Final Exam

- This is a 180 minutes exam.
- You are allowed to bring in 3 cheat sheets in addition to the thermodynamic tables.
- You are advised to read the whole exam before you start.
- Make sure you state all the assumptions you make and that you clearly identify any control mass or control volume you utilize in your analysis.
- Good luck!

Name:

Section:

Solve 5 of the following 6 problems. Make sure your choice is clear.

Problem 1 [20 points]

One ton of water is in a fixed-volume container. Initially, the temperature is $T_1 = 190$ °C and the pressure is $p_1 = 10$ bar. It is desired to raise the pressure to $p_2 = 30$ bar. This is achieved by interactions with another system that transfer energy and 1000 kJ/K of entropy into the container.
(a) How much energy is transferred?
(b) How entropy is generated by irreversibility?
(c) What are the possible types of interactions that could result in the given transfers of energy and entropy?

correction: [control mass : 2 pts] [state 1: 2 pts], [state 2: 3 pts], [energy: 5 pts], [irrversibility : 5 pts], [part (c): 3 pts].

Problem 1 Solution

(a) Take the container as the control mass. Initial state is

$p_1 = 10$ bar
$T_1 = 190$ °C
$s_1 = 6.641$ kJ/kg.K
$v_1 = 0.2002$ m^3/kg
$u_1 = 2603$ kJ/kg

Since the container is closed, the specific volume remains unchanged, so that the final state is given

$p_2 = 30$ bar
$v_2 = 0.2002$ m^3/kg
$T_2 = 1030$ °C
$s_2 = 8.461$ kJ/kg.K
$u_2 = 4108.1$ kJ/kg

Applying the first law for a control mass, the energy transferred into the container is

$$W^{\leftarrow} + Q^{\leftarrow} = U_2 - U_1 = m(u_2 - u_1) = 1000(4108.1 - 2603) = 1505100 \text{ kJ}$$

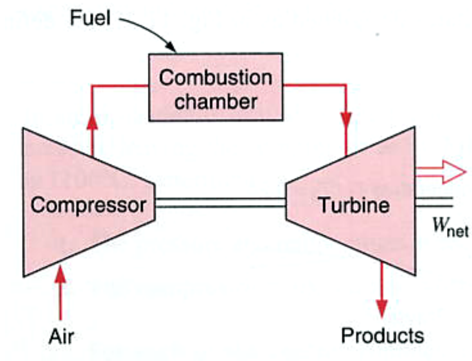
(b) Applying the second law for a control mass,

$$\Delta S = S^{\leftarrow} + S_{irr} \Rightarrow S_{irr} = m(s_2 - s_1) - S^{\leftarrow} = 1000(8.461 - 6.641) - 1000 = 820 \text{ kJ/K}$$

(c) The energy transferred into the container has to be heat since it is the only way to transport entropy across the boundary of a closed system.

Problem 2 [20 points]

Consider a gas turbine consisting of a compressor, combustion chamber and a turbine as shown in the figure below. The compressor and turbine are coupled by a common shaft as shown in the figure. If the net work of the cycle is zero meaning that the work produced by the turbine is equal to the work consumed by the compressor, find the pressure of the air leaving given the following information. The air enters the compressor at 0.1 MPa, 15 °C . The pressure leaving the compressor is 1.0 MPa and the temperature of air entering the turbine is 1100 °C . The pressure drop in the combustion chamber is negligible. Assume the efficiency of the compressor 80% and the turbine efficiency 85%.



Problem 3 [20 points]

Geothermal saturated steam at 100 °C is being considered for heating a building. You are only to consider the latent heat of the geothermal steam in the heating schemes you are asked to come up with. The temperature of the building is to be maintained at 20 °C and the average outdoor temperature is 5 °C . Also available to you is the electricity grid so that you can connect it to any machinery you decide to employ. Electricity from the grid is valued at $10x$ Lebanese pounds per kWh. Energy from the steam is valued at x Lebanese pounds per kWh.
(a) What is the smallest energy cost per unit of energy delivered to the building? How is the smallest energy cost achieved? Make a sketch of the interacting systems involved.
(b) Under the conditions of part (a), how much electricity is transferred per unit energy delivered to the building? Is this electricity transferred to the building or to the grid?

