

## Problem 1

$$R_F = R_{S1} = R_{D2} = 3K\Omega$$

The current through  $R_F$  is zero

$$I_{S1} = I_{D3} = 0.5mA$$

$$\Rightarrow V_{S1} = V_{RS1} = V_{DS} = V_{RD2} = 0.5 \times 3 = 1.5V$$

M1:

$$I_{D1} = \frac{1}{2} K'_n \frac{W}{L} V_{OV1}^2 \Rightarrow 0.5m = \frac{1}{2} \times 180\mu \times \frac{50}{0.5} V_{OV1}^2$$

$$V_{OV1} = 0.235V = V_{GS1} - V_{TN}$$

$$\Rightarrow V_{GS1} = 0.885V = V_{in} - V_{S1}$$

$$\Rightarrow V_{in} = 2.385V$$

M2:

$$I_{D2} = \frac{1}{2} K'_p \frac{W}{L} V_{OV2}^2 \Rightarrow V_{OV2} = 0.392V$$

$$V_{OV2} = V_{SG2} - |V_{TP}| \Rightarrow V_{SG2} = 1.142V$$

$$V_{SG2} = V_{DD} - V_b \Rightarrow V_b = 1.858V$$

b)

M1:

$$g_{m1} = \frac{2I_{D1}}{V_{OV1}} = \frac{2 \times 0.5m}{0.235} = 4.255mA/V$$

$$r_{o1} = \frac{1}{\lambda_N I_{D1}} = \frac{1}{0.15 \times 0.5m} = 13.33K\Omega$$

M2:

$$g_{m2} = \frac{2I_{D2}}{V_{OV2}} = \frac{2 \times 0.5m}{0.392} = 2.55mA/V$$

$$r_{o2} = \frac{1}{\lambda_p I_{D2}} = \frac{1}{0.2 \times 0.5m} = 10K\Omega$$

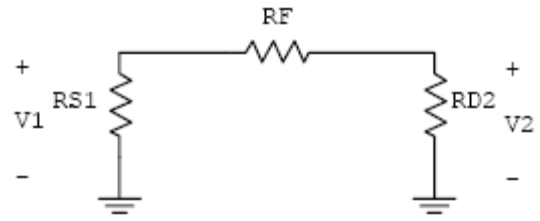
M3:

$$g_{m3} = \sqrt{2k'_p \frac{W}{L} I_{D3}} = \sqrt{2 \times 65\mu \times \frac{50}{0.5} \times 0.5m} = 2.55mA/V$$

$$r_{O3} = \frac{1}{\lambda_p I_{D3}} = 10K\Omega$$

$\beta$  Block: Series – Shunt Amplifier

$$\beta = \frac{V_1}{V_2} = \frac{R_{S1}}{R_{S1} + R_F} = \frac{3}{3+3} = 0.5V/V$$



A Block:

$$R_S = R_{S1} \parallel R_F = 1.5K\Omega$$

$$R_L = R_{D2} \parallel (R_F + R_{S1}) = 2K\Omega$$

$$A_1 = \frac{V}{V_{in}} \quad (\text{Gain of common source with source resistance})$$

$$A_1 = \frac{-g_{m1}r_{O1}r_{O2}}{r_{O2} + r_{O1}(1 + g_{m1}R_S)}$$

$$= \frac{-4.255 \times 13.33K \times 10K}{10K + 13.33K(1 + 4.255m \times 1.5K)}$$

