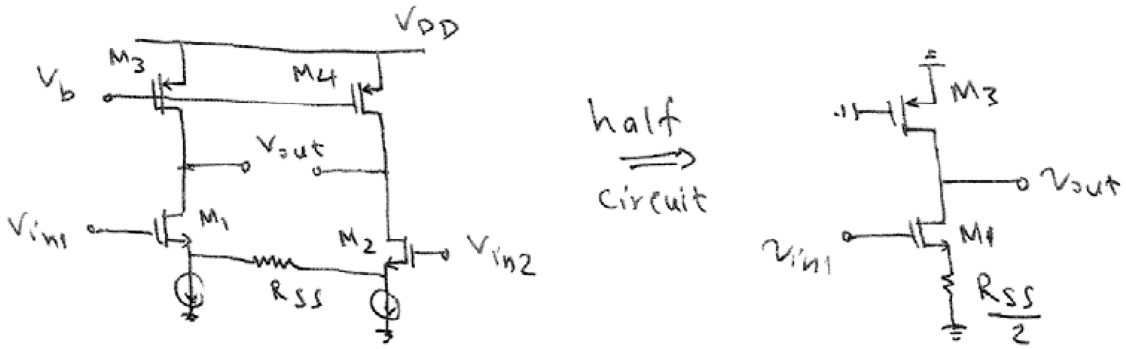
Homework 4 Solution

Problem 1



$v_{out1}/v_{in1} = -g_{m1}\, r_{o1}\, r_{o3} / (r_{o3} + R_{out})$ Equation 6.145 in textbook

$R_{out} = r_{o1} + (1 + g_{m1}\, r_{o1})\, R_{SS}/2$ Equation 6.142 in textbook

$v_{out1}/v_{in1} = -g_{m1}\, r_{o1}\, r_{o3} / (r_{o1} + r_{o3} + (1 + g_{m1}\, r_{o1})\, R_{SS}/2)$

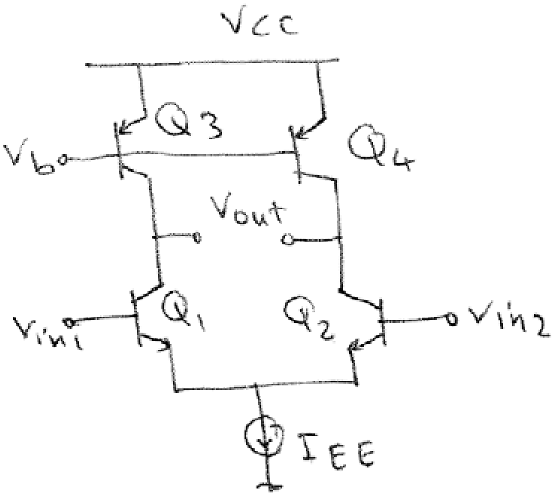
Similarly, and assuming symmetry

$v_{out2}/v_{in2} = -g_{m1}\, r_{o1}\, r_{o3} / (r_{o1} + r_{o3} + (1 + g_{m1}\, r_{o1})\, R_{SS}/2)$

Therefore

$(v_{out2} - v_{out1}) / (v_{in1} - v_{in2}) = g_{m1}\, r_{o1}\, r_{o3} / (\, r_{o1} + r_{o3} + (1 + g_{m1}\, r_{o1})\, R_{SS}/2\,)$

Problem 2



$A_v = 100$

$P = 1mW$

$V_{A,n} = 6$

$V_{CC} = 2.5\ V$

$$P = 1mW = I_{EE} V_{CC} \Rightarrow I_{EE} = \frac{10^{-3}}{2.5} = 0.4mA$$

$$r_{o_N} = \frac{V_{A_n}}{I_{EE}/2} = \frac{6}{0.2 \times 10^{-3}} = 30k\Omega, \quad g_{m_N} = \frac{I_{EE}/2}{V_T} = \frac{0.2}{26}\ S$$

$$A_v = g_{m_N}(r_{o_N} \| r_{op}) \Rightarrow$$

$$100 = \frac{0.2}{26}(30 \times 10^3 \| r_{op}) \Rightarrow r_{op} = 22.94k\Omega$$

$$\Rightarrow V_{A,p} = r_{op}\frac{I_{EE}}{2} = 4.588\ V$$

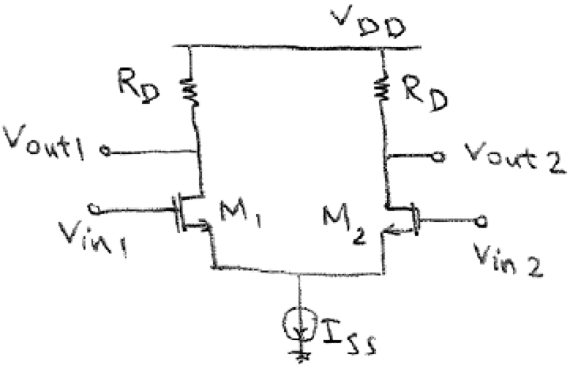
<u>Note:</u> For V_T = 25 mV, the answer is slightly different at 4.286 V.

