MECH 310 Thermodynamics I
American University of Beirut, Fall 2007

January 11, 2007
Handout # Quiz 2

Quiz 2

- This is a 120 minutes exam.
- You allowed to bring in 3 cheat sheets in addition to the thermodynamic tables.
- You are advised to read the whole exam before you start.
- Make sure you state all the assumptions you make and that you clearly identify any control mass or control volume you utilize in your analysis.
- Good luck!

Problem 1[30 points] - *Hydroelectric Power*

In the hydroelectric power plant sketched in Figure 1, water flows to the turbine from a large reservoir at a rate of 10^5 kg/s. The difference in elevation between the water reservoir and the turbine exit plane is 180 m. The turbine is well insulated and its efficiency is 97%. The water velocity at the exit plane of the turbine is 6 m/s. The water reservoir temperature is $T_0 = 280$ K.

(a) What is the water temperature at the turbine exit plane?
(b) How much entropy is generated by irreversibility in the turbine?
(c) If the difference in elevation between the turbine exit plane and the discharge pond is 3 m, what is the water temperature at the discharge pond? How much entropy is generated by the irreversbility in the discharge?

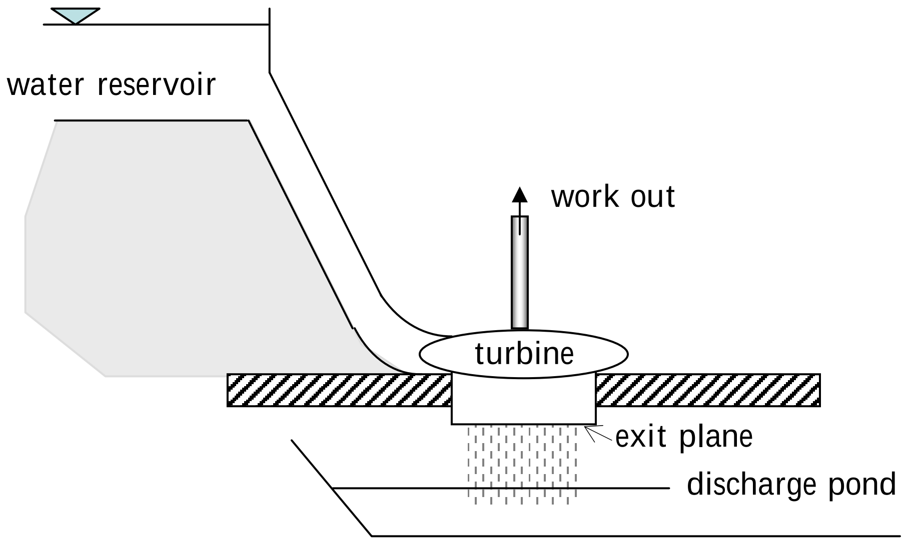

Figure 1: Schematic for problem 1.

Solution

(a) Applying the first law for a control volume consisting of the water reservoir and the falling water until the inlet to the turbine (CV1 in Figure 2) and noting that the flow is steady, no heat or work interactions are taking place, the velocity of water in the reservoir is negligible, and that the flow is isothermal, we get

$$\frac{(V_{in}^2)_{\text{turbine}}}{2} = gH \Rightarrow (V_{in}^2)_{\text{turbine}} = \sqrt{2gH} = 60 \text{ m/s}$$

where g is gravity.