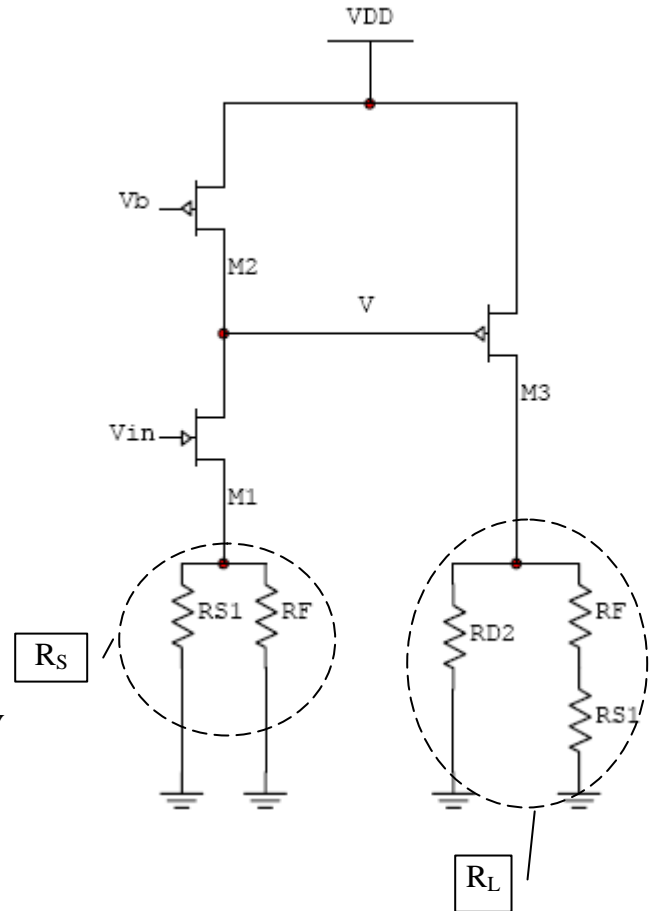
$$= -5.23V/V$$

$$A_2 = \frac{V_{out}}{V} \quad (\text{Gain of common source})$$

$$= -g_{m3}(r_{O3} \parallel R_L) = -2.55m(10K \parallel 2K) = -4.25V/V$$

$$A = A_1 A_2 = (-5.23) \times (-4.25) = 22.23V/V$$

$$A\beta = 22.23 \times 0.5 = 11.115 > 0 \Rightarrow \text{Negative Feedback}$$



d) A-Circuit (See figure below)

$$A_1 = \frac{V}{V_{in}} \quad (\text{Gain of differential amplifier stage formed by } M1 \text{ \& } M2)$$

$$A_2 = \frac{V_{out}}{V} \quad (\text{Gain of common source stage } M3)$$

$$g_{m1} = g_{m4} = 4.255mA/V$$

$$g_{m2} = g_{m3} = 2.55mA/V$$

$$r_{O1} = r_{O4} = 13.33K\Omega$$

$$r_{O2} = r_{O3} = 10K\Omega$$

$$A_1 = -\frac{1}{2} g_{m1} (r_{o1} \parallel r_{o2}) = -\frac{1}{2} \times 4.255m(13.33K \parallel 10K)$$

