Quiz 1

- This is an open book 90 minutes exam.
- It is recommended that you read the whole exam before you start solving.
- Make sure that the units are consistent.
- Write your name and section number on both the question and answer sheets.
- Clearly identify your control mass / control volume.
- State any assumptions you need and provide a convincing justification.

Problem 1 (40 points)

A rigid cylinder of volume 1 m^3 contains 1 kg of a saturated liquid+vapor water mixture at 100 C. The water undergoes a process with a final state of saturated vapor. The cylinder is fitted with a piston which is either free to move or held stationary, depending on the process. For each of the processes (a), (b) and (c):

(1) Find the initial and final states, work produced by and heat transferred into water,
(2) Sketch the process on a $p - v$ and $T - v$ diagrams,
(3) Explain whether the piston is free to move or held stationary.

(a) An isobaric process.
(b) An isothermal process.
(c) An isometric process.

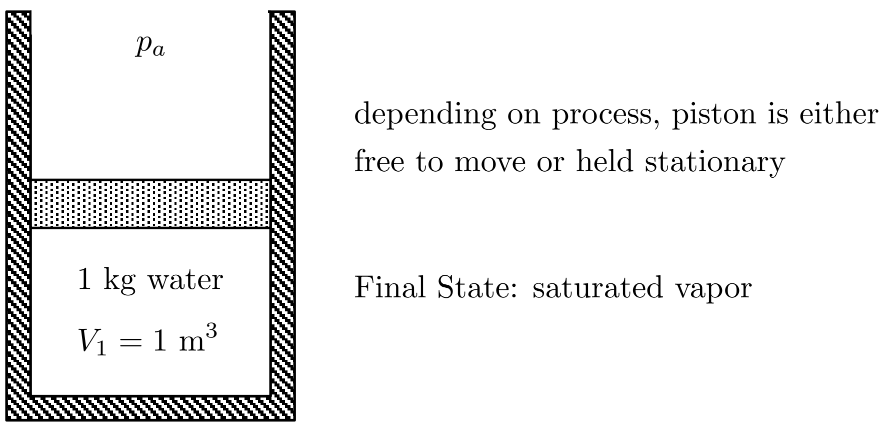

Figure 1: Schematic for problem 1.

Problem 2 (30 points)

A well-insulated rigid tank contains 5 kg of saturated liquid+vapor mixture of water at 100 kPa at an initial unknown quality, x_1. An electric resistor, placed in the tank, is connected to a 110-V source and a current of 8 A flows through the resistor when the switch is turned on. It took 10 minutes to completely vaporize the liquid. Determine the following:

(a) The volume of the tank in m^3.
(b) The initial quality x_1
(c) Sketch the process on a p-v diagram

Note that when electric current I flows through a resistor, it dissipates heat at a rate of $I\,V$ (Watts), where V is the potential difference across the resistor.

Note: This problem may require an iterative solution. Do not carry out more than two iterations.

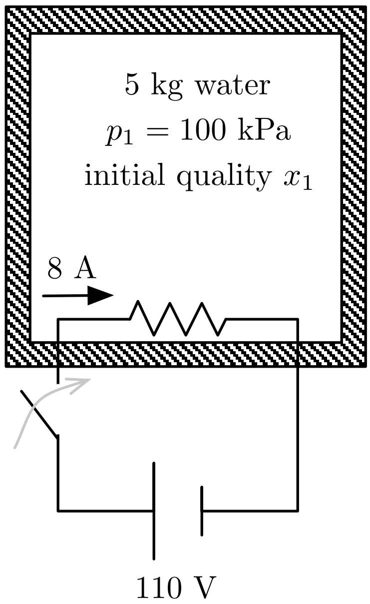

Figure 2: Schematic for problem 2.

