American University of Beirut

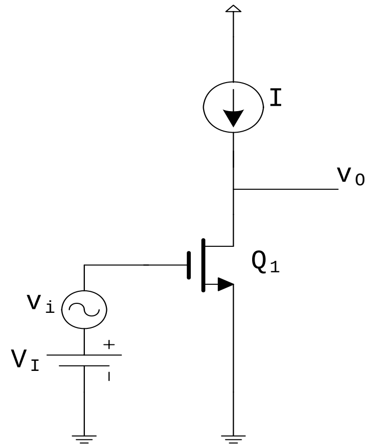
Department of Electrical and Computer Engineering

EECE 311 – Electronics II

Fall 2005 – 2006 (Section 2)

Quiz 1 – Solution

Problem 1:



a) I = ½ k'$_n$ (W/L) $(V_{GS} – V_{tn})^2$

=> 40 u = 0.5 (0.2m)(5)$(V_I – 0.5)^2$

=> V_I = 0.5 + sqrt(40u/0.5m) = 0.783 V

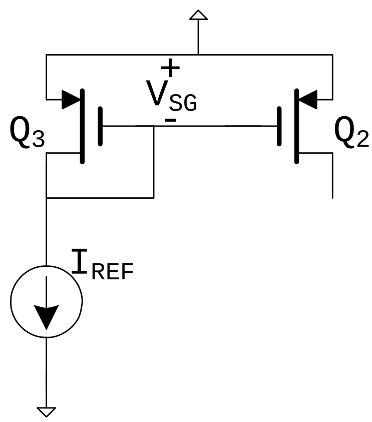
b) $v_O = v_{DS}$ => $v_{Omin} = v_{DSmin} = v_{GS} – V_{tn}$

Assuming $v_{GS} = V_{GS}$ (DC value) => v_{Omin} = 0.783 – 0.5 = 0.283 V

c) $g_m = I_D/(V_{OV}/2)$ = 40u/(0.283/2) = 0.283 mA/V

$r_o = V_A/I_D$ = (1/0.1) / 40u = 250 K

gain = $v_o/v_i = –g_m(r_o$ // R_o of current source) = –0.283(100//250) = –20.2 V/V



d) I_D(Q2) = ½ k'$_p$ (W/L) $(V_{OV2})^2$

This current is equal to 40u

=> 40u = 0.5(0.08m)(W/L)$(0.2)^2$ => (W/L) = 25 for Q_2

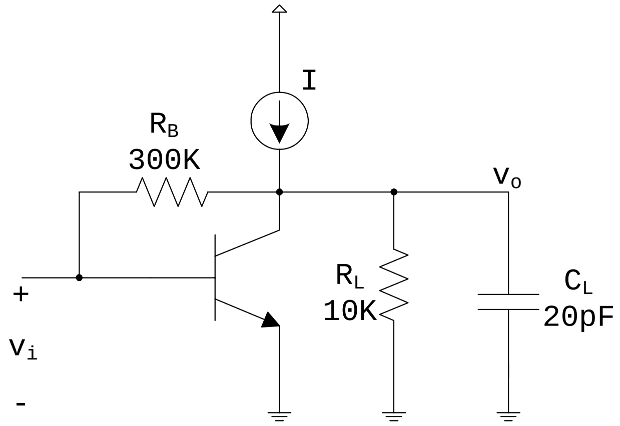
(W/L) = 25 for Q_3, since Q_2 and Q_3 are matched.

e) $V_{SG} = |V_{tp}| + |V_{OV2}|$ = 0.54 + 0.2 = 0.74 V

f) Output resistance is $r_{o2} = V_{AP} / I_{D2} = (1/0.12) / 40u = 208.33$ K

g) For Q2: $|V_{DS}| \geq |V_{GS}| - |V_{tp}| \Rightarrow V_{DD} - V_O \geq 0.2$ V $\Rightarrow V_{Omax} = V_{DD} - 0.2$ V

