

**American University of Beirut**  
 Department of Electrical and Computer Engineering  
**EECE 311 – Electronics II**  
 Fall 2005 – 2006 (Section 2)  
**Quiz 2 – December 22, 2005**  
 Closed Book – 90 minutes

NAME: \_\_\_\_\_ ID Number: \_\_\_\_\_

*I have neither given nor received aid on this exam*

**SIGNATURE**

**Problem 1 [40 points]**

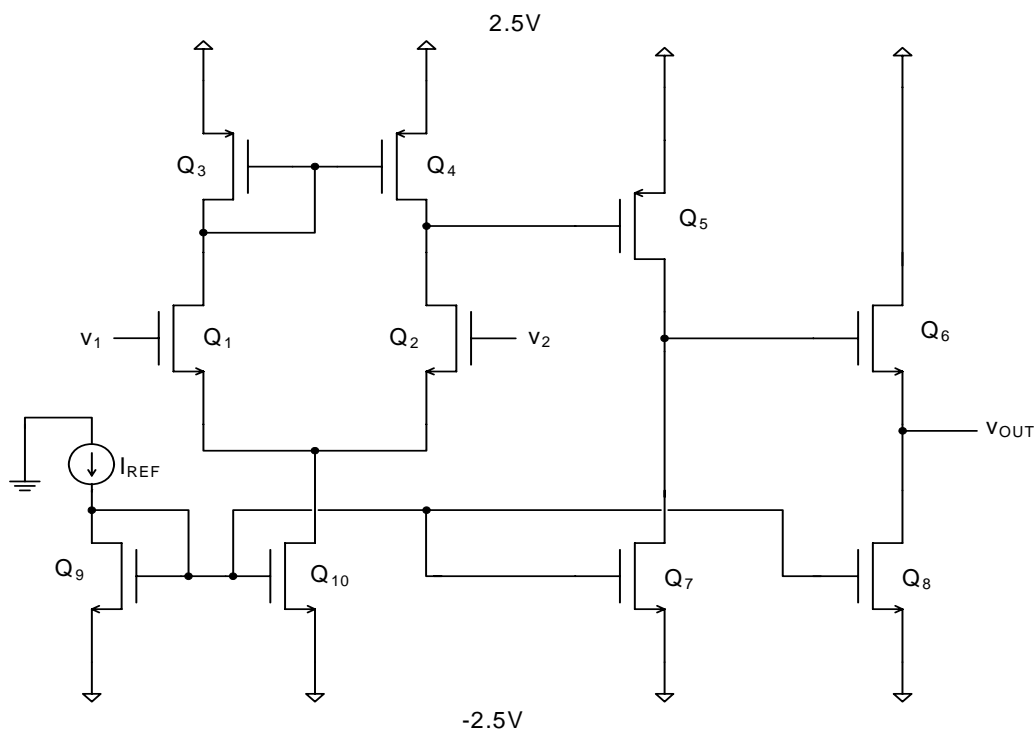


Figure 1

Refer to the MOSFET amplifier shown in Figure 1.

**a) [5 points]** How many stages are there? For each stage, identify the type of the stage, the amplifier transistor(s) and the load transistor(s).

There are three stages.  
 The first stage is a differential stage with an active load. The amplifier transistors are  $Q_1$  and  $Q_2$ , and the load transistors are  $Q_3$  and  $Q_4$ .  
 The second stage is a common-source amplifier. The amplifier transistor is  $Q_5$  and the load transistor is  $Q_7$ .

The third stage is a common-drain (or source follower) stage. The amplifier transistor is  $Q_6$  and the load transistor is  $Q_8$ . Transistors  $Q_9$  and  $Q_{10}$  form a current mirror.

Assume in the following that  $k'_n = 250 \mu\text{A}/\text{V}^2$ ,  $V_{tn} = 0.5 \text{ V}$ ,  $\lambda_n = 0.05 \text{ V}^{-1}$ ,  $k'_p = 100 \mu\text{A}/\text{V}^2$ ,  $V_{tp} = -0.6 \text{ V}$ , and  $\lambda_p = 0.1 \text{ V}^{-1}$ .

