## American University of Beirut

Department of Electrical and Computer Engineering
EECE 311 - Electronic Circuits
Spring 2008
Midterm - April 11, 2008
Open Book - 90 minutes

NAME: $\qquad$ ID Number: $\qquad$

## Problem 1 [60 points]

Mark the answers for questions 1 to 15 below on the Scantron (computer) sheet.
Consider the circuit shown in Figure 1. Assume $V_{\mathrm{DD}}=2.5 \mathrm{~V}$. The MOSFET parameters are $k^{\prime}=200 \mu \mathrm{~A} / \mathrm{V}^{2}, V_{\mathrm{t}}=0.5 \mathrm{~V},(W / L)_{1}=20 \mu \mathrm{~m} / 0.25 \mu \mathrm{~m}$, and $(W / L)_{2}=10 \mu \mathrm{~m} / 0.25 \mu \mathrm{~m}$.
Assume that the DC drain current of a MOSFET in SAT is given by $I_{D} \simeq \frac{1}{2} k^{\prime}\left(\frac{W}{L}\right) V_{O V}^{2}$.


Figure 1

1. [4 points] Find the resistance $R$ (in $\mathrm{K} \Omega$ ) in the circuit if the current $I_{\text {REF }}$ is $100 \mu \mathrm{~A}$.
a) 6.02
b) 18.88
c) 12.42
d) 9.21
e) 7.29
2. [4 points] Find the drain current of $M_{2}, I_{\text {OUT }}$ (in $\mu \mathrm{A}$ ).
a) 75
b) 100
c) 125
d) 150
e) 50
3. [4 points] Find the largest resistance (in $K \Omega$ ) that can be connected between the drain of $M_{2}$ and $V_{\mathrm{DD}}$ to maintain saturation region operation for $M_{2}$.
a) 15.38
b) 23.42
c) 47.76
d) 31.51
e) 18.59
4. [4 points] Find the variation in $I_{\text {OUT }}\left(\right.$ in $\mu \mathrm{A}$ ) when $V_{\text {OUT }}$ increases from 1.5 to 2.5 V . At $L=$ $0.25 \mu \mathrm{~m}, V_{\mathrm{A}}$ is 10 V .
a) 5
b) 7.5
c) 10
d) 12.5
e) 15
5. [4 points] Repeat question 4 , assuming that $(W / L)_{2}$ is changed from $10 \mu \mathrm{~m} / 0.25 \mu \mathrm{~m}$ to $40 \mu \mathrm{~m} / 1 \mu \mathrm{~m}$.
a) 3.75
b) 1.25
c) 1.88
d) 2.50
e) 3.13
6. [4 points] A P-channel MOSFET $M_{3}$ with $k^{\prime}=100 \mu \mathrm{~A} / \mathrm{V}^{2}$, and $V_{\mathrm{t}}=-0.6 \mathrm{~V}$ is connected with MOSFET $M_{1}$ of question 1, as shown in Figure 2. Find $(W / L)_{3}$ if the current $I_{\text {REF }}$ is 200 $\mu \mathrm{A}$ and $R=4.5 \mathrm{~K} \Omega$.
a) 108.6
b) 34.2
c) 46.9
d) 68.4
e) 20.5


Figure 2

Consider the differential amplifier shown in Figure 3. The reference current is $I_{\text {REF }}=240 \mu \mathrm{~A}$. Assume $V_{\mathrm{DD}}=2.5 \mathrm{~V}$. The MOSFET parameters are as follows.

For NMOS:
$k_{\mathrm{n}}^{\prime}=200 \mu \mathrm{~A} / \mathrm{V}^{2}, V_{\mathrm{tn}}=0.5 \mathrm{~V}$
$(W / L)_{1}=20 \mu \mathrm{~m} / 0.25 \mu \mathrm{~m}$
$(W / L)_{2}=10 \mu \mathrm{~m} / 0.25 \mu \mathrm{~m}$
$(W / L)_{4}=10 \mu \mathrm{~m} / 0.25 \mu \mathrm{~m}$
$(W / L)_{5}=10 \mu \mathrm{~m} / 0.25 \mu \mathrm{~m}$

## For PMOS:

$k_{\mathrm{p}}=100 \mu \mathrm{~A} / \mathrm{V}^{2}, V_{\mathrm{tp}}=-0.6 \mathrm{~V}$
$(W / L)_{3}=40 \mu \mathrm{~m} / 0.25 \mu \mathrm{~m}$
$(W / L)_{6}=10 \mu \mathrm{~m} / 0.25 \mu \mathrm{~m}$
$(W / L)_{7}=10 \mu \mathrm{~m} / 0.25 \mu \mathrm{~m}$


Figure 3
7. [4 points] Find the DC voltage (in mV ) at node $X$ when the two inputs are at 0.9 V .
a) 258.6
b) 250.0
c) 277.5
d) 267.7
e) 288.2
8. [4 points] Find the total power dissipation (in mW ) in the circuit.
a) 0.90
b) 1.05
c) 1.20
d) 1.35
e) 0.75

