Homework 3 Solution

Problem 1

a- Find the value of V_{SG} for Q_3.

$I_{Ref} = \frac{1}{2} K_p' \left(\frac{W}{L}\right)_p (V_{SG} - |V_{tp}|)^2 (1 + |\lambda| V_{G})$

$\Rightarrow 15\mu = \frac{1}{2} \times 100\mu \times 10 \, (V_{SG} - 0{,}5)^2 \, (1 + 0{,}35\, V_{SG})$

$\Rightarrow 0{,}35\, V_{SG}^3 + 0{,}65\, V_{SG}^2 - 0{,}9125\, V_{SG} + 0{,}22 = 0$

$\Rightarrow \boxed{V_{SG} = 0{,}656V}$

b- Find I_{D_2} when $V_o = \frac{V_{DD}}{2}$

$I_{D_2} = \frac{1}{2} K_p' \left(\frac{W}{L}\right)_p (V_{SG_2} - |V_{tp}|)^2 \, (1 + 0{,}35\, V_{SD_2})$

$V_o = \frac{V_{DD}}{2} = 0{,}9V$

$V_{SD_2} = V_S - V_{D_2} = V_{DD} - V_o = 0{,}9V$

$V_{SG_2} = V_{SG_3} = 0{,}656V$

$I_{D_2} = \frac{1}{2} \times 100\mu \times 10 \times (0{,}656 - 0{,}5)^2 \, (1 + 0{,}35 \times 0{,}9)$

$\Rightarrow \boxed{I_{D_2} = 16\mu A}$

c- $V_{sig} = 0$; $V_o = \frac{V_{DD}}{2} = 0{,}9V$

$I_{D_2} = 16\mu A = I_{D_1}$

$I_{D_1} = \frac{1}{2} K_n' \left(\frac{W}{L}\right)_n (V_{GS} - V_t)^2 \, (1 + 0{,}4 V_{DS})$

$V_{GS} = V_{Bias} - V_S$

$V_S = R_S . I_{D_1} = 50 \times 16.10^{-6} = 8.10^{-4} V$

$V_{DS} = V_D - V_S = V_o - V_S = 0{,}9 - 8.10^{-4} = 0{,}8992V$

$16\mu A = \frac{1}{2} \times 350\mu \times 2 \times (V_{GS} - 0{,}45)^2 \, (1 + 0{,}4 \times 0{,}8992)$

$\Rightarrow V_{GS} = 0{,}6334V$

$V_{Bias} - V_S = V_{GS} = 0{,}6334V$

$\Rightarrow V_{Bias} = 0{,}6334 + 8.10^{-4} \Rightarrow \boxed{V_{Bias} = 0{,}6342\ V}$

d- Find the values of g_{m1}, r_{o1}, r_{o2}, R_{in} and R_{out}

$$g_{m1} = \frac{2 I_{D1}}{V_{ov}} = \frac{2 \times 16\mu}{V_{GS} - V_t} = \frac{2 \times 16\mu}{0,6334 - 0,45} = \boxed{0,1744 \ mA/V}$$

$$r_{o1} = \frac{V_A + V_{DS}}{I_{D1}} = \frac{1/0,4 + 0,8992}{16\mu} = \boxed{212,45 \ K\Omega}$$

$$r_{o2} = \frac{V_A + V_{SD2}}{I_{D2}} = \frac{1/0,35 + 0,9}{16\mu} = \boxed{234,82 \ K\Omega}$$

$$R_{in} = \frac{V_i}{i_1} = \frac{r_{o1} + R_L}{1 + (g_{m1} + x g_{m1}) r_{o1}} = \frac{r_{o1} + r_{o2}}{1 + g_{m1}(1+x) r_{o1}}$$

$$\Rightarrow R_{in} = \frac{212,45 + 235,82}{1 + (0,174 + 0,2 \times 0,174) \times 212,45} = \boxed{9,882 \ K\Omega}$$

$$R_{out} = r_{o1} + (1 + g_{m1}(1+x) r_{o1}) R_s$$

$$R_{out} = 212,45 \ (1 + (0,174 + 0,2 \times 0,174) \times 212,45) \times 50 . 10^{-3}$$

$$\boxed{R_{out} = 214,71 \ K\Omega}$$

e- Find $\frac{V_o}{V_{sig}}$

$$G_v = \frac{V_o}{V_{sig}} = (1 + g_{m1}(1+x) r_{o1}) \cdot \frac{r_{o2}}{r_{o1} + r_{o2}} \times \frac{R_{in}}{R_{in} + R_s} = \boxed{23,77 \ V/V}$$