$$= -12.156V/V$$

$$A_2 = -g_{m3} (r_{o3} \parallel R_2) = -2.55m(10K \parallel 2K)$$

$$= -4.25V/V$$

$$A = A_1 \times A_2 = (-12.156) \times (-4.25) = 51.663V/V$$

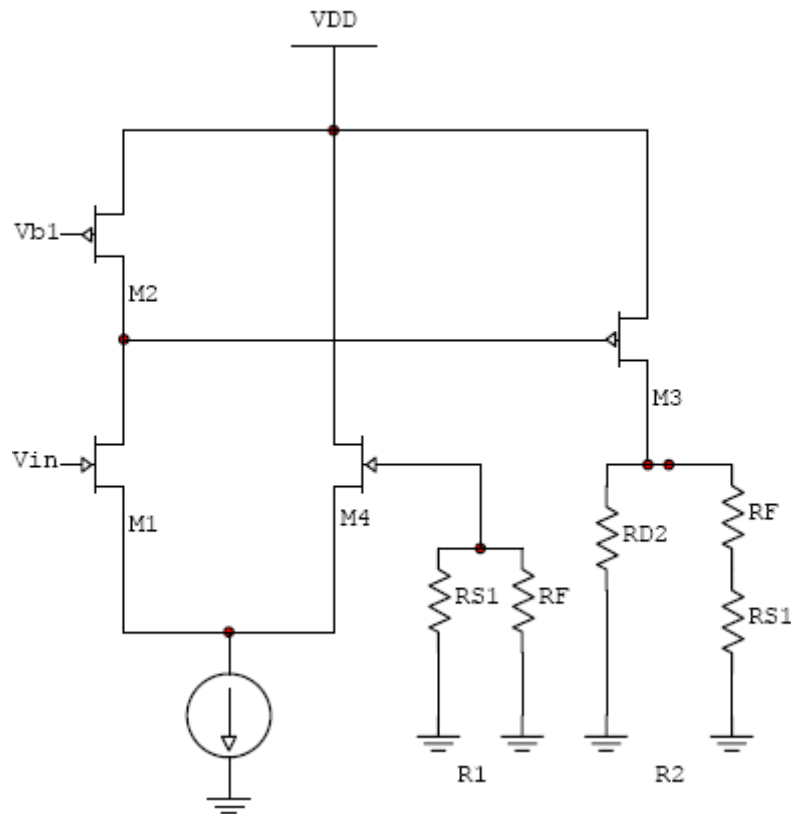
$$i) A_F = \frac{A}{1 + A\beta} = \frac{51.663}{1 + 51.663 \times 0.5} = 1.925V/V$$

ii)

$$R_o = R_2 \parallel r_{o3} = 2K \parallel 10K = 1.667K\Omega$$

$$R_F = \frac{R_o}{1 + A\beta} = 62.13\Omega$$

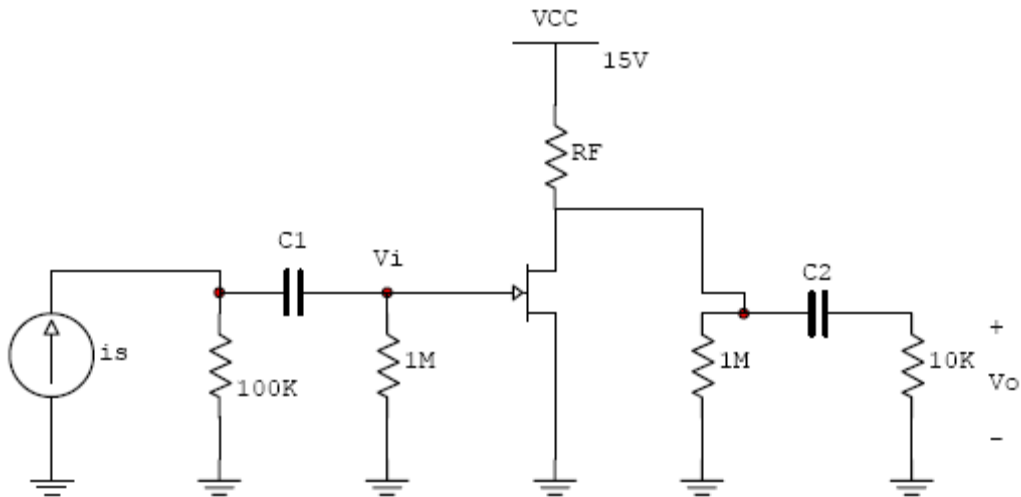
One can see that the closed loop gain is higher than that of part c and that the output resistance is much smaller than that of part c. This is due to the large amount of feedback.