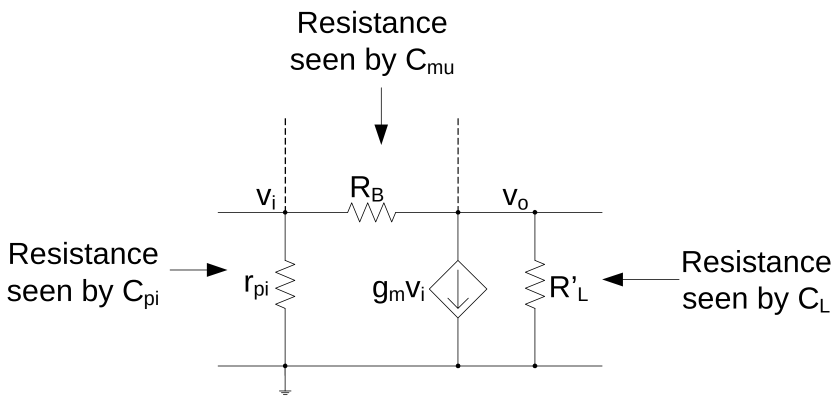
Problem 2:
In this problem, there is an implicit assumption that the signal source (not shown) has a large source resistance, and that the source does not affect the DC bias.



a) DC analysis: current in 300K resistor = I_B
KCL at collector: 0.5 mA = $I_B + I_C + V_O/10K$
But $V_O = 300K \times I_B + V_{BE} = 300K \times I_B + 0.7$ V
$\Rightarrow$ 0.5 mA = $I_B + \beta I_B + (300K \times I_B + 0.7 \text{ V})/10K$
$\Rightarrow$ 0.5 mA – 0.07 mA = 91 $I_B \Rightarrow I_B = 4.725$ uA
$I_C = 60 I_B = 0.2835$ mA
$V_O = 300K \times 4.725$ uA + 0.7 V = 2.118 V

b) The Miller constant is the gain from base to collector without R_B, and is given by $-g_m R'_L$ with $R'_L = r_o$ (BJT) // R_o (current source) // R_L
r_o (BJT) = V_A/I_C = 80/0.2835m = 282.2 K
R'_L = 282.2 K // 100 K // 10K = 8.807 K
$g_m = I_C/V_T$ = 0.2835m/25m = 11.34 mA/V
Miller's K = $-g_m R'_L$ = –11.34×8.807 = –99.87
Equivalent resistance from base to ground due to R_B: $R_1 = R_B/(1 - K)$ = 300K/(1+99.87) = 2.974 K
The input resistance is $R_{in} = r_\pi // R_1$
$r_\pi = V_T/I_B$ = 25m/4.725u = 5.291 K
R_{in} = 5.291 // 2.974 = 1.904 K

c) The resistance seen by C_π is R_{in} (we can use the approximation from part (b)) = 1.904 K. The more accurate estimate is obtained as follows:
$i_{RB} = (v_i - v_o)/R_B = (1 - \text{gain})v_i/R_B \Rightarrow v_i/i_{RB} = R_B/(1 - \text{gain})$ = 300K/(1 + 97) = 3.061 K [note: the value of the voltage gain is found in part (d)]
Resistance seen by C_π is equal to: $r_\pi // (v_i/i_{RB})$ = 5.291 K // 3.061 K = 1.939 K



The resistance seen by C_L is equal to:

R'_L // (resistance due to controlled source) // ($R_B + r_\pi$):

$v_\pi = v_i = v_x\, r_\pi/(r_\pi+R_B) = v_x\times5.291/(5.291+300) = v_x/57.7$

The controlled current source is therefore equivalent to a resistance equal to $57.7/g_m$ = 5.088 K

The resistance seen by C_L is therefore: 8.807 // 5.088 // 305.29 = 3.191 K

The resistance seen by C_μ is equal to: R_B // ($R'_L + r_\pi + g_m\times r_\pi\times R'_L$) = 300 K // (8.807 K + 5.291 K + 11.34×8.807×5.291 K) = 193.2 K.