f- Q_2 saturated if: $V_{SDp} \geq V_{SGp} - |V_{tp}|$

$$V_{SDp} \geq 0,656 - 0,5$$

$$V_{SDp} \geq 0,156 \ V$$

$$V_{DD} - V_{Dp} \geq 0,156 V$$

$$V_{DD} - V_o \geq 0,156$$

$$V_o \leq 1,8 - 0,156$$

$$V_o \leq 1,644 V$$

Q_1 is saturated if: $V_{DS} \geq V_{GS_n} - V_{tn}$

$$V_{DSn} \geq 0,6334 - 0,45$$

$$V_{DSn} \geq 0,1834$$

$$V_o - V_s \geq 0,1834$$

$$V_o - 16\mu \times 50 \geq 0,1835$$

$$V_o \geq 0,1843 \ V$$

V_o peak-to-peak $= 1,644 - 0,1843 = 1,4597 V$

V_{sig} peak-to-peak $= V_{o \, peak\text{-}to\text{-}peak} = 1,4597 / 23,64 = 61,747 mV$

g- Scan it from Rami Fatayri's Homework

h- $f_{P1} = \dfrac{1}{2\pi (C_{gs}+C_{sb})\left(R_s // \dfrac{1}{g_m+g_{mb}}\right)}$

$$f_{P1} = \frac{1}{2\pi (25+10).10^{-15}\left(50 // \dfrac{1}{0,174(1+0,2).10^{-3}}\right)} = 3,9.10^{11} Hz = 390 GHz$$

$$f_{P2} = \frac{1}{2\pi (C_L+C_{gd}+C_{db}).(R_L // R_{out})} = \frac{1}{2\pi(50+5+10).10^{-15} \times (234,82 // 214,7.10^3)}$$

$$f_{P2} = 21,831 MHz$$

$$f_H = \frac{1}{2\pi \left(R_1(C_{gs}+C_{sb}) + R_2(C_{gd}+C_{db}+C_L)\right)}$$

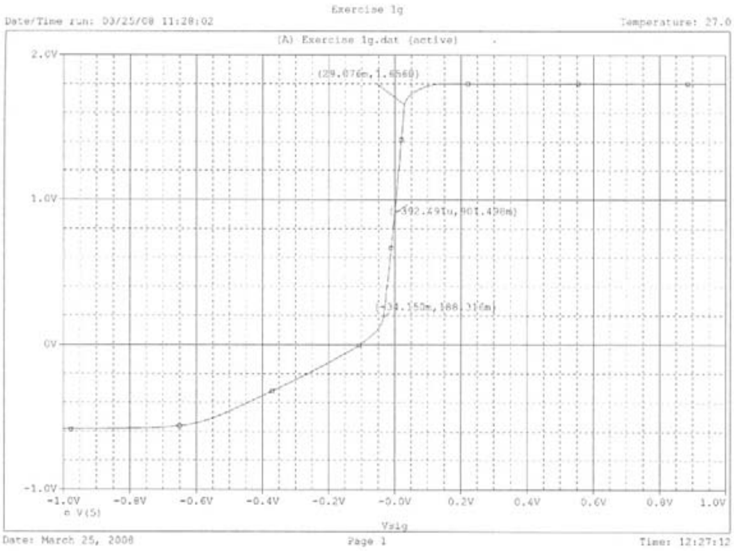
$R_1 = R_{in} // R_s = 9,832 // 0,05 = 49,74\ \Omega$

$R_2 = R_{out} // R_L = 234,82 // 214,7K = 112,154\ K\Omega$

$\Rightarrow \boxed{f_H = 21,326\ MHz}$

i- Scan it from Rami Fatayri's Homework

g)



$$Here\ from\ the\ graph\ V_{sig}\ peak\ to\ peak = 29.076m + 34.150m = 63.226\ mV$$

$$and\ G_v = slope = \frac{1.6569 - 0.1883}{63.226} = 23.23\frac{V}{V}$$

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Exercise 1g

VDD 1 0 1.8V
IREF  2 0 15u
Vbias  4 0 0.634V
Vsig  7 0 DC 0V

Rs 6 7 50

M1 5 4 6 0 cmosn W=10u L=5u
M2 5 2 1 1 cmosp W=10u L=1u
M3 2 2 1 1 cmosp W=10u L=1u

.DC Vsig -1V 1V 1mV
.model cmosn nmos kp=350u vto=0.45 lambda=0.4 gamma=0.4 phi=1
.model cmosp pmos kp=100u vto=-0.5 lambda=0.35

