American University of Beirut
Department of Electrical and Computer Engineering
EECE 311 – Electronic Circuits
Summer 2008
Midterm – July 22, 2008
Open Book – 90 minutes

NAME:__________Solution_________________ **ID Number:** ______________________

Problem 1 [30 points]

Consider the current mirror shown in Figure 1. The MOSFET parameters are $k' = 220\ \mu A/V^2$, $V_t = 0.5$ V, $V_A' = 20$ V/μm, and $(W/L)_1 = 10\mu m / 0.25\mu m$.
Assume that the DC drain current of a MOSFET in SAT is given by $I_D \simeq \frac{1}{2}k'\left(\frac{W}{L}\right)V_{OV}^2$.

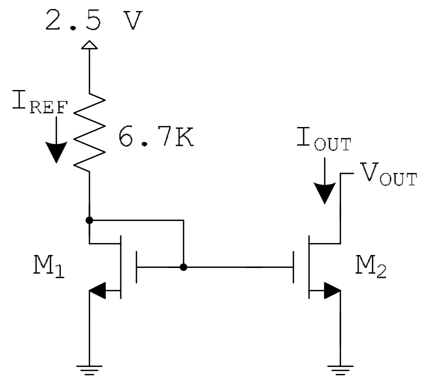


Figure 1

a) [6 points] Find the value of the reference current I_{REF}.

$I_{REF} = (V_{DD} - V_{DS})/R = (V_{DD} - V_{GS})/R = (2.5 - x)/6.7K$ where $x = V_{GS}$

Also: $I_{REF} = ½\, k'(W/L)(V_{GS} - V_t)^2 = ½(220e\text{-}6)(10/0.25)(V_{GS} - 0.5)^2$ =>

$0.5(220e\text{-}6)(10/0.25)(x - 0.5)^2 = (2.5 - x)/10K$

Solve to get: $x = V_{GS} = 0.744$ V => $I_{REF} = 262\ \mu A$

b) The output current should nominally be equal to I_{REF}. It is also specified that the output current I_{OUT} should not change by more than +3% when the output voltage V_{OUT} changes by +1 V.

i) [6 points] What should be the output resistance of the current mirror?

$I_{OUT} = 262\ \mu A$. A change of 3% is therefore $\Delta I_{OUT} = 7.86\ \mu A$ for $\Delta V_{OUT} = 1V$.

The output resistance is $\Delta V_{OUT}/\Delta I_{OUT} = 1/7.86\ \mu A = 127.2\ K\Omega$.

ii) [6 points] Determine the values of W and L for M_2.

$R_{out} = r_{o2} = V_{A2}/I_{D2} = V_{A2}/\ 262\mu A = 127.2$ K => $V_{A2} = 33.33$ V.

$L_2 = V_{A2}/V_A' = 33.33/20 = 1.666$ µm.

$W_2/L_2 = 10/0.25 \Rightarrow W_2 = 10/0.25 \times 1.666 = 66.65$ µm.

c) A resistor R_P is now inserted between the source terminal of M2 and ground, in order to make the value of the output current $I_{OUT} = I_{REF}/10$.

i) [6 points] Find R_P.

$I_{OUT} = 26.2\ \mu A = I_{D2}$.

$V_{GS1} = 0.744$ V

$I_{D2} = ½(220e\text{-}6)(40)(V_{GS2} – 0.5)^2 \Rightarrow V_{GS2} = 0.5772$ V

$V_{GS1} – V_{GS2} = I_{OUT}\times R_P \Rightarrow$

$R_P = (0.744 – 0.5772)/26.2\mu = 6.366$ K

ii) [6 points] Find the new change in I_O (as a *percentage*) when V_{OUT} changes by +1 V.

Output resistance is $r_{o2} + R_P + g_{m2}\, r_{o2}\, R_P$

$r_{o2} = V_{A2}/I_{OUT} = 33.33/26.2\mu = 1272.1$ K

$g_{m2} = I_{OUT}/(0.5(V_{GS2} – V_t)) = 26.2\mu /(0.5(0.5772 – 0.5)) = 0.6788$ mA/V

Output resistance = $r_{o2} + R_P + g_{m2}\, r_{o2}\, R_P$ =

1272.1 K + 6.366 K + 0.6788 m×6.366 K×1272.1 K = 6775.5 K

$\Delta I_{OUT} = \Delta V_{OUT}/R_{out} = 1/6775.5\text{K} = 0.1476\ \mu A$

As a percentage this is: 0.1476/26.2 = 0.56%

Problem 2 [30 points]

The BJT shown in the amplifier circuit of Figure 3 is biased such that g_m = 40 mA/V, r_π = 4 KΩ, and r_o = 15 KΩ. The resistance values are R_B = 10 KΩ, R_C = 5 KΩ, and R_{sig} = 20 KΩ.

