The maximum efficiency corresponds to a Carnot cycle operating between the high temperature reservoir (saturated steam at 100 °C) and the low temperature reservoir (atmosphere at 27 °C),

$$\eta_c = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{373}$$

(b) The cyclic engine receives heat $\dot{Q}_H$ from the high temperature reservoir, reject heat $\dot{Q}_L$ to the low temperature reservoir and produces work

$$\dot{W} = \dot{Q}_H - \dot{Q}_L$$

The heat received from the saturated steam is obtained by taking the steam component of the heat exchanger as a control volume operating at steady conditions. Then

$$\dot{Q}_H = \dot{m}(h_o - h_i) = \dot{m}\,(h_{fg})_{1\,atm}$$

where $\dot{m} = 100$ kg/s. Then $\dot{W}_{max} = \eta_c \dot{Q}_H$

(c) The conditions for satisfying the Carnot cycle requirements are to have no external or internal irreversibilities. External irreversibilities are avoided by requiring the heat transfer interaction between the high temperature reservoir and the evaporator or boiler to occur across a very small temperature difference. Similar condition applies to heat transfer from the condenser to the low temperature reservoir. This means that if these reservoirs are at fixed temperature then the only way to exchange heat with them without violating the very small temperature difference requirement is the have isothermal phase change in both evaporator and condenser. Also expansion in the turbine must be reversible and adiabatic (isentropic). Similar condition applies to compression in the compressor. Insulation of all the components including the pipes must be perfect to avoid undesired heat losses. Finally flow in the pipes my be frictionless.

Problem 3 (35%)
Refrigerant R-134a enters a compressor at 200 kPa, 0 °C and exits at 1200 kPa and 50 °C. The diameters of the inlet and exit pipes are 4 and 2 cm. The volumetric flow rate of the refrigerant at the entrance is 0.38 m^3/min and the compressor experiences a heat loss to cooling water circulating through a water jacket enclosing the compressor. The cooling water flow rate is 0.25 kg/s and its temperature rise from inlet to exit is 2 °C. Determine the following:
(a) (15%) The power input to the compressor
(b) (15%) The compressor is powered by an adiabatic steam turbine that is directly coupled to the refrigerant compressor. If the steam enters the turbine at 3 MPa and 400 °C and expands to 6 kPa with quality (x = 0.9), determine the flow rate of steam required to drive the compressor.

Problem 3 Solution

Given: $T_i, p_i, T_o, p_o, D_i, D_o, \dot{V}_i,$ compressor loses heat to circulating water with $\dot{m}_w$ and ΔT_w

(a) Applying first law for compressor operating at steady state, and noting that conservation of mass at S.S. implies that the inlet mass flow rate is equal to the outlet mass flow rate, then (neglecting changes in PE)

$$\dot{m}(h_o - h_i) + \dot{m}\left(\frac{V_o^2}{2} - \frac{V_i^2}{2}\right) = -\dot{Q}^{\rightarrow} + \dot{W}^{\leftarrow}$$

Noting that $\dot{m} = \dot{V}_i/v_i, V_i = \dot{V}_i/(\pi D_i^2/4), V_o = v_o\dot{m}/(\pi D_o^2/4)$

and applying the first law for the cooling water, we then get

$$\dot{Q}_w^{\leftarrow} = \dot{Q}^{\rightarrow} = (\dot{m}c_p\Delta T)_w$$

The only unknown is the power input to the compressor.

(b) Now we apply first law for the turbine operating at steady state, in the absence of heat transfer and neglecting changes in P.E. and K.E., the power produced by the turbine, which is equal to the power input to the compressor, is

$$\dot{W}_t^{\rightarrow} = \dot{W}_c^{\leftarrow} = (\dot{m}(h_o - h_i))_t$$