.OP
.Probe
.End
```

i)

The 3db frequency from the bode plot is 21.892 MHz which is really close to the one we calculated which is 21.84 MHz.

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Exercise 1i

VDD 1 0 1.8V
IREF  2 0 15u
Vbias  4 0 0.634V
Vsig  7 0 AC 30.71mV

Rs 6 7 50
Cgs 4 6 25f
Cgd 4 5 5f
Cdb 5 0 10f
Csb 6 0 10f
CL 5 0 50f

M1 5 4 6 0 cmosn W=10u L=5u
M2 5 2 1 1 cmosp W=10u L=1u
M3 2 2 1 1 cmosp W=10u L=1u

.AC DEC 10 1 1e8
.model cmosn nmos kp=350u vto=0.45 lambda=0.4 gamma=0.4 phi=1
.model cmosp pmos kp=100u vto=-0.5 lambda=0.35

.OP
.Probe
.End
```
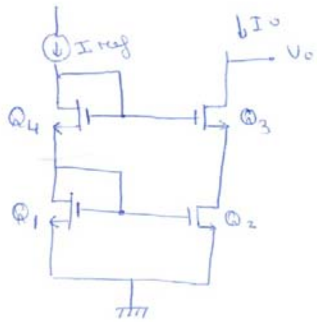

Problem 2.



$I_{Ref} = 10\mu A$
$V_o = 2{,}5V$
$K'_n \frac{W}{L} = 1 mA/V^2$
$V_t = 0{,}5V$
$V_A = 10V$

a) • $I_{REF} = I_{D4} = \frac{1}{2} K'_n \frac{W}{L} (V_{GS4} - V_t)^2 (1 + \frac{V_{GS4}}{V_A})$

$\Rightarrow 10\mu A = 1/2 \times 1 mA/V^2 (V_{GS4} - 0{,}5V)^2 (1 + \frac{V_{GS4}}{10})$

$\Rightarrow 10^{-2} = 1/2 (V_{GS4} - 0{,}5)^2 (1 + \frac{V_{GS4}}{10})$

$\Rightarrow \frac{V_{GS4}^3}{10} + \frac{9}{10} V_{GS4}^2 - 0{,}975 V_{GS4} + 0{,}23 = 0$

$\Rightarrow V_{GS4} = 0{,}6371 V > V_t = 0{,}5V \Rightarrow$ accepted

$V_{GS4} = 0{,}361V < V_t \Rightarrow$ rejected

$V_{GS4} = -9{,}998 < V_t \Rightarrow$ rejected.

$\Rightarrow \boxed{V_{GS4} = 0{,}6371 V}$

• $I_{Ref} = 1/2 K'_n \frac{W}{L} (V_{GS1} - V_t)^2 (1 + \frac{V_{GS1}}{V_A})$

$\Rightarrow \boxed{V_{GS1} = V_{GS4} = 0{,}6371V}$

• $I_o = I_{D2} = I_{D3}$

$\Rightarrow 1/2 K'_n \frac{W}{L} (V_{GS2} - V_t)^2 (1 + \frac{V_{DS2}}{V_A}) = 1/2 K'_n \frac{W}{L} (V_{GS3} - V_t)^2 (1 + \frac{V_{DS3}}{V_A})$

$V_{GS2} = V_{GS1} = V_{GS4} = 0{,}6371$

$V_{GS3} = V_G - V_{S3} = V_{GS4} + V_{GS1} - V_{S3} = 2 \times 0{,}6371 - V_{DS2}$

$\Rightarrow V_{GS3} = 1{,}2742 - V_{DS2}$

$V_{DS3} = V_O - V_{DS2} = 2{,}5 - V_{DS2}$

Replacing

$\frac{1}{2} K'_n \frac{W}{L}(V_{GS1} - V_t)^2 \left(1 + \frac{V_{DS2}}{V_A}\right)$

$\Rightarrow -0,1 V_{DS_2}^3 + 1,4084 V_{DS_2}^2 - 1,997317 V_{DS_2} + 0,730435 = 0$

$\Rightarrow V_{DS_2} = 0,6492 \text{ V} \Rightarrow V_{GS3} = 0,625 > V_t$ (accepted)

$V_{DS_2} = 0,8974 \text{ V} \Rightarrow V_{GS3} = 0,3768 < V_t$ (rejected)

$V_{DS2} = 12,537 \text{ V} \Rightarrow V_{GS3} = -11,262 < V_t$ (rejected)

$\Rightarrow \boxed{V_{DS_2} = 0,6492}$

$I_O = \frac{1}{2} K'_n \frac{W}{L} (V_{GS_1} - V_t)^2 \left(1 + \frac{V_{DS2}}{V_A}\right)$