Problem 3 Solution

What we can do is to employ a reversible cyclic power engine between the saturated steam and the building (which is considered as thermal reservoir at 20 °C), and in the process we produce work, i.e. we provide electricity to the grid. See the sketch below.
Using this design, we provide power to the building from steam at the cost of x Lebanese pounds per kWh. At the same time we produce work that is equal to

$$\dot{W}^{\rightarrow} = \dot{Q}^{\leftarrow}_{steam} - \dot{Q}^{\rightarrow}_{building} = \eta_{carnot}\,\dot{Q}^{\leftarrow}_{steam} = \eta_{carnot}\,\dot{m}_{steam}\,h_{fg@100^{\circ}\mathrm{C}}$$

Noting that

$$\eta_{carnot} = 1 - \frac{T_{building}}{T_{steam}}$$

Then

$$\begin{aligned}\frac{\dot{W}^{\rightarrow}}{\dot{m}_{steam}} &= \left(1 - \frac{T_{building}}{T_{steam}}\right) h_{fg@100^{\circ}\mathrm{C}} \\ &= \left(1 - \frac{273+20}{273+100}\right) 2257 \text{ kJ/kg} = 484 \text{ kJ/kg}\end{aligned}$$

Problem 4 [20 points]

Consider an air compressor that receives air at 100 kPa, 25 °C . It compresses the air to a pressure of 1 MPa, where it exits at a temperature of 540 K. The compressor heat losses are 50 kJ for each kg of air flowing through the compressor. Find the following:
a) Work of the compressor assuming variable specific heats.
b) The efficiency of the compressor assuming variable specific heat.
c) The amount of entropy generated.
d) Draw the $T - s$ diagram.

Problem 4 Solution
(a) From the thermodynamic tables, the inlet state is $T_{in} = 298$ K, $p_{in} = 100$ kPa, $h_{in} = 298.44$ kJ/kg, $s_{in} = 6.864$ kJ/kg K. The outlet state $T_{out} = 540$ K, $p_{out} = 1$ MPa, $h_{out} = 550.12$ kJ/kg, $s_{out} = 6.8524$ kJ/kg K. Applying first law for a control volume consisting of the compressor, for steady state,

$$
\begin{aligned}
0 &= \dot{Q}^{\leftarrow} + \dot{m}(h_{in} - h_{out}) + \dot{W}^{\leftarrow} \\
\Rightarrow \quad \frac{\dot{W}^{\leftarrow}}{\dot{m}} &= (h_{out} - h_{in}) + \frac{\dot{Q}^{\rightarrow}}{\dot{m}} = 550.12 - 298.44 - 50 = 201.68 \text{kJ/kg}
\end{aligned}
$$

(b) Assuming isentropic compression to $p_2 = 1$ MPa, $s_2 = s_1 = 6.864$ kJ/kg K, we get $h_{out,s} = 583.4$ kJ/kg, leading to

$$
\eta = \frac{\dot{W}^{\leftarrow}}{\dot{W}^{\leftarrow}_{isentropic}} = \frac{201.68}{583.4 - 298.44} = 0.708
$$

(c) Noting that the compressor boundary temperature at which heat is transferred to the surrounding is unknown, we take both compressor plus surrounding as a control mass and apply the second law of thermodynamics,

$$
\frac{dS_c}{dt} + \frac{dS_{sur}}{dt} = \dot{S}_{irr}
$$

note that compressor is undergoing steady operation, then $dS_c/dt = 0$. The surrounding is a thermal reservoir for which

$$
\begin{aligned}
\frac{dS_{sur}}{dT} &= \dot{S}^{\leftarrow}_{heat} + \dot{m}(s_{in} - s_{out})_{sur} \\
\Rightarrow \quad \dot{S}_{irr} &= \frac{dS_{sur}}{dT} = \frac{\dot{Q}^{\leftarrow}}{T_0} - \dot{m}(s_{in} - s_{out})_c \\
\Rightarrow \quad \frac{\dot{S}_{irr}}{\dot{m}} &= \frac{\dot{Q}^{\leftarrow}}{\dot{m}T_0} - (s_{in} - s_{out})_c = 0.156 \text{ kJ/kg K}
\end{aligned}
$$

It is assumed that heat is transferred to the surrounding at the inlet temperature. In reality this temperature is higher.

Problem 5 [20 points]

Consider two large blocks of copper, A and B, of masses m_A and m_B (kg). Initially, block A is at temperature of $T_{A,1}$ K and block B at $T_{B,1}$. The specific heat of copper is c (kJ/kg K). For this problem, we neglect expansion of copper as a function of temperature.
(a) If perfect machinery is available, what is the largest work that can be extracted using the two copper blocks?
(b) If perfect machinery and a reservoir at T_o are available, what is the largest work that can be extracted from the two blocks?
(c) Which scenario produces more work (a) or (b)? Explain.