Problem 3 (30 points)

The rigid container shown in Figure 1 consists of two air compartments separated by a piston of area $A = 250$ cm^2. The piston conducts heat but experiences no friction with the container walls. Compartment B is thermally insulated from the surrounding. The piston is supported by linear spring with a spring constant $k = 100$ kN/m. Initially the temperature in both compartments is 300 K and the volumes of the two compartments are $V_{A1} = 4$ liters and $V_{B1} = 6$ liters. The mass of air in compartment A is 20 grams and that in compartment B is 10 grams. Air is assumed to behave as an ideal gas with $c_v = 0.72$ kJ/kg.K and $R = 0.287$ kJ/kg.K.
(a) Find the initial air pressure in each compartment.
(b) Is the spring in the initial state under tension or compression? Find $l_1 - l_0$ where l_1 is length of spring at the initial state and l_0 is its length when it is under no stress.

Heat is slowly added to compartment A (while compartment B is kept insulated) until the final volume of air in compartment B is $V_{B2} = 5.5$ liters.
(c) Find final temperature and pressure in each compartment, $T_{A2}, p_{A2}, T_{B2}, p_{B2}$.
(d) Find the work produced by air in both compartments $W^{\rightarrow}_{A+B}$ and the heat added $Q^{\leftarrow}_{A}$.

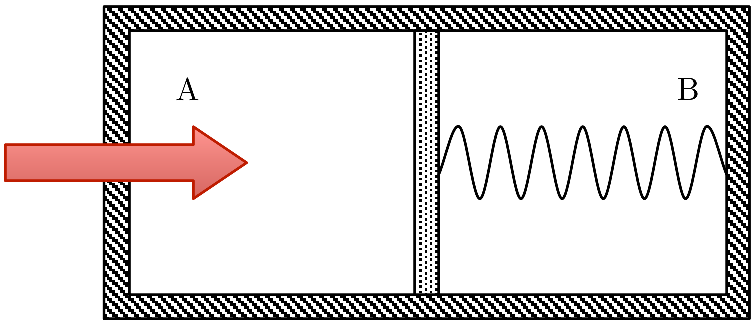

Figure 3: Schematic for problem 3.

Problem 1: Solution
Initial state: $T_1 = 100C$, $v_1 = V_1/m = 1\,m^3/kg$. From tables $p_1 = 1$ atm, $v_f = 0.0010435\,m^3/kg$, $v_g = 1.673\,m^3/kg$, so that $x_1 = (v - v_f)/v_{fg} = 0.597$. We can then get $u_1 = u_f + xu_{fg} = 418.94 + 0.597(2506.5 - 418.94) = 1665.2$ kJ/kg

(a) For an isobaric process, $p = p_1$, and in particular the final state is saturated vapor at 100 C and 1 atm, so that $v_2 = v_g = 1.673\,m^3/kg$. The work done is $W^{\rightarrow} = p(V_2 - V_1) = p(mv_2 - V_1) = 101.4(1.673 - 1) = 68.24$ kJ. The change in internal energy is $\Delta U = m(u_2 - u_1) = 1(2506.5 - 1665.2) = 841.28$ kJ. The heat added is then $Q^{\leftarrow} = \Delta U + W^{\rightarrow} = 909.52$ kJ. To maintain a constant pressure, the piston must be free to move with a negligible acceleration.

(b) Same as for part (a). Why?

(c) Final state is saturated vapor with $v_g = 1m^3/kg$. From the tables $T_2 = 116$ C, $p_2 = 175$ kPa and $u_2 = 2524$ kJ/kg. Since both mass and specific volume are constant during this process, the volume must be constant which requires the piston to be held stationary. The produced work is zero $W^{\rightarrow} = 0$ and the heat added to the water is $Q^{\leftarrow} = \Delta U = 1(2524 - 1665.2) = 860$ kJ.