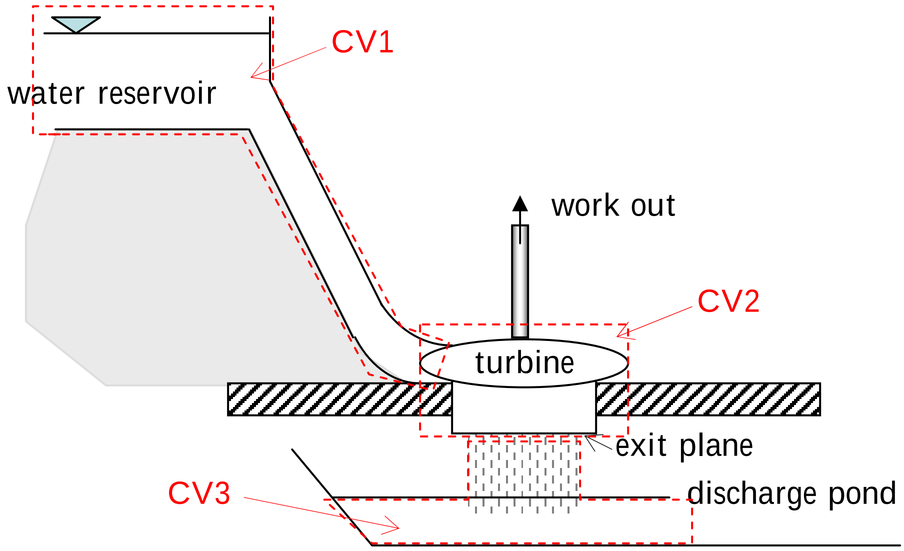

Figure 2: Control volumes for problem 1.

Next we apply the first law for a control volume consisting of the turbine (CV2 in Figure 2). Noting that the turbine is thermally insulated, the flow is steady, and changes in potential energy across the turbine are negligible, we get

$$0 = -(W^{\rightarrow})_{\text{turbine}} + \dot{m}\,(h_{in} - h_{out}) + \frac{\dot{m}}{2}\left((V_{in}^2)_{\text{turbine}} - (V_{out}^2)_{\text{turbine}}\right)$$

The efficiency of the turbine is the ratio of the actual work produced to the maximum work that can be produced. The maximum work corresponds to the isentropic process in the turbine and to zero outlet velocity,

$$\eta = \frac{W^{\rightarrow}}{(W^{\rightarrow})_{max}} = \frac{2\dot{m}\,(h_{in} - h_{out}) + \dot{m}\,((V_{in}^2)_{\text{turbine}} - (V_{out}^2)_{\text{turbine}})}{2\dot{m}\,(h_{in} - h_{out,s}) + \dot{m}\,((V_{in}^2)_{\text{turbine}})}$$

Now using the relation $dh = T\,ds + v\,dp$ and noting that the turbine inlet and outlet pressure are atmospheric, then for an isentropic process $h_{in} = h_{out,s}$. Also note that $h_{in} - h_{out} = c_p(T_{in} - T_{out})$, then

$$\eta = \frac{W^{\rightarrow}}{(W^{\rightarrow})_{max}} = \frac{2c_p\,(T_{in} - T_{out}) + (V_{in}^2 - V_{out}^2)}{V_{in}^2}$$

Which leads to

$$T_{out} = T_{in} + \frac{1}{2c_p}\left((1-\eta)V_{in}^2 - V_{out}^2\right) = 288.6 \text{ K}$$

(b) Applying 2nd law for the turbine control volume and noting that the turbine is thermally insulated, then for an isobaric process of an incompressible liquid

$$\dot{S}_{irr} = \dot{m}(s_{out} - s_{in}) = \dot{m}c_p \ln \frac{T_{out}}{T_{in}} = 12663 \text{ kJ/s K}$$

(c) Assuming that the discharge process (CV3 in Figure 2) is adiabatic (this claim is not entirely true since some heat is transferred from the water to the surrounding), the first lay yields for a steady process ,

$$0 = \dot{m}(h_{in} - h_{out}) + \frac{\dot{m}}{2}\left(V_{in}^2 - V_{out}^2\right) + \dot{m}g(z_{in} - z_{out})$$

with $V_{out} = 0$ and $h_{in} - h_{out} = c_p(T_{in} - T_{out})$, we get

$$0 = c_p(T_{in} - T_{out}) + \frac{V_{in}^2}{2} + g(z_{in} - z_{out})$$

$$\Rightarrow \quad T_{out} = T_{in} + \frac{V_{in}^2}{2c_p} + \frac{g}{c_p}(z_{in} - z_{out}) = 288.6 + 11.46 = 300 \text{ K}$$

The second law applied for the control volume consisting of the turbine discharge and the discharge pond assuming adiabatic process leads to

$$\dot{S}_{irr} = \dot{m}(s_{out} - s_{in}) = \dot{m}c_p \ln \frac{T_{out}}{T_{in}} = 16310 \text{ kJ/s K}$$

Problem 2 [20 points] - *Homer Simpson: "Lisa... In this house, we obey the laws of thermodynamics"*

US Patent No.: US 6,962,052 B2 (2005) by "inventor" Haim Goldenblum.