Input bias current: $I_B = I_{EE}/2(\beta+1) = 0.995\ \mu A$.

Input offset current: $I_{OS} = I_B\ \Delta\beta/\beta = 39.8\ nA$.

Problem 3

Assuming V_{SS} = 0 in Figure 7.1. In the following $(V_{GS} - V_{TH})_{equil}$ is V_{OV}



$A_v = 5$

$P = 1mW$

$(V_{GS} - V_{TH})_{equil} = 150 mV$

$\lambda = 0,\ \mu_n C_{ox} = 100\ \mu A/V^2$

$V_{DD} = 1.8\ V$

$$P = 1mW = V_{DD} I_{SS} = 1.8 I_{SS} \Rightarrow I_{SS} = 0.556 mA$$

$$g_{m_1} = \frac{2 I_{D1}}{(V_{GS} - V_{TH})_{equil}} = \frac{I_{SS}}{(V_{GS} - V_{TH})_{equil}} = \frac{0.556 \times 10^{-3}}{0.15}$$

$$= 3.704\ mS$$

$$A_v = g_{m_1} R_D \Rightarrow 5 = 3.704 \times 10^{-3} \times R_D \Rightarrow R_D = 1.35\ k\Omega$$

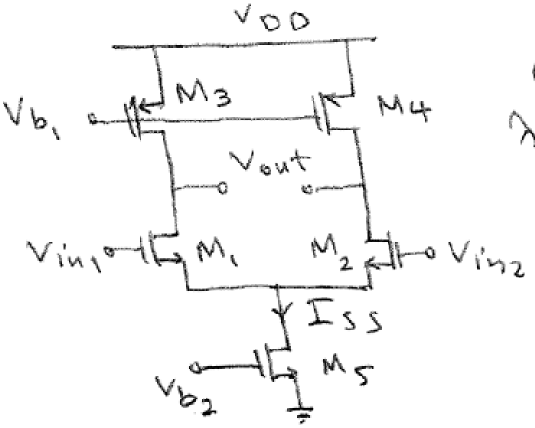
$$(V_{GS} - V_{TH})_{equil} = \sqrt{\frac{2 I_{D1}}{\mu_n C_{ox} \frac{W}{L}}} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} \Rightarrow$$

$$0.15 = \sqrt{\frac{0.556 \times 10^{-3}}{10^{-4} \times \frac{W}{L}}} \Rightarrow \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 246.91$$

Solution for V_{SS} = –1.8 V is similar, and will be considered correct.

Problem 4

In the following $(V_{GS} - V_{TH})_{equil}$ is the same as V_{OV}



$A_V = 40$

$(V_{GS} - V_{TH})_{equil} = ?$

$\lambda_n = 0.1 V^{-1}$ $\quad \lambda_p = 0.2 V^{-1}$

$\mu_n C_{ox} = 100\ \mu A/V^2$

$\mu_p C_{ox} = 50\ \mu A/V^2$

$V_{DD} = 1.8$

$P = 2mw$

$$A_V = -g_{m_N}(r_{o_p} \| r_{o_N}) = -\frac{I_{SS}}{(V_{GS_1} - V_{TH})_{equil}}\left(\frac{1}{\frac{I_{SS}}{2}\lambda_n} \| \frac{1}{\frac{I_{SS}}{2}\lambda_p}\right)$$

$$= -\frac{2}{(V_{GS_1} - V_{TH})_{equil}}\left(\frac{1}{\lambda_n} \| \frac{1}{\lambda_p}\right) \Rightarrow$$

$$\frac{2}{(V_{GS_1} - V_{TH})_{equil}}(10 \| 5) = 40 \Rightarrow (V_{GS_1} - V_{TH})_{equil} = 166.67\ mV$$

$$P = 2\times10^{-3} = V_{DD} I_{SS} \Rightarrow I_{SS} = \frac{2\times10^{-3}}{1.8} = 1.11\ mA$$

$$\left(\frac{W}{L}\right)_{1,2} = \frac{I_{SS}}{\mu_n C_{ox} (V_{GS_1} - V_{TH})^2_{equil}} = \frac{1.11\times 10^{-3}}{10^{-4} \times (0.16667)^2} = 400$$

$$\left(\frac{W}{L}\right)_{3,4} = \frac{I_{SS}}{\mu_p C_{ox} (V_{GS} - V_{TH})_{equil}} = \frac{1.11\times 10^{-3}}{0.5\times 10^{-4} \times (0.16667)^2} = 800$$

$$\left(\frac{W}{L}\right)_{5} = \frac{2 I_{SS}}{\mu_n C_{ox} (V_{GS} - V_{TH})_{equil}} = \frac{2\times 1.11\times 10^{-3}}{10^{-4} \times (0.16667)^2} = 800$$