Problem 2: Solution
Applying first law for control mass consisting of water inside the container, and noting that the volume is constant ($p{-}v$ work is zero), no heat is transferred across the boundary, and electric work crosses the boundary

$$\Delta U = W^{\leftarrow}_{electrical} \quad (1)$$

or

$$m(u_2 - u_1) = IVt$$

Initial state is saturated liquid/vapor at 100 kPa so that $u_1 = (u_f + xu_{fg})_1 = 417.36 + x_1(2506.1 - 417.3) = 417.36 + 2088.8x_1$. Final state is saturated vapor, with $v_2 = v_1 = (v_f + xv_{fg})_1 = 0.0010431 + x_1(1.694 - 0.0010431) = 0.001 + 1.693x_1$. So the first law looks like

$$5(u_g(v_g = 0.001 + 1.693x_1) - (417.36 + 2088.8x_1)) = 8(110)(600)/1000$$

or

$$2088.8x_1 = (u_g(v_g = 0.001 + 1.693x_1) - 522.96$$

Let's start with an initial guess $x_1 = 0.5$, then $v_g = 0.001 + 1.693x_1 = 0.8475\,m^3/kg$, then $u_2 = 2533$ kJ/kg, updating $x_1 = (2533 - 552.96)/2088.8 = 0.95$. Then $v_g = 0.001 + 1.693x_1 = 1.606\,m^3/kg$, then $u_2 = 2505$ kJ/kg. Updating $x_1 = (2505 - 552.96)/2088.8 = 0.934$.

Problem 3: Solution

(a) Ideal gas law $pV = mRT$ for each compartment yields $p_{A1} = 430.5$ kPa and $p_{B1} = 143.5$ kPa.

(b) Force balance on the piston, assuming zero acceleration, yields

$$p_A = p_B + \frac{k(l_0 - l)}{A}$$

so that $l_0 - l = A(p_A - p_B)/k$ which for the initial state gives $l_0 - l = 7.2$ cm. Since $l = V_{B1}/A$, we get $l_0 = 31.2$ cm.

(c) Force balance on the piston yields

$$dp_A = dp_B - \frac{k}{A^2} dV_B \Rightarrow (p_A - p_B) = (p_A - p_B)_1 - \frac{k}{A^2}(V_B - V_{B1}) \qquad (2)$$

First law for compartments A and B respectively

$$\begin{aligned} \Delta U_A &= Q_A^{\leftarrow} - W_A^{\rightarrow} \\ \Delta U_B &= -W_B^{\rightarrow} \end{aligned}$$

or

$$\begin{aligned} m_A c_v (T_2 - T_1) &= Q_A^{\leftarrow} - \int_{V_{A1}}^{V_{A2}} p_A \, dV_A \\ m_B c_v (T_2 - T_1) &= - \int_{V_{B1}}^{V_{B2}} p_B \, dV_B \end{aligned}$$

Noting that $dV_A = -dV_B$, and adding the two equations, we get

$$(m_A + m_B) c_v (T_2 - T_1) = Q_A^{\leftarrow} + \int_{V_{B1}}^{V_{B2}} (p_A - p_B) \, dV_B$$

So the net work output from air in both compartments is

$$\begin{aligned} W_{A+B}^{\rightarrow} &= - \int_{V_{B1}}^{V_{B2}} (p_A - p_B) \, dV_B \\ &= - \int_{V_{B1}}^{V_{B2}} \left((p_A - p_B)_1 - \frac{k}{A^2}(V_B - V_{B1}) \right) dV_B \\ &= 0.1235 \text{ kJ} \end{aligned}$$

Ideal gas law at state 2,

$$RT_2 = \frac{p_{A2} V_{A2}}{m_A} = \frac{p_{B2} V_{B2}}{m_B} \Rightarrow \frac{p_{A2}(V - V_{B2})}{m_A} = \frac{p_{B2} V_{B2}}{m_B} \Rightarrow p_{A2} = p_{B2} \frac{V_{B2}}{V - V_{B2}} \frac{m_A}{m_B}$$

leading to $p_{A2} = 2.444 p_{B2}$. Substituting in Equation (2),

$$p_{A2} - p_{B2} = (p_A - p_B)_1 - \frac{k}{A^2}(V_{B2} - V_{B1}) = 367 \text{ kPa}$$

resulting in $p_{B2} = 254$ kPa and $p_{A2} = 621$ kPa. Using ideal gas law, we get $T_2 = 487$ K.

Finally first law, yields

$$Q_A^{\leftarrow} = (m_A + m_B) c_v (T_2 - T_1) + W_{A+B}^{\rightarrow} = 4.16 \text{ kJ}$$