Assume  $I_{REF} = 100 \mu\text{A}$ , and that all MOSFETs have a  $(W/L)$  ratio equal to 20, except for of  $Q_5$  which has  $(W/L) = 40$ .

**Neglect channel length modulation in DC analyses.**

**b) [6 points]** For each MOSFET, find the drain current and the overdrive voltage  $V_{OV}$  when  $V_1 = V_2 = 0$ . This condition ( $V_1 = V_2 = 0$ ) results in  $V_{OUT} = 0$ . Show your results in the following format:

Transistor	Drain Current ( $\mu\text{A}$ )	Overdrive Voltage (V)
Q1		
Q2		
:		

Since  $Q_9$  and  $Q_{10}$  are matched, this means that  $I_{D9} = I_{D10} = 100 \mu\text{A}$ . The current splits equally in  $Q_1$  and  $Q_2$  since they are matched and have the same  $V_{GS}$  (and channel length modulation is being neglected). Therefore  $I_{D1} = I_{D2} = 50 \mu\text{A}$ .  $I_{D1} = I_{D3}$  and  $I_{D2} = I_{D4} \Rightarrow I_{D3} = I_{D4} = 50 \mu\text{A}$ . The current in  $Q_7$  is  $100 \mu\text{A}$ , since it is mirroring  $I_{REF}$ . This same current flows in  $Q_5 \Rightarrow I_{D5} = I_{D7} = 100 \mu\text{A}$ . Same for  $Q_6$  and  $Q_8$ :  $I_{D6} = I_{D8} = 100 \mu\text{A}$ .

$V_{OV}$  for the various MOSFETs is calculated from

$$I_D = \frac{1}{2} k' \left(\frac{W}{L}\right) V_{OV}^2 \Rightarrow V_{OV} = \sqrt{\frac{2I_D}{k' \left(\frac{W}{L}\right)}}$$

where  $k'$  is  $k'_n$  for NMOS and  $k'_p$  for PMOS. For

PMOS transistors,  $V_{OV}$  is negative. The table shown below also shows the value of  $V_{GS}$  of the transistors, and the values of  $g_m$  and  $r_o$ .

	$k'$ ( $\mu\text{A}/\text{V}^2$ )	W/L	$V_T$ (V)	Lambda ( $1/\text{V}$ )	$I_D$ ( $\mu\text{A}$ )	VOV (V)	VGS (V)	$g_m$ (mA/V)	$r_o$ (kOhm)
Q1	250	20	0.5	0.05	50	0.1414	0.6414	0.7071	400
Q2	250	20	0.5	0.05	50	0.1414	0.6414	0.7071	400
Q3	100	20	-0.6	0.1	50	-0.2236	-0.8236	0.4472	200
Q4	100	20	-0.6	0.1	50	-0.2236	-0.8236	0.4472	200
Q5	100	40	-0.6	0.1	100	-0.2236	-0.8236	0.8944	100
Q6	250	20	0.5	0.05	100	0.2	0.7	1	200
Q7	250	20	0.5	0.05	100	0.2	0.7	1	200
Q8	250	20	0.5	0.05	100	0.2	0.7	1	200
Q9	250	20	0.5	0.05	100	0.2	0.7	1	200
Q10	250	20	0.5	0.05	100	0.2	0.7	1	200

**c) [4 points]** Find the total power dissipation in the circuit.

The total power dissipated in the circuit comes from the two supplies.

The current in the (+2.5V) supply is equal to  $I_{D3} + I_{D4} + I_{D5} + I_{D6} = 50 + 50 + 100 + 100 = 300 \text{ uA}$ .

The current in the (-2.5V) supply is  $I_{D9} + I_{D10} + I_{D7} + I_{D8} = 100 + 100 + 100 + 100 = 400 \text{ uA}$ , into the supply.

The total power dissipation is therefore  $(2.5)(0.3\text{m}) + (2.5)(0.4\text{m}) = 1.75 \text{ mW}$ .