$\Rightarrow \boxed{I_O = 10,0083 \mu A}$

Output resistance

$g_{m3} = \frac{2 I_O}{V_{ov}} = \frac{2 I_O}{V_{GS3} - V_t} = 160,13 \ \mu A/V$

$r_{o3} = \frac{V_A + V_{DS3}}{I_O} = 1184 \ K\Omega$

$r_{o2} = \frac{V_A + V_{DS2}}{I_O} = \frac{V_A}{I_{O2}} = 1064 \ K\Omega$

$R_O \simeq g_{m3} \, r_{o3} \, r_{o2} = 201,56 \ M\Omega$

b) Minimum value of V_O

Q_3 saturates when $V_{DS3} \geq V_{GS3} - V_t$

$\Rightarrow V_{DS3\,min} = V_{GS3} - V_t$

$(V_D - V_{S3})_{min} = (V_G - V_{S3})_{min} - V_t$

$V_{D\,min} = V_{G\,min} - V_t$

$V_{G\,min} = V_G = 1,2742 \text{V}$

$\Rightarrow \boxed{V_{O\,min} = 0,7742 \text{V}}$

Problem 3

a) $\frac{I_2}{I_0} = \frac{(W/L)_2}{(W/L)_1} = \frac{20}{5} = 4$

$I_2 = 4 I_0$

$\boxed{I_0 = \frac{1}{2}K_n' \left(\frac{W}{L}\right)_3 (V_{GS_3} - V_t)^2}$ ①

$V_{G_3} = 3 - 10.10^{-3} \times 4 I_2$

$\boxed{V_{G_3} = 3 - 4.10^4 I_0}$ ②

$\boxed{I_0 = \frac{1}{2} K_n' \left(\frac{W}{L}\right)_1 (V_{GS_1} - V_t)^2}$ ③

$\frac{①}{③} \Rightarrow 1 = \frac{(W/L)_3 \; (V_{GS_3} - V_t)^2}{(W/L)_1 \; (V_{GS_1} - V_t)^2}$

$4 (V_{GS_3} - V_t)^2 = (V_{GS_1} - V_t)^2$

$V_{GS_1} = V_{S_3}$

$4 (V_{GS_3} - V_t)^2 = (V_{S_3} - V_t)^2$

$V_{S_3} - V_t = 2(V_{GS_3} - V_t)$

$V_{S_3} - V_t = 2 V_{G_3} - 2 V_{S_3} - V_t$

$3 V_{S_3} = 2 V_{G_3} - V_t = 2 V_{G_3} - 0{,}4$

$I_0 = \frac{1}{2} K_n' \left(\frac{W}{L}\right)_3 \left(V_{G_3} - \frac{2}{3} V_{G_3} + \frac{1}{3} \times 0{,}4 - 0{,}4\right)^2$

$= \frac{1}{2} \times 200.10^{-6} \times 20 \left(\frac{V_{G_3}}{3} - \frac{2}{3} \times 0{,}4\right)^2 = \frac{2}{9}.10^{-3} (V_{G_3} - 0{,}8)^2$

Replacing V_{G3}:

$I_0 = \frac{2.10^{-3}}{9} (3 - 4.10^4 I_0 - 0{,}8)^2 = \frac{2}{9}.10^{-3} (2{,}2 - 4.10^4 I_0)^2$

$I_0 = \frac{2.10^{-3}}{9} (4{,}84 + 1{,}6.10^9 I_0^2 - 1{,}76.10^5 I_0)$

$\Rightarrow \frac{3{,}9}{9}.10^6 I_0^2 - \frac{361}{9} I_0 + \frac{9{,}68}{9}.10^{-3} = 0$

$I_{0,1} = 6{,}89.10^{-5} A$

$\boxed{I_{0,2} = 4{,}388.10^{-5} A}$

with $I_o = 6,89 \cdot 10^{-5}$ A, we can calculate

$V_{G_3} = 3 - 4 \cdot 10^4 I_o = 0,244$ V

$V_{GS_1} = 0,771$ V

inacceptable since V_{G_3} must be greater than V_{GS_1}

$I_o = 4,388 \cdot 10^{-5}$ A

$I_o = \frac{1}{2} K_n' \left(\frac{W}{L}\right)_1 (V_{GS_1} - V_t)^2 \Rightarrow V_{GS_1} = V_{DS_1} = V_{D_1} = 0,696$ V

balancing the drain voltages of Q_1 and Q_2, we should make $V_{D_2} = V_{D_1} = 0,696$ V