(57) **ABSTRACT**

The present invention is converting any environmental heat into a useful form of energy, by utilizing kinetic energy of randomly moving particles, by use a mechanism which selectively block particles by their direction related said mechanism, for creating a force, or a pressure difference or a flow of particles, and etc, for example, in FIG. 6*e* of the Patent, we use a permeable membrane-50 that divides a box-35 into two sub-compartments, A and B, the membrane is covered only on one side of it by tiny unidirectional gates-54, that let passing through particles-15 from subcompartment B to subcompartment A, but block the passing through back of particles from subcompartment A to subcompartment B, so as a result there will be generated a general flow-V of gas from subcompartment B to subcompartment A through the membrane-50, and this flow will rotate the turbine-70 and return back in circulation, and since the particles lost some of their kinetic energy during their pass through the turbine, we use a heat-absorber-133 for returning the particles heat energy from the surrounding environment.

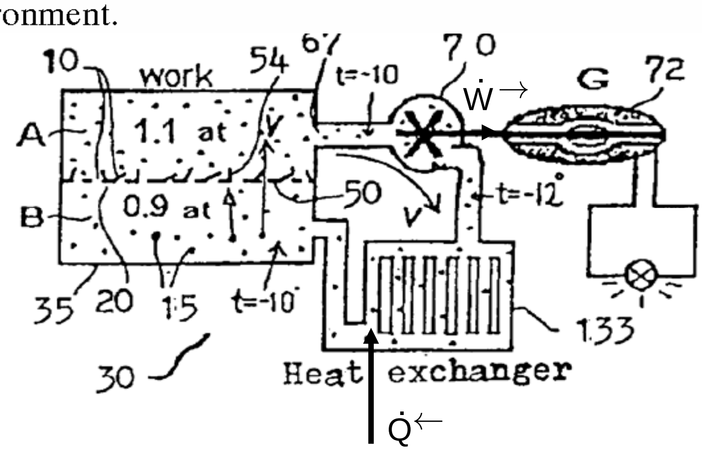

Figure 3: Schematic for problem 2.

(a) Does the proposed device obey the first law of thermodynamics? If heat $\dot{Q}$ enters the heat exchanger, how much work per unit time does the device produce?
(b) Does the device obey the second law of thermodynamics? Why?

Solution

(a) Applying first law for the cyclic device, we get

$$W^{\rightarrow} = Q^{\leftarrow}$$

So the device obeys the first law of thermodynamics.

(b) The device violated the Kelvin-Planck statement of the second law since it consists of a cyclic device producing work while exchanging heat with a single thermodynamic reservoir.

Problem 3 [25 points]

Air is contained in the insulated cylinder shown in the figure below. At this point the air is at 140 kPa, 25 °C and the cylinder volume is 15 L. The piston cross-sectional area is 0.045 m^2, and the spring is linear with spring constant 35 kN/m. The valve is opened, and air from the line at 700 kPa, 25 °C flows into the cylinder until the pressure reaches 700 kPa, and the valve is closed. Find the final temperature.

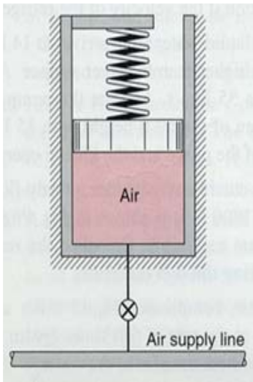

Figure 4: Schematic for problem 3.