Problem 5 Solution

(a) As seen from quiz 2 solution, the maximum work that can be obtained from employing a carnot engine between the two blocks and noting that work will continue to be produced until both blocks reach the same temperature. Assuming that $T_{A,1} > T_{B,1}$, we get

$$W^{\rightarrow} = Q_H^{\leftarrow} - Q_L^{\rightarrow} = m_A c \left[T_{A,1} - T_f\right] - m_B c \left[T_f - T_{B,1}\right]$$

where the final temperature is

$$T_f = \left(T_{B,1}^{m_B c} \, T_{A,1}^{m_A c}\right)^{\frac{1}{c(m_A + m_B)}}$$

(b) In this case, we employ a carnot engine between A and atmosphere and B and atmosphere, assuming $T_A > T_0$ and $T_B > T_0$, then

$$\begin{aligned} W^{\rightarrow} &= W_{A,atm}^{\rightarrow} + W_{B,atm}^{\rightarrow} \\ &= (m_A c) \left[(T_{A,1} - T_o) - T_o \ln \frac{T_{A,1}}{T_o}\right] + (m_B c) \left[(T_{B,1} - T_o) - T_o \ln \frac{T_{B,1}}{T_o}\right] \end{aligned}$$

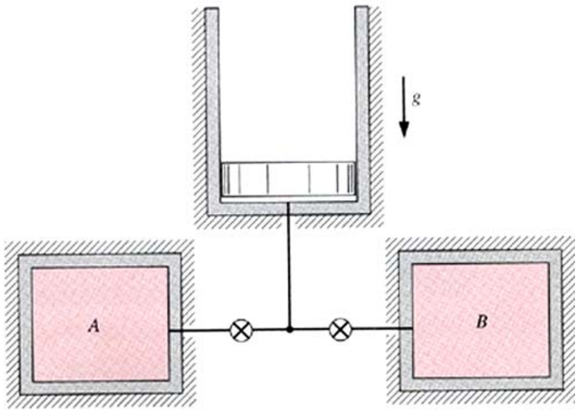
(c) If $T_A > T_B > T_0$, then scenario (b) produces more work than scenario (a) because on way to achieve scenario (b) is to employ first scenario (a) at the end of which $T_A = T_B = T_f$ and then put a cyclic engine between T_f and environment at $T_0 < T_f$, in this case,

$$W_{(b)}^{\rightarrow} = W_{(a)}^{\rightarrow} + W_{T_f, T_0}^{\rightarrow}$$

Actually, as long as $T_f = \left(T_{B,1}^{m_B} \, T_{A,1}^{m_A}\right)^{\frac{1}{m_A + m_B}}$ is less larger than T_0, scenario (b) produces more work than scenario (a).

Problem 6 [20 points]

Two tanks contain steam, and they are both connected to a piston/cylinder, as shown in the figure below. Initially the piston is at the bottom and the mass of the piston is such that a pressure of 1.4 MPa is required to lift it. Steam in tank A is 4 kg at 7 MPa, 700 °C and tank B has 2 kg at 3 MPa, 350 °C . The two valves are opened, and the water comes to a uniform state. Find the final temperature and the total entropy generation assuming no heat transfer.



Problem 6 Solution
Taking a control mass that consists of steam inside A, B, cylinder (C) and connecting pipes, and applying the first law for control mass between initial and final state, we get

$$(U_2 - U_1)_A + (U_2 - U_1)_B + (U_2 - U_1)_C = Q^{\leftarrow} + W^{\leftarrow}$$

Noting that $(U_1)_C = 0$ since cylinder is initially empty, thew control mass is insulated, and that steam inside cylinder expands isobarically, we get

$$(m_2u_2 - m_1u_1)_A + (m_2u_2 - m_1u_1)_B + (m_2u_2)_C = -p_C(m_2v_2)_C$$

where $p_C = 1.4$ MPa.

Note that conservation of mass yields $(m_2)_C = (m_1 - m_2)_A + (m_1 - m_2)_B$.

Note also at the final state, all the temperatures are the same so that $(T_2)_A = (T_2)_B = (T_2)_C = T_2$ and all the pressures are the same $(p_2)_A = (p_2)_B = (p_2)_C = p_C$.

We get

$$V_A\left(\frac{u_2}{v_2} - \frac{u_1}{v_1}\right)_A + V_B\left(\frac{u_2}{v_2} - \frac{u_1}{v_1}\right)_B + \left[V_A\left(\frac{1}{v_2} - \frac{1}{v_1}\right)_A + V_B\left(\frac{1}{v_2} - \frac{1}{v_1}\right)_B\right](h_2)_C = 0$$

where $(h_2)_C = (u_2)_C + p_C(v_2)_C$. The initial states and mass are known. The final pressure is known, the only unknown in the above equation is the final temperature. So the above equation is solved by trial and error or by plotting the quantity on the left hand side of the equation versus T and the final T is the interception with the T axis.

The entropy generated is obtained by applying the second law

$$(m_2 s_2 - m_1 s_1)_A + (m_2 s_2 - m_1 s_1)_B + (m_2 s_2)_C = S_{irr}$$