**d) [7 points]** Find the differential gain  $A_d = v_{out}/(v_1 - v_2)$  by calculating  $v_{d2}/(v_1 - v_2)$ ,  $v_{d5}/v_{d2}$ , and  $v_{out}/v_{d5}$ .

The first stage gain is given by  $v_{out}/(v_1 - v_2) = g_{m2}(r_{o2}/r_{o4}) = 0.7071(200/400) = 94.28 \text{ V/V}$ .

The second stage gain is given by  $v_{d5}/v_{d2} = -g_{m5}(r_{o5}/r_{o7}) = 0.8944(100/200) = -59.63 \text{ V/V}$ .

The third stage gain is given by  $v_{out}/v_{d5} = (g_{m6}R'_L)/(1 + g_{m6}R'_L)$  with  $R'_L = (r_{o6}/r_{o8}) = 100 \text{ K}$ . Therefore  $v_{out}/v_{d5} = 100/101 = 0.99 \text{ V/V}$ .

The overall gain is  $94.28 \times (-59.63) \times (0.99) = -5565.7 \text{ V/V}$

**e) [6 points]** Find the common-mode gain  $A_{cm} = v_{out}/v_{icm}$  when  $v_1 = v_2 = v_{icm}$ . What is the common-mode rejection ratio of the amplifier (in dB)?

Since  $r_{o3} = r_{o4}$  and  $g_{m3}r_{o3} \gg 1$ , the first stage common-mode gain is given by  $-1/(2g_{m3}r_{o10}) = -1/(2 \times 0.4472 \times 200) = -0.00559 \text{ V/V}$

The second and third stages have the same gain as in part (d).

The common-mode gain is therefore  $(-0.00559) \times (-59.63) \times 0.99 = 0.33 \text{ V/V}$

The CMRR is  $20 \log(5566/0.33) = 84.54 \text{ dB}$

**f) [4 points]** Find the input common-mode range by calculating the maximum and minimum values of  $V_1 = V_2 = V_{ICM}$  for which all MOSFETs are saturated.

The maximum  $V_{ICM}$  is determined by  $Q_1$  leaving SAT  $\Rightarrow V_{DS1} = V_{OV1}$  or  $V_{GD1} = V_{tn}$   
 $\Rightarrow V_{ICMmax} - V_{G3} = V_{tn} \Rightarrow V_{ICMmax} = V_{GS3} + 2.5 + V_{tn} = -0.8236 + 2.5 + 0.5 = 2.1764 \text{ V}$

The minimum  $V_{ICM}$  is determined by  $Q_{10}$  leaving SAT  $\Rightarrow V_{DS10} = V_{OV10} \Rightarrow$   
 $V_{S1} - (-2.5) = V_{OV10} \Rightarrow V_{ICMmin} - V_{GS1} + 2.5 = 0.2 \Rightarrow V_{ICMmin} = 0.6414 - 2.5 + 0.2 = -1.659 \text{ V}$

**g) [2 points]** Find the minimum output voltage  $V_{OUT}$  for which *all* MOSFETs remain saturated. What transistor determines this limit?

The minimum output voltage corresponds to the condition  $Q_8$  at edge of SAT  $\Rightarrow V_{DS8} = V_{OV8} \Rightarrow V_{OUTmin} - (-2.5) = 0.2 \Rightarrow V_{OUTmin} = -2.3 \text{ V}$ .

**h) [2 points]** Find the maximum output voltage  $V_{OUT}$  for which *all* MOSFETs remain saturated. What transistor determines this limit?

The maximum output voltage corresponds to the condition  $Q_5$  at edge of SAT since  $Q_5$  enters the linear region before  $Q_6$  does as  $V_{OUT}$  increases  $\Rightarrow |V_{DS5}| = |V_{OV5}| \Rightarrow 2.5 - (V_{OUTmax} + V_{GS6}) = 0.2236 \Rightarrow V_{OUTmax} = 2.5 - 0.7 - 0.2236 = 1.576 \text{ V}$

**i) [4 points]** A capacitor  $C_C$  is connected from the output node to the drain of  $Q_2$ . Neglecting all other capacitances in the circuit, find the frequency of the resulting zero and pole.