Solution

Applying the first law for the control volume consisting of the air inside the cylinder and noting that the process is unsteady and the cylinder is insulated, then

$$m_2u_2 - m_1u_1 = -W^{\rightarrow} + m_{in}h_{in} \tag{1}$$

where kinetic and potential energy effects are neglected. Now conservation of mass requires

$$m_{in} = m_2 - m_1$$

Assuming the process to be quasi-static, the work done by the system must balance the work done on the environment plus the work done in compressing the spring

$$W^{\rightarrow} = \int_{\mathcal{V}_1}^{\mathcal{V}_2} p\, d\mathcal{V} = \int_{\mathcal{V}_1}^{\mathcal{V}_2} \left(p_1 - \frac{k_{\text{spring}}(\Delta z)}{A_{\text{piston}}} \right) d\mathcal{V}$$

Noting that $\Delta z = z - z_1 = \frac{\mathcal{V}_1 - \mathcal{V}}{A_{\text{piston}}}$, where z_1 and V_1 correspond height of piston and volume at initial state

$$W^{\rightarrow} = p_1(\mathcal{V}_2 - \mathcal{V}_1) + \frac{1}{2}\frac{k_{\text{spring}}}{A_{\text{piston}}^2}(\mathcal{V}_2 - \mathcal{V}_1)^2$$

Equation (1) becomes

$$m_2u_2 - m_1u_1 = p_1(\mathcal{V}_2 - \mathcal{V}_1) + \frac{k_{\text{spring}}}{2A_{\text{piston}}^2}(\mathcal{V}_2 - \mathcal{V}_1)^2 + (m_2 - m_1)h_{in}$$

Using the ideal gas law $p\mathcal{V} = mRT$ and noting $u = c_vT$, $h = c_pT$,

$$\frac{c_v}{R}(p_2\mathcal{V}_2 - p_1\mathcal{V}_1) = p_1(\mathcal{V}_2 - \mathcal{V}_1) + \frac{k_{\text{spring}}}{2A_{\text{piston}}^2}(\mathcal{V}_2 - \mathcal{V}_1)^2 + \frac{c_p}{R}\left(\frac{p_2\mathcal{V}_2}{T_2} - \frac{p_1\mathcal{V}_1}{T_1}\right)T_{in} \tag{2}$$

The unknowns in the above equation are the volume and temperature at the final state.

The additional equation needed is naturally the second law of thermodynamics applied for the control volume between initial and final states. Noting that the cylinder is insulated and assuming that the process is irreversible, we get

$$\begin{aligned}
& m_2s_2 - m_1s_1 = m_{in}s_{in} = (m_2 - m_1)s_{in} \\
\Rightarrow\ & m_2(s_2 - s_{in}) = m_1(s_1 - s_{in}) \\
\Rightarrow\ & m_2\left(c_p \ln\frac{T_2}{T_{in}} - R\ln\frac{p_2}{p_{in}}\right) = m_1\left(c_p \ln\frac{T_1}{T_{in}} - R\ln\frac{p_1}{p_{in}}\right) \\
\Rightarrow\ & \frac{p_2\mathcal{V}_2}{RT_2}\left(c_p \ln\frac{T_2}{T_{in}} - R\ln\frac{p_2}{p_{in}}\right) = \frac{p_1\mathcal{V}_1}{RT_1}\left(c_p \ln\frac{T_1}{T_{in}} - R\ln\frac{p_1}{p_{in}}\right) \qquad (3)
\end{aligned}$$

So basically we solve for T_2 by substituting expression for $\mathcal{V}_2$ obtained from equation (3) in equation (2) to get an equation in which the only unknown is T_2.

Problem 4 [25 points]

A thermal storage is made of a rock bed having a volume of 2 m^3, density 2750 kg/m^3 and specific heat of 0.89 kJ/kg K. The bed is heated by solar energy until its temperature reaches 400 K. If a reversible heat engine is connected between the rock bed and the environment thermal reservoir at temperature T_o.

a) Find the amount of work produced as the temperature of the rock bed decreases to T_o.

b) If the environmental thermal reservoir is replaced by a rigid vessel containing 10 kg of air that is initially at T_o, what would be the amount of work produced?

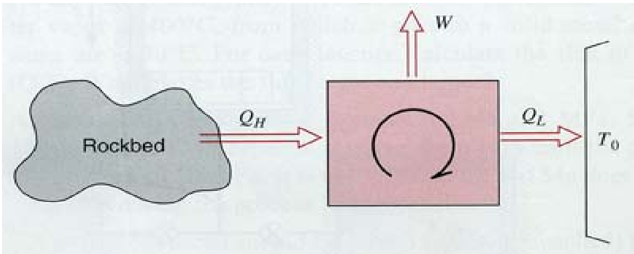

Figure 5: Schematic for problem 4.