## Problem 2

Shunt – Shunt amplifier

A-circuit



$$A = \frac{V_o}{i_s}$$

$$\begin{aligned} \frac{V_o}{V_i} &= -g_m (r_o \parallel 1M \parallel 10K \parallel 10K) \\ &= -1.8m(45K \parallel 1M \parallel 10K \parallel 10K) \\ &= -8.064V/V \end{aligned}$$

$$\frac{V_i}{i_s} = 100K \parallel 1M = 90.91K\Omega$$

$$A = \frac{V_o}{i_s} = \frac{V_o}{V_i} \times \frac{V_i}{i_s} = -8.064 \times 90.91K = -733.07K\Omega$$

$$\beta = \frac{-1}{1M} = -1\mu A/V$$

$$A_F = \frac{A}{1 + A\beta} = \frac{-733.07K}{1 + 733.07K \times 1\mu} = -423K\Omega$$

$$R_i = 100K \parallel 1M = 90.91K\Omega$$

$$R_{iF} = \frac{R_i}{1 + A\beta} = \frac{90.91K}{1 + 733.07 \times 10^{-3}} = 52.45K\Omega$$

$$R_{in} = \frac{1}{\left(\frac{1}{R_{iF}} - \frac{1}{R_S}\right)} = 110.3K\Omega$$

$$R_O = r_o \parallel 1M \parallel 10K \parallel 10K = 4.48K\Omega$$

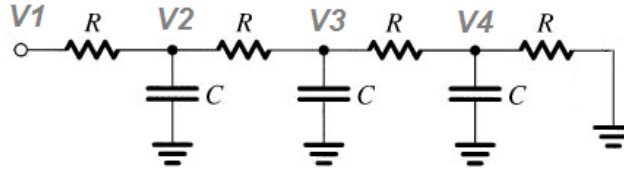
$$R_{OF} = \frac{R_O}{1 + A\beta} = \frac{4.48}{1 + 733.07 \times 10^{-3}} = 2.58K\Omega$$

$$R_{out} = \frac{1}{\left(\frac{1}{R_{OF}} - \frac{1}{R_L}\right)} = 3.477K\Omega$$

### PROBLEM 3

The A block is the inverting op-amp amplifier. The value of A is given by  $A = -\frac{R_f}{R}$ .

For the  $\beta$  block, we have to calculate  $\beta = V_4/V_1$ , taking into consideration the input resistance of the A block.



We solve the three node equations:

$$\begin{aligned} \left(\frac{2}{R} + sC\right)V_2 - \frac{1}{R}V_3 &= \frac{V_1}{R} \\ -\frac{1}{R}V_2 + \left(\frac{2}{R} + sC\right)V_3 - \frac{1}{R}V_4 &= 0 \\ -\frac{1}{R}V_3 + \left(\frac{2}{R} + sC\right)V_4 &= 0 \end{aligned}$$

to get:

$$\beta = \frac{V_4}{V_1} = \frac{1}{4 + 10sRC + 6s^2R^2C^2 + s^3R^3C^3}$$

Therefore, the loop gain is:

$$L(s) = \frac{-\frac{R_f}{R}}{4 + 10sRC + 6s^2R^2C^2 + s^3R^3C^3}$$

For  $s = j\omega$ :

$$L(j\omega) = \frac{-\frac{R_f}{R}}{(4 - 6\omega^2R^2C^2) + j\omega RC(10 - \omega^2R^2C^2)}$$

$L$  is real when

$$10 - \omega^2 R^2 C^2 = 0$$

$$\Rightarrow \omega^2 = \frac{10}{(RC)^2} \Rightarrow \omega = \omega_0 = \frac{\sqrt{10}}{RC} \Rightarrow f_0 = \frac{\sqrt{10}}{2\pi RC}$$

At the frequency of oscillation ( $\omega_0$  rad/sec or  $f_0$  Hz)

$$L = \frac{-\frac{R_f}{R}}{(4 - 6 \times 10)} = \frac{R_f}{56R} = 1 \Rightarrow R_f = 56R.$$

For  $f_0 = 8$  KHz,

$$f_0 = \frac{\sqrt{10}}{2\pi RC} = 8000 \Rightarrow C = 6.3 \text{ nF}$$

$$R_f = 56R = 560 \text{ K}\Omega$$