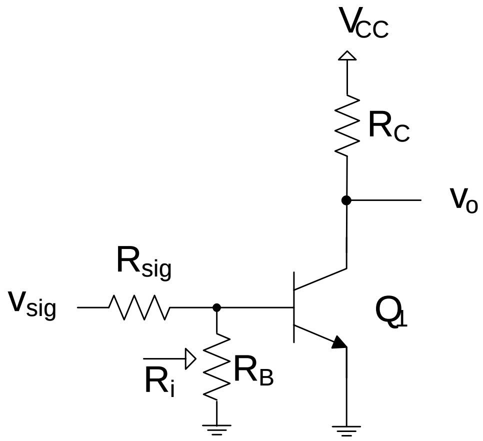


Figure 3

a) Find input resistance R_i, as shown in the figure.

$R_i = R_B \,//\, r_\pi = 10 \,//\, 4 = 2.857$ KΩ.

b) Find the output resistance. Assume that all elements of the circuit are an internal part of the amplifier.

$R_o = R_C \,//\, r_o = 5 \,//\, 15 = 3.75$ KΩ.

c) Find the voltage gain v_o/v_{sig}.

$v_o/v_{sig} = (v_o/v_b)\,(v_b/v_{sig}) = -g_m\, R'_L(R_i/(R_i+R_{sig}) = -g_m\,(r_o//R_C)(R_i/(R_i+R_{sig}))$

$\Rightarrow v_o/v_{sig} = -40(3.75)(2.857/22.857) = -18.75$ V/V

Assume in the following that for the BJT, $C_\pi = 20$ fF, $C_\mu = 10$ fF, and that a load capacitor $C_L = 50$ fF is connected from the output node to ground.

d) Using Miller's theorem, and assuming that <u>the input circuit</u> determines the upper 3-dB frequency f_H, find the value of f_H.

Miller's constant $K = -g_m\, R'_L = -g_m\,(r_o// R_C) = -40(3.75) = -150$

$C_{in} = C_\pi + (1-K)C_\mu = 20 + (151)10 = 1530$ fF

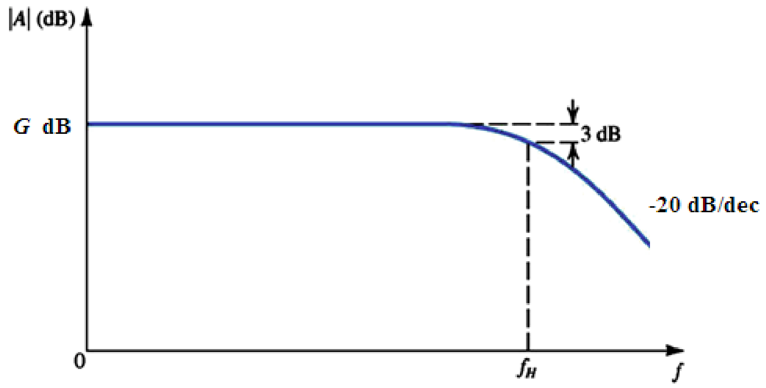
$R_{in} = R_B \,//\, r_\pi \,//\, R_{sig} = 10 \,//\, 4 \,//\, 20 = 2.5$ K

$f_H = 1/(2\pi\, R_{in}\, C_{in}) = 1/(2\pi\times 2.5\text{ K}\times 1530\text{ fF}) = 41.6$ MHz

e) Based on the results of (c) and (d), show and *label* the Bode plot for the magnitude of the voltage gain (v_o/v_{sig}).

$G = 20\log|A_M| = 20\log(18.75) = 25.5$ dB

$f_H = 41.6$ MHz



f) Using the open-circuit time constants methods, find the resistance seen by each capacitor, and estimate the value of f_H.

$R_\pi = R_B \,//\, r_\pi \,//\, R_{sig} = 2.5$ K

$R_{CL} = R_o = 3.75$ K

$R_\mu = R_\pi + R_o + g_m R_\pi R_o = 381.25$ K

$f_H = 1/(2\pi(C_\pi R_\pi + C_L R_{CL} + C\mu R\mu) = 39.3$ MHz

Problem 3 [20 points]

a) [12 points] Design a non-inverting opamp amplifier for the following specifications:
Voltage gain = 5, gain error = 1%, bandwidth = 10 MHz.
Determine the *ratio* of the two resistors in the circuit, the opamp open-loop gain A_M, and the opamp unity-gain frequency f_t.