Solution

(a) First law for the cyclic engine gives

$$\Delta E = 0 = Q_H^{\leftarrow} - Q_L^{\rightarrow} - W^{\rightarrow}$$

Second law for the reversible cyclic engine gives

$$\Delta S = 0 = S_H^{\leftarrow} - S_L^{\rightarrow} = \int \frac{\delta Q_H^{\leftarrow}}{T} - \frac{Q_L^{\rightarrow}}{T_o}$$

where the temperature of the rock bed T decreases from T_H =400 K to T_o.

Now applying the first law for the rock bed over an infinitesimal process from T to $T - dT$, we get

$$\delta Q_H^{\leftarrow} = -m_{\mathrm{rb}} c_{\mathrm{rb}}\, dT = -(\rho \mathcal{V} c)_{\mathrm{rb}}\, dT \tag{4}$$

Substituting in the entropy balance equation, we get

$$\begin{aligned} \int_{T_H}^{T_o} \frac{-(\rho \mathcal{V} c)_{\mathrm{rb}}\, dT}{T} - \frac{Q_L^{\rightarrow}}{T_o} &= 0 \\ \Rightarrow \quad Q_L^{\rightarrow} &= (\rho \mathcal{V} c)_{\mathrm{rb}}\, T_o \ln \frac{T_H}{T_o} \end{aligned}$$

Also from equation (4),

$$Q_H^{\leftarrow} = (\rho \mathcal{V} c)_{\mathrm{rb}} (T_H - T_o)$$

Finally the work output is

$$W^{\rightarrow} = Q_H^{\leftarrow} - Q_L^{\rightarrow} = (\rho \mathcal{V} c)_{\mathrm{rb}} \left[(T_H - T_o) - T_o \ln \frac{T_H}{T_o} \right]$$

(b) In this case, not only the temperature of the rock bed is decreasing as it is giving heat to the cyclic engine, but also the temperature of the water reservoir is increasing as it is receiving heat from the cyclic engine. The process will start from the initial state and continue until both reservoirs reach the same final temperature T_f.

First law for the rock bed, we have

$$\begin{aligned} \delta Q_H^{\leftarrow} &= -m_{\mathrm{rb}} c_{\mathrm{rb}}\, dT \\ \Rightarrow \quad Q_H^{\leftarrow} &= m_{\mathrm{rb}} c_{\mathrm{rb}} (T_H - T_f) \end{aligned}$$

First law for the water reservoir

$$\begin{aligned} \delta Q_L^{\rightarrow} &= m_{\mathrm{w}} c_{\mathrm{w}}\, dT \\ \Rightarrow \quad Q_L^{\rightarrow} &= m_{\mathrm{w}} c_{\mathrm{w}} (T_f - T_o) \end{aligned}$$

Entropy balance for an infinitesimal reversible process of the cyclic engine yields,

$$\begin{aligned} \int_{T_H}^{T_f} \frac{\delta Q_H^{\leftarrow}}{T} - \int_{T_o}^{T_f} \frac{\delta Q_L^{\rightarrow}}{T} &= 0 \\ \Rightarrow \quad (mc)_{\text{rb}} \ln \frac{T_H}{T_f} &= (mc)_{\text{w}} \ln \frac{T_f}{T_o} \\ \Rightarrow \quad T_f &= \left(T_o^{C_w}\, T_H^{C_{rb}}\right)^{\frac{1}{C_w + C_{rb}}} \end{aligned}$$

where $C_w = m_{\text{w}} c_{\text{w}}$ and $C_{rb} = m_{\text{rb}} c_{\text{rb}}$.

Substituting T_f in $Q_H^{\leftarrow}$ and $Q_L^{\rightarrow}$, we get

$$W^{\rightarrow} = Q_H^{\leftarrow} - Q_L^{\rightarrow} = C_{rb} \left[T_H - \left(T_o^{C_w}\, T_H^{C_{rb}}\right)^{\frac{1}{C_w + C_{rb}}}\right] - C_w \left[\left(T_o^{C_w}\, T_H^{C_{rb}}\right)^{\frac{1}{C_w + C_{rb}}} - T_o\right]$$