The zero is at  $G_{m2}/C_C$  where  $G_{m2}$  is the transconductance of the combined stage 2 – stage 3 amplifier. The pole is at a  $1/(R_1R_2G_{m2}C_C)$  where  $R_1$  is the output resistance of stage 1, and  $R_2$  is the output resistance of stage 3.

$R_1$  is therefore  $r_{o2}/r_{o4} = 133.3 \text{ K}\Omega$

$R_2$  is therefore  $(1/g_{m6})//r_{o6}/r_{o8} = 1 // 200 // 200 = 0.99 \text{ K}\Omega$

$G_{m2}$  is  $g_{m5}(r_{o5}/r_{o7})g_{m6} = 0.8944(100/200)1 = 59.63 \text{ mA/V}$

The zero is therefore at  $59.63/(2\pi C_C) = 9.49/C_C \text{ GHz}$  with  $C_C$  in pF.

The pole is therefore at  $1000/(133.3 \cdot 0.99 \cdot 59.63 \cdot 2\pi \cdot C_C) = 0.0202/C_C \text{ MHz}$  with  $C_C$  in pF.

### **Problem 2 [10 points]**

For a BJT differential amplifier, the value of  $\beta$  for the transistors varies between 100 and 140, around a mean value of 120. The transistors are biased using a  $100 \mu\text{A}$  current source.

**a) [4 points]** Find the input bias and the input offset currents.

The input bias current is  $I_B = (I/2)/(\beta+1)$  where  $I$  is the source current.  $I_B$  is therefore  $(100/2)/(120+1) = 0.413 \mu\text{A}$ .

The input offset current is given by  $I_B\Delta\beta/\beta = 0.413 (40/120) = 0.138 \mu\text{A}$ .

**b) [2 points]** The circuit uses collector resistors, each equal to  $10 \text{ K}\Omega$ , and emitter resistors, each equal to  $750 \Omega$ . Show the circuit diagram.

See figure 7.15(a) in textbook.

**c) [4 points]** Find the range of values for the differential input resistance.

The differential input resistance is  $R_{id} = 2(\beta+1)(r_e+R_e)$ , where  $r_e = V_T/I_E = V_T/(I/2) = 25\text{mV}/50\mu\text{A} = 500 \Omega$ . At  $\beta = 100$ ,  $R_{id} = 2(101)(500+750) = 252.5 \text{ K}\Omega$ ; at  $\beta = 140$ ,  $R_{id} = 2(141)(500+750) = 352.5 \text{ K}\Omega$ . The range of values for  $R_{id}$  is therefore 252.5 to 352.5  $\text{K}\Omega$ .

**Problem 3 [20 points]**

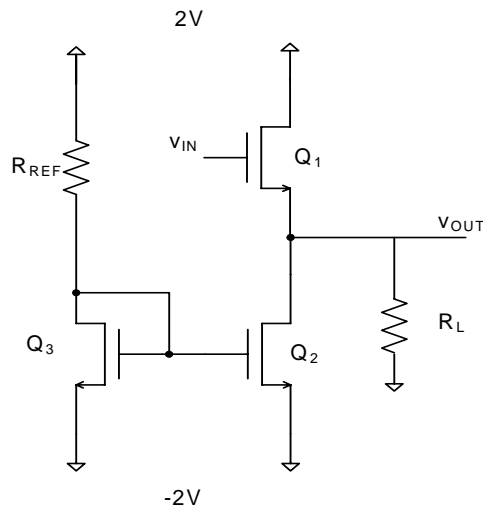


Figure 2

Refer to the circuit shown in Figure 2. The load resistor is  $R_L = 1000 \Omega$ . The MOSFETs have  $k'_n = 200 \mu\text{A}/\text{V}^2$  and  $V_{tn} = 0.5 \text{ V}$ . The input voltage range is unrestricted.

a) [3 points] What type of output stage is this circuit?