Assume in the following that $V_{\mathrm{AN}}=10 \mathrm{~V}$, and $\left|V_{\mathrm{AP}}\right|=8 \mathrm{~V}$.
9. [4 points] Find the small-signal differential gain $v_{\text {out }} /\left(v_{\mathrm{in} 1}-v_{\mathrm{in} 2}\right)$.
a) -79.5
b) -72.6
c) -67.2
d) -62.9
e) -59.3
10. [4 points] Find the small-signal differential gain $v_{\text {out }} /\left(v_{\text {in } 1}-v_{\mathrm{in} 2}\right)$ when a $1000 \mathrm{~K} \Omega$ resistor is connected across the $v_{\text {out }}$ terminals.
a) -53.9
b) -56.6
c) -67.5
d) -63.2
e) -59.6
11. [4 points] Find the small-signal common-mode gain when the output is taken singleendedly (at the drain of $M_{4}$ or $M_{5}$ ). In this case, $v_{1}=v_{2}=v_{\text {icm }}$.
a) -2.8
b) -0.008
c) -0.08
d) -0.8
e) -1.8
12. [4 points] Find the small-signal common-mode gain when the output is taken differentially. In this case, $v_{1}=v_{2}=v_{\text {icm }}$.
a) 0
b) 0.08
c) -0.08
d) -0.008
e) 0.008

Assume in the following two questions that when the input is common-mode, the DC voltage at the drains of $M_{4}$ and $M_{5}$ is 1.5 V .
13. [4 points] Find the maximum common-mode input voltage (in V ) that maintains saturation-region operation for all MOSFETs in the circuit.
a) 2.4
b) 2.0
c) 2.1
d) 2.2
e) 2.3
14. [4 points] Find the minimum common-mode input voltage (in V ) that maintains saturation-region operation for all MOSFETs in the circuit.
a) 0.86
b) 0.80
c) 0.82
d) 0.84
e) 0.77
15. [4 points] The two inputs are $v_{\mathrm{in} 1}(t)=4.5 \sin (\omega t) \mathrm{mV}$ and $v_{\mathrm{in} 2}(t)=4.9 \sin (\omega t) \mathrm{mV}$. If the differential gain is 62 dB , and the CMRR is 50 dB , find the amplitude of the output voltage (in V).
a) 0.52
b) 0.33
c) 0.37
d) 0.41
e) 0.47

## Problem 2 [40 points]

Solve this problem on the answer booklet.
The common-emitter amplifier shown in Figure 4 is biased using $I_{\text {REF }}=140 \mu \mathrm{~A}$, and has $V_{\mathrm{An}}=10 \mathrm{~V},\left|V_{\mathrm{Ap}}\right|=5 \mathrm{~V}$, and large $\beta . Q_{2}$ and $Q_{3}$ are matched. In DC analysis, assume that the collector current in ACTIVE is given by $I_{C} \simeq I_{S} e^{\frac{V_{\mathrm{VE}}}{T}}$, where $V_{\mathrm{T}}=25 \mathrm{mV}$.


Figure 4
a) [5 points] Find the output resistance of the amplifier. Assume all transistors are an internal part of the amplifier.

The output resistance is $r_{\mathrm{o}(\mathrm{Q} 1)} / / r_{\mathrm{o}(\mathrm{Q} 2)}$
Since $\mathrm{Q}_{2}$ and $\mathrm{Q}_{3}$ are matched, and $\beta$ is very large: $I_{\mathrm{C} 1}=I_{\mathrm{C} 2}=I_{\mathrm{REF}}=140 \mu \mathrm{~A}$.
$r_{\mathrm{o}(\mathrm{Q} 1)}=V_{\mathrm{AN}} / I_{\mathrm{C} 1}=10 / 0.14=71.43 \mathrm{~K} \Omega$
$r_{\mathrm{o}(\mathrm{Q} 2)}=\left|V_{\mathrm{AP}}\right| / I_{\mathrm{C} 1}=5 / 0.14=35.71 \mathrm{~K} \Omega$
Rout $=r_{\text {o(Q1) }} / / r_{\text {o(Q2) }}=71.43 / / 35.71=23.81 \mathrm{~K} \Omega$
b) [5 points] Find the open-circuit voltage gain $v_{0} / v_{\mathrm{i}}$.