τ_H = 1.939 K×1 pF + 3.191 K×20 pF + 193.2 K×1 pF = 258.96 nsec

$f_H = 1/(2\pi\, \tau_H)$ = 614.6 KHz

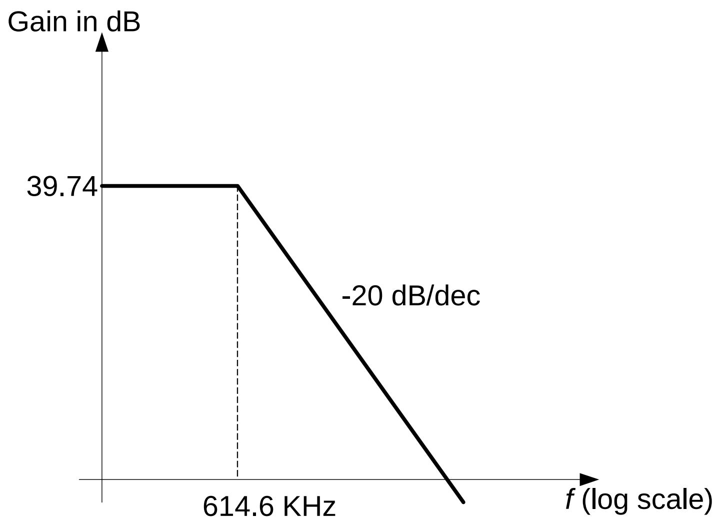
d) The low-frequency voltage gain is determined *without* using Miller's theorem. The current in R_B is $(v_i - v_o)/R_B$. This current is equal to $g_m v_i + v_o/R'_L$. Therefore:

$(1/R_B - g_m)\, v_i = v_o(1/R'_L + 1/R_B)$ => The voltage gain is

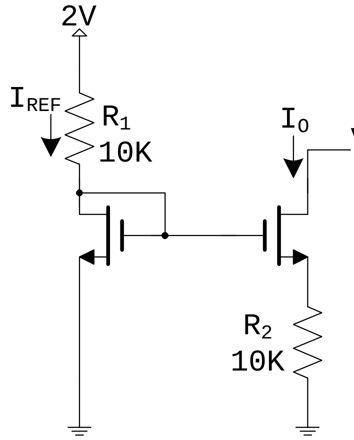
$v_o/v_i = (1/R_B - g_m)/ (1/R'_L + 1/R_B) = (1 - g_m R_B)/(R_B/R'_L + 1)$

= (1 – 11.34×300)/(300/8.807+1) = –97 V/V or 39.74 dB

e) Asymptotic Bode Plot



Problem 3



a) $I_{REF} = (2 - V_{GS1})/10K = ½\ k'_n\ (W/L)\ (V_{GS1} - V_{tn})^2 = (0.5)(0.2m)(10)(V_{GS1} - 0.4)^2$.
Solving the quadratic equation we get: $V_{GS} = 0.753$ V (the other root is negative).
I_{REF} is therefore $(2 - 0.753)/10K = 0.1247$ mA

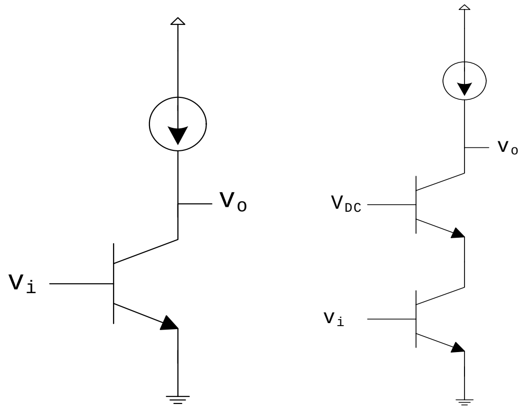
b) $V_{GS2} = V_{GS1} - I_O \times R_2$ and
$I_O = ½\ k'_n\ (W/L)\ (V_{GS2} - V_{tn})^2 = ½\ k'_n\ (W/L)(V_{GS1} - I_O \times R_2 - V_{tn})^2$
Therefore $I_O = (0.5)(0.2m)(10)(0.753 - I_O \times 10K - 0.4)^2$
=> $I_O = 20.86$ uA
The value of V_{GS2} is $0.753 - 20.86u \times 10K = 0.5444$ V

c) The minimum output voltage corresponds to $V_{DS2} = V_{GS2} - V_{tn}$ => $V_O - I_O R_2 = V_{GS2} - V_{tn}$ => $V_{Omin} = 20.86u \times 10K + 0.5444 - 0.4 = 0.353$ V

d) The output resistance is given by $r_{o2} + R_2 + g_{m2}\ r_{o2}\ R_2$
$r_{o2} = V_A/I_O = (1/0.08)/20.86u = 599.23$ K
$g_{m2} = I_O/(V_{OV}/2) = 20.86u/((0.5444\text{-}0.4)/2) = 0.289$ mA/V
R_O is therefore equal to: $10K + 599.23K + 599.23K \times 0.289m \times 10K = 2340.5$ K

Problem 4

In this problem, there is an implicit assumption that the signal source (not shown) has a large source resistance.



a) Using Miller's theorem, the time constant at the input of the circuit is given by

$\tau_H = r_\pi (C_\pi + C_\mu (1 + g_m r_o))$ since the only "load" is the output resistance of the BJT itself.