This is a class-A output stage.

b) [10 points] Calculate the values of  $R_{REF}$  and the  $(W/L)$  ratios for the MOSFETs so that the output ranges from  $-0.5$  to  $0.5$  volts with the highest efficiency and *smallest area*. All MOSFETs should be in saturation over the range of operation. Assume that  $Q_2$  and  $Q_3$  are matched.

The current  $I$ , flowing in  $R_{REF}$  and mirrored in  $Q_2$ , should be equal to  $|V_{OUTmin}|/R_L = 0.5/1000 = 0.5 \text{ mA}$ , for maximum efficiency.  
 $Q_2$  is always SAT  $\Rightarrow v_{OUT} - (-2) > V_{OV2} \Rightarrow V_{OV2} < v_{OUT} + 2 \Rightarrow V_{OV2} = -0.5 + 2 = 1.5 \text{ V}$ , for minimum area. Therefore,  $(\frac{W}{L})_2 = \frac{I_{D2}}{\frac{1}{2}k'_n V_{OV}^2} = 0.5\text{m}/(.1\text{m } 1.5^2)$   
 $\Rightarrow (W/L)_2 = 2.22$ .  
 Since  $Q_2$  and  $Q_3$  are matched  $\Rightarrow (W/L)_3 = 2.22$ .  
 Since  $V_{OV3} = V_{OV2} = 1.5\text{V}$ , the voltage at the drain of  $Q_3$  is  $V_{G3} = V_{GS3} - 2 = V_{OV3} + V_{tn} - 2 = 1.5 + 0.5 - 2 = 0 \text{ V}$ .  
 The voltage across  $R_{REF}$  is  $2 \text{ V}$ , therefore  $R_{REF} = 2\text{V}/0.5\text{mA} = 4 \text{ K}\Omega$ .  
 $Q_1$  should be saturated  $\Rightarrow V_{OV1} = V_{DS1min} = 2 - v_{OUTmax} = 2 - 0.5 = 1.5 \text{ V}$   
 When  $v_{OUT} = v_{OUTmax}$ ,  $I_{D1} = I_L + I_{D2} = 1 \text{ mA} \Rightarrow (\frac{W}{L})_1 = \frac{I_{D1}}{\frac{1}{2}k'_n V_{OV1}^2} = 4.44$ .

c) [7 points] Calculate the sine wave efficiency (power\_into\_load / total\_power) for a sine wave output with an amplitude of 0.5 V. The total\_power should include the power dissipated in all circuit elements.

The power in the load is  $P_L = \frac{V_{out\max}^2}{2R_L} = \frac{0.5^2}{2000} = 0.125 \text{ mW}$ .

The power in the two supplies is  $2 \text{ V} (1 \text{ mA}) + 2 \text{ V} (1 \text{ mA}) = 4 \text{ mW}$ .