$V_{DS_4} = V_{Q_4} = V_{G_3} - V_{D_2} = (3 - 4 \cdot 10^4 I_o) - 0,696$

$V_{DS_4} = 1,2448 - 0,696 = 0,5488$ V

$V_{DS_4} = V_{GS_4} = 0,5488$ V

$V_{ov_4} = V_{GS_4} - V_t = 0,5488 - 0,4 = 0,1488$ V

$I_2 = \frac{1}{2} K_n' \left(\frac{W}{L}\right)_4 V_{ov_4}^2$

$$\left(\frac{W}{L}\right)_4 = \frac{I_2}{\frac{1}{2} K_n' V_{ov_4}^2} = \frac{4 \times 4,1388 \cdot 10^{-5}}{0,5 \times 0,2 \cdot 10^{-3} \times 0,1488^2}$$

$$\boxed{\left(\frac{W}{L}\right)_4 = 79,272}$$

b) $g_{m_3} = \dfrac{2 I_{D_3}}{(V_{GS_3} - V_t)} = 594 \cdot 10^{-6}$ A/V

$r_{o1} = \dfrac{1}{\lambda \cdot I_{D_1}} = 151,86 \times 10^3\ \Omega$

$r_{o3} = \dfrac{1}{\lambda \cdot I_{D_3}} = 151.86 \cdot 10^3\ \Omega$

$\Rightarrow R_o = r_{o1} \cdot r_{o3} \cdot g_{m3} = 13,7 \cdot 10^6\ \Omega$

$$\boxed{R_o = 13,7 \cdot 10^6\ \Omega}$$

c- $V_{omin} = V_{GS_3} + V_{GS_2} - V_t = 0,84\,V$

$\Rightarrow \Delta V_o = 3 - V_{omin} = 3 - 0,844 = 2,156\,V$

$\Rightarrow \Delta I_o = \frac{\Delta V_o}{R_o} = \frac{2,156}{13.7.10^6} = 0,16\,\mu A$

$\Rightarrow \% = \frac{\Delta I_o}{I_o} \times 100\% = 0,36\%$

Problem 4

a - $A_v = \frac{v_o}{v_i} = \frac{-R_2/R_1}{1 + \left(\frac{1 + R_2/R_1}{A_{vo}}\right)} = \frac{-560/5.6}{1 + \left(\frac{1+100}{8000}\right)} = \boxed{-98.7\ V/V}$

b - At $\omega = 2\pi f = 94,25$ Krad/s

$\Rightarrow A = \frac{A_v}{1 + \frac{j\omega}{\omega_p}} = 78.71 \angle 2,49\ \frac{V}{V}$

where $\omega_p = \frac{\omega_t}{1 + \frac{R_2}{R_1}} = 124,4$ Krad/s

$\Rightarrow \boxed{v_o = 157.4 \cdot \sin(\omega t + 2,49)}$

Problem 5

$P = 10W$

$R_L = 8\Omega$

$F = 20\,KHz$

$P = \frac{V_o^2}{R_L} \Rightarrow V_o^2 = R_L \cdot P = 8 \times 10 = 80$

$V_{orms} = \sqrt{80} \Rightarrow V_{omax} = \sqrt{2}\,.\,V_{orms} = \sqrt{2}\,.\,\sqrt{80} = 12.64\,V$

To avoid distorsion.

$SR = V_{omax} \cdot \omega_t = 12.64 \times 20.10^3 \times 2\pi = 1588774\ V/S$

$\boxed{SR = 1,59\,V/\mu s}$

Problem 6

$v_o(t) = 2400\, v_1(t) - 2391\, v_2(t)$ $\quad$ $v_1(t)$, $v_2(t)$ amplifier inputs.

$$CMRR = 20 \log \left| \frac{A_d}{A_{cm}} \right|$$

$$v_o(t) = A_d\, v_{Id} + A_{cm}\, v_{Icm}$$

$$\begin{cases} v_{Id} = v_{I2} - v_{I1} \\ v_{ICM} = \frac{1}{2}(v_{I1} + v_{I2}) \end{cases} \Rightarrow \begin{cases} v_{I1} = v_{ICM} - \frac{v_{Id}}{2} \\ v_{I2} = v_{ICM} + \frac{v_{Id}}{2} \end{cases}$$

$$\Rightarrow v_o(t) = 2400\, v_1(t) - 2391\, v_2(t)$$

$$= -4791 \left(\frac{v_{Id}}{2} \right) + 9\, v_{Icm}$$

$$\Rightarrow A_d = \frac{-4791}{2} \qquad A_{cm} = 9$$

$$\Rightarrow CMRR = 20 \log \frac{4791}{18} = 48{,}5 \text{ dB}$$