CMRR when output is single-ended

Common-mode gain is $g_{m1}\, r_{o1}\, r_{o3} / (\, r_{o1} + r_{o3} + (1 + g_{m1}\, r_{o1})\,(2r_{o5})\,)$. This result is similar to Problem 1 with $R_{SS}/2$ replaced with $(2r_{o5})$.

r_{o3} = 9 KΩ

r_{o5} = 9 KΩ (current folwing in M_5 is I_{SS})

r_{o1} = 18 KΩ

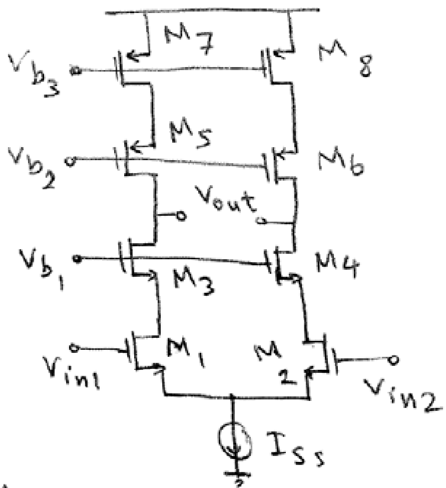
g_{m1} = 6.667 mA/V

A_{cm} = 0.49 V/V

$A_{d[\text{single-ended}]}$ = 40/2 = 20 V/V

CMRR = 20 $\log_{10}$(20/0.49) = 32.2 dB

Problem 5



$A_V = 600$

$P = 4mW$

$(V_{GS} - V_{TH})_{NMOS} = 100mV$

$(V_{GS} - V_{TH})_{PMOS} = 150mV$

$\mu_n C_{ox} = 100\ \mu A/V^2$

$\mu_p C_{ox} = 50\ \mu A/V^2$

$\lambda_n = 0.1\ V^{-1}$

$$A_V \simeq g_{m_1}\left[(g_{m_3} r_{o_3} r_{o_1}) \| (g_{m_5} r_{o_5} r_{o_7})\right] = 600$$

$$P = 4mW = I_{SS} V_{DD} \Rightarrow I_{SS} = \frac{4\times10^{-3}}{1.8} = 2.22\,mA$$

$$g_{m_{1-4}} = \frac{2I_{D_1}}{(V_{GS}-V_{TH})_{NMOS}} = \frac{I_{SS}}{(V_{GS}-V_{TH})_{NMOS}} = \frac{2.22\times10^{-3}}{0.1} = 22.22\,mS$$

$$g_{m_{5-8}} = \frac{2I_{D_5}}{(V_{GS}-V_{TH})_{PMOS}} = \frac{I_{SS}}{(V_{GS}-V_{TH})_{PMOS}} = \frac{2.22\times10^{-3}}{0.15} = 14.815\,mS$$

$$r_{o_{1-4}} = \frac{1}{\lambda_n \frac{I_{SS}}{2}} = \frac{1}{0.1 \times \frac{2.22}{2} \times 10^{-3}} = 9\,k\Omega$$

$$r_{o_{5-8}} = \frac{1}{\lambda_p \frac{I_{SS}}{2}} = \frac{1}{\lambda_p \times \frac{2.22 \times 10^{-3}}{2}} = \frac{0.9 \times 10^{3}}{\lambda_p}$$

$\Rightarrow$ in A_v equation

$$22.22 \times 10^{-3} \left[(22.22 \times 10^{-3} \times 81 \times 10^{6}) \,||\, (14.815 \times 10^{-3} \times \frac{0.81 \times 10^{6}}{\lambda_p^{2}}) \right] = 600 \Rightarrow$$

$$\lambda_p = 0.66\ V^{-1}$$

$$\left(\frac{W}{L}\right)_{NMOS} = I_{SS} / (\mu_n C_{ox} (V_{GS} - V_{TH})^2_{NMOS}) = 2222.2$$

$$\left(\frac{W}{L}\right)_{PMOS} = I_{SS} / (\mu_p C_{ox} (V_{GS} - V_{TH})^2_{PMOS}) = 1975.31$$

$V_{Ap} = 1/\lambda_p = 1.5$ V.