Gain = $(1+R_2/R_1)/(1+(1+R_2/R_1)/A_M)$

Nominal gain is $(1+ R_2/R_1)$ => $R_2/R_1 = 4$

Error = $(1+ R_2/R_1)/A_M = 5/ A_M = 0.01$ => $A_M = 500$

Bandwidth is f_b. Gain-bandwidth product is approximately fixed

~~=> 5 f_b = $A_M f_t$ => f_t = 50 MHz/500 = 0.1 MHz.~~

b) [8 points] What is the largest peak-to-peak sinusoidal swing at the input of an opamp integrator with an integrator time constant of 16 nsec, that produces an output free from slewing? The slew rate of the opamp is 0.1 V/nsec.

Hint: find $\left.\frac{dv_{out}}{dt}\right|_{max}$ when the input is $V_p\sin(\omega t)$.

$v_{out} = 1/(RC)\, dv_i/dt$ => $v_{out} = V_p/\omega RC \cos(\omega t)$

$dv_{out}/dt = -V_p/RC \sin(\omega t)$

the maximum value is V_p/RC; this should be $\leq$ slew_rate

Therefore $V_p = RC \times$ slew_rate = 16 nsec × 0.1 V/nsec = 1.6 V

The maximum peak-to-peak input is therefore 2×1.6 = 3.2 V

Problem 4 [20 points]

Design the source follower of Figure 2 (determine R_S and W/L) for a power budget of 1 mW and a voltage gain v_{out}/v_{in} of 0.9 when R_L = 10 K.
The MOSFET parameters are k' = 220 μA/V^2, V_t = 0.5 V, and V_A = 5 V.
Assume that the DC drain current of a MOSFET in SAT is given by $I_D \simeq \frac{1}{2} k' \left(\frac{W}{L}\right) V_{OV}^2$, and that all capacitors are very large.

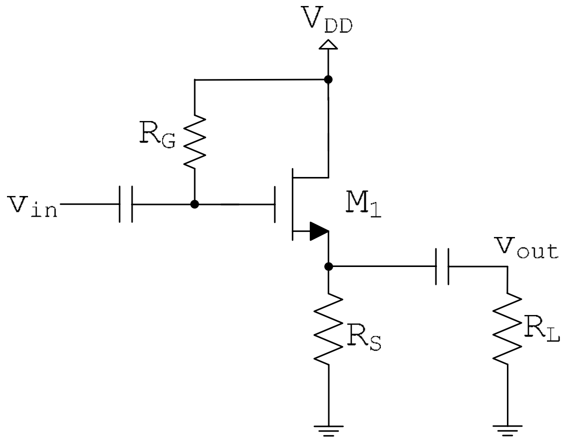


Figure 3

V_{DD} = 2.5V

I_D = 1 mW/2.5V = 0.4 mA

$r_o = V_A/I_D$ = 5/0.4 = 12.5K

$2\, I_D/V_{OV} = g_m \Rightarrow V_{OV} = 2I_D/g_m$

Gain = $v_{out} / v_{in} = R'/(1/g_m + R')$ where $R' = R_S \,//\, r_o \,//\, R_L$

$\Rightarrow 0.9 = R'/(1/g_m + R') \Rightarrow g_m = 9/R'$

Solve by iterations with an initial guess of V_{OV} = 0.2 V.

Find

$V_{GS} = V_{OV} + V_t$;

$V_S = V_G - V_{GS} = V_{DD} - V_{GS}$;

$R_S = V_S/I_D$;

$R' = R_S \,//\, r_o \,//\, R_L$;

$g_m = 9/R'$;

$V_{OV} = 2I_D/g_m$;

Repeat above steps until convergence.

VOV	VGS	VS	RS	R'	gm
0.2000	0.7000	1.8000	4500.0000	2486.1878	0.00362
0.2210	0.7210	1.7790	4447.5138	2470.0828	0.00364
0.2196	0.7196	1.7804	4451.0927	2471.1864	0.00364
0.2197	0.7197	1.7803	4450.8475	2471.1108	0.00364

W/L is calculated from the current equation:

$I_D = ½k'\ (W/L)V_{OV}^2 \Rightarrow (W/L) = 2I_D/\ (k'\ V_{OV}^2) = 2\times0.4\text{e-}3\ /\ (220\text{e-}6\times0.2197^2)$

$\Rightarrow\ (W/L) = 75.3$