$\tau_H = r_\pi (C_\pi + C_\mu (1 + g_m r_o)) = r_\pi C_\pi + r_\pi C_\mu + r_\pi C_\mu g_m r_o = r_\pi C_\pi + r_\pi C_\mu + \beta r_o C_\mu$

The third term is much larger than the first two, and therefore τ_H is approximately $\beta r_o C_\mu$.

$f_H = 1/(2\pi\ \tau_H) = 1/(2\pi\ \beta r_o C_\mu)$

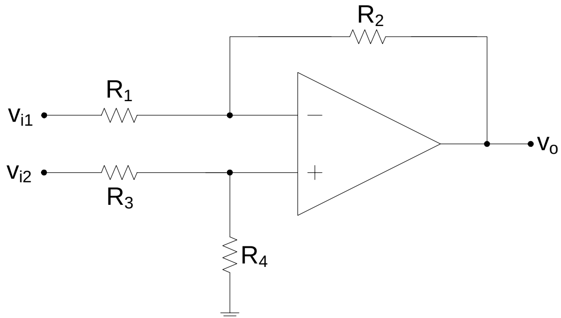
b) The current source output resistance is approximately r_o. The output resistance of the amplifier becomes $R_o \approx r_o // r_o = r_o/2$. The open-circuit voltage gain becomes $-g_m r_o/2 = -A_0/2$. The 3-dB frequency becomes $1/(2\pi\ \tau_H) = 1/(2\pi\ \beta(r_o/2)C_\mu) = 2\ f_H$

c) The current source output resistance is approximately βr_o. The output resistance of the amplifier becomes $R_o \approx \beta r_o // r_o \approx r_o$. The open-circuit voltage gain becomes $-g_m r_o = -A_0$. The 3-dB frequency becomes $1/(2\pi\ \tau_H) = 1/(2\pi\ \beta r_o C_\mu) = f_H$

d) For an unloaded cascode amplifier, the output resistance of the *unloaded* amplifier is given by βr_o. The output resistance of the amplifier becomes $R_o \approx \beta r_o // r_o \approx r_o$. The open-circuit voltage gain becomes (see Fig. 6.41 in textbook) $-\beta A_0\ r_o/(r_o+\beta r_o) \approx -A_0$. The 3-dB frequency becomes $1/(2\pi\ \tau_H) \approx 1/(2\pi\ r_o C_\mu) = \beta f_H$ (see Fig. 6.42 in textbook)

e) The output resistance of the amplifier becomes $R_o \approx \beta r_o // \beta r_o = \beta r_o/2$. The open-circuit voltage gain becomes $-\beta A_0\ \beta r_o/(\beta r_o+\beta r_o) = -\beta A_0/2$. The 3-dB frequency becomes $1/(2\pi\ \tau_H) \approx 1/(2\pi\ (\beta r_o/2)C_\mu) = 2f_H$

Problem 5



For the difference amplifier, the output voltage is given by:

$$v_o = \frac{-R_2}{R_1} v_{i1} + \frac{R_4}{R_3+R_4}\left(1+\frac{R_2}{R_1}\right) v_{i2}$$

=> $v_o = -4.7 v_{i1} + 4.674 v_{i2} = A_{v1}\ v_{i1} + A_{v2}\ v_{i2} = A_d\ (v_{i1} - v_{i2}) + A_{cm}\ (v_{i1}+v_{i2})/2$

=> $A_d = (A_{v1} - A_{v2})/2 = -4.687$ V/V

and $A_{cm} = A_{v1} + A_{v2} = -0.026$ V/V

=> CMRR = 20 log|4.687/0.026| = 45.12 dB