The open circuit voltage gain $v_{\mathrm{o}} / v_{\mathrm{i}}$ is $-g_{\mathrm{m} 1} R_{\text {out }}$
$g_{\mathrm{m} 1}=I_{\mathrm{C} 1} / V_{\mathrm{T}}=0.14 / 25=5.6 \mathrm{~mA} / \mathrm{V}$
$v_{0} / v_{\mathrm{i}}=-g_{\mathrm{ml}} R_{\text {out }}=-5.6 \times 23.81=-133.34 \mathrm{~V} / \mathrm{V}$
Assume in the following that a signal source is connected to the input of the amplifier. It is a voltage source $v_{\text {sig }}$ with a source resistance $R_{\text {sig }}=25 \mathrm{~K} \Omega$. Assume also that a $10 \mathrm{~K} \Omega$ load resistor is connected from the output to ground. The signal source and the load do not affect the DC bias.
c) [4 points] Find the input resistance $v_{\mathrm{i}} / I_{\mathrm{i}}$ of the amplifier if $\beta=200$.

The input resistance $v_{\mathrm{i}} / i_{\mathrm{i}}$ is $r_{\pi}=V_{\mathrm{T}} / I_{\mathrm{B}}=\beta V_{\mathrm{T}} / I_{\mathrm{C}}=200 \times 25 / 0.14=35.71 \mathrm{~K} \Omega$
d) [4 points] Find the voltage gain $v_{0} / v_{\text {sig }}$.

The voltage gain is $-g_{\mathrm{m} 1} R^{\prime}{ }_{L} \times$ (voltage divider ratio at input).
$R^{\prime}{ }_{L}=R_{\text {out }} / / R_{\mathrm{L}}=23.81 / / 10=7.04 \mathrm{~K} \Omega$
The voltage divider ratio at the input is $r_{\pi} /\left(r_{\pi}+R_{\text {sig }}\right)=35.71 /(35.71+25)=0.5882$
The voltage gain $v_{0} / v_{\text {sig }}=-5.6 \times 7.04 \times 0.5882=-23.19 \mathrm{~V} / \mathrm{V}$

Assume now that for $Q_{1}, C_{\pi}=25 \mathrm{fF}, C_{\mu}=10 \mathrm{fF}$, and that a load capacitance $C_{\mathrm{L}}=40 \mathrm{fF}$ is connected from the output to ground. Neglect all other capacitances.
e) [5 points] Using Miller's theorem, and assuming that the input circuit determines the upper $3-\mathrm{dB}$ frequency $f_{\mathrm{H}}$, find the value of $f_{\mathrm{H}}$.

The Miller constant $K$ is $v_{\mathrm{c}} / v_{\mathrm{b}}=-g_{\mathrm{m} 1} R^{\prime}{ }_{L}=-5.6 \times 7.04=-39.42$
We apply Miller's theorem to obtain the input capacitance:
The input capacitance is $C_{\mathrm{in}}=C_{\pi}+(1-K) \mathrm{C}_{\mu}=25+(1+39.42) \times 10=429.2 \mathrm{fF}$
The resistance seen by this input capacitance is $r_{\pi} / / R_{\text {sig }}=35.71 / / 25=14.71 \mathrm{~K} \Omega$
The 3-dB frequency due to the input circuit is
$f_{\mathrm{H}}=1 /\left(2 \pi C_{\text {in }}\left(r_{\pi} / / R_{\text {sig }}\right)\right)=1 /(2 \pi \times 429.2 \mathrm{f} \times 14.71 \mathrm{~K})=25.21 \mathrm{MHz}$
f) [5 points] Based on the results of (d) and (e), show and label the Bode plot for the magnitude of the voltage gain $\left(v_{0} / v_{\text {sig }}\right)$.

The low-frequency gain is $-23.19 \mathrm{~V} / \mathrm{V}$, which corresponds to 27.3 dB The 3-dB frequency is 25.21 MHz

g) [12 points] Using the open-circuit time constants methods, find the resistance seen by each capacitor [3 points each], and estimate the value of $f_{\mathrm{H}}$ [3 points].

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\begin{array}{rll}
\mathrm{R}\left(\text { seen by } C_{\pi}\right) & =r_{\pi} / / R_{\text {sig }} & =14.71 \mathrm{~K} \Omega \\
\mathrm{R}\left(\text { seen by } C_{\mathrm{L}}\right) & =R^{\prime}{ }_{L} & =7.04 \mathrm{~K} \Omega \\
\mathrm{R}\left(\text { seen by } C_{\mu}\right) & =\left(r_{\pi} / / R_{\text {sig }}\right)+R^{\prime}{ }_{L}+g_{\mathrm{m} 1} R^{\prime}{ }_{L}\left(r_{\pi} / / R_{\text {sig }}\right) & \\
& =14.71+7.04+5.6 \times 14.71 \times 7.04 & \\
& =601.7 \mathrm{~K} \Omega & \text { frequently-used resistance calc. }
\end{array}
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$$
f_{\mathrm{H}}=1 /(2 \pi(25 \mathrm{f} \times 14.71 \mathrm{~K}+40 \mathrm{f} \times 7.04 \mathrm{~K}+10 \mathrm{f} \times 601.7 \mathrm{~K}))=23.87 \mathrm{MHz}
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