The efficiency is therefore:  $0.125/4 = 3.125\%$

**Problem 4 [30 points]**

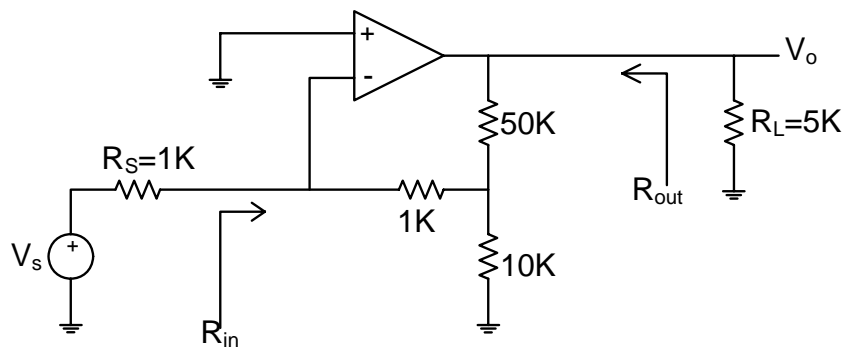


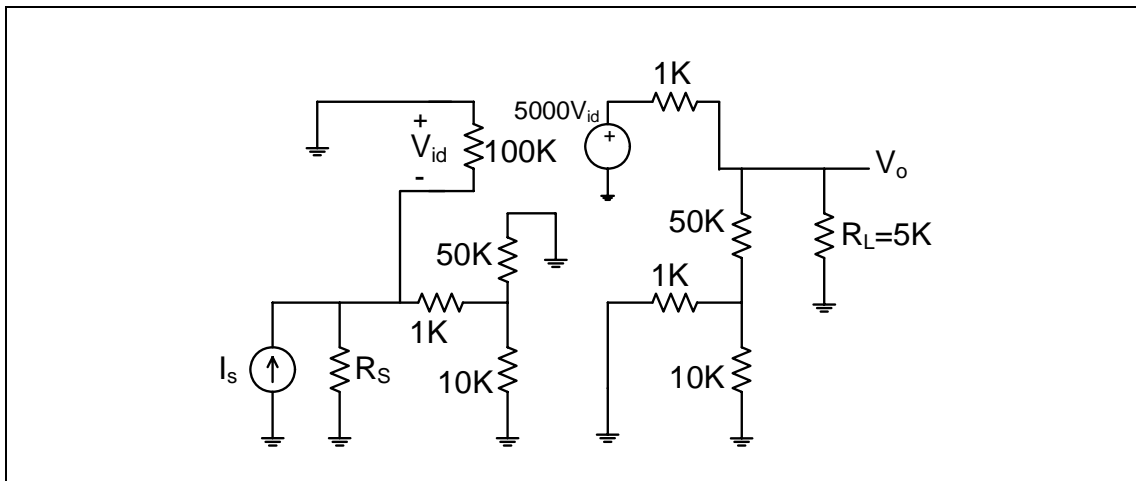
Figure 3

Consider the op-amp circuit shown in Figure 3.

a) [3 points] Determine the type of feedback used.

The output is a voltage that is sampled using a shunt. The input is mixed using currents, which is also a shunt. The type of feedback is therefore shunt-shunt.

b) [7 points] The op-amp is modeled by a differential input resistance  $R_{id} = 100 \text{ K}\Omega$ , an open-circuit voltage gain of 5000, and an output resistance of  $1 \text{ K}\Omega$ . Using feedback techniques, find the open-loop gain  $A$ . What are the units of  $A$ ?



The input is transformed into a current source using Norton's equivalent since the feedback is shunt-shunt.

The feedback circuit is modified (opened) to calculate the open-loop gain  $V_o/I_s$ .

From the circuit,  $V_{id} = -I_s(R_s//100K//(1K+10K//50K)) = I_s(-895.14)$ .

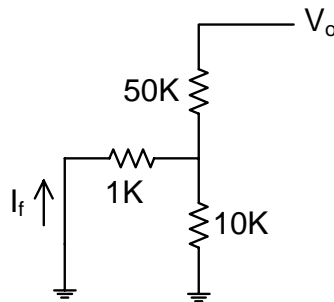
The output voltage is  $V_o = 5000 V_{id} R'_L/(1K + R'_L)$ .

$R'_L$  is given by  $5K//(50K+1K//10K) = 4.553K$ .

Therefore  $V_o = 4099.45 V_{id} = 4099.45 I_s(-895.14)$ .

$A = V_o/I_s = -3669.58 K\Omega$ .

**c) [4 points]** Find the feedback factor  $\beta$ . What are the units of  $\beta$ ? What is the loop gain?



The circuit to find  $\beta$  is shown above. The value of  $\beta$  is given by  $I_f/V_o = (1K//10K)/(1K//10K+50K)(-1/1K) = -17.857 \mu A/V$

The loop gain is  $A\beta = (-3669.58 K\Omega)(-17.857 \mu A/V) = 65.53$

**d) [3 points]** Find the closed-loop gain  $A_f$ .

$$A_f = \frac{A}{1 + \beta A} = \frac{-3669.58}{1 + 65.53} = -55.16 K\Omega$$

**e) [3 points]** Find the voltage gain  $V_o/V_s$ .

$$\frac{V_o}{V_s} = \frac{V_o}{I_s R_s} = \frac{A_f}{R_s}. \text{ The voltage gain is therefore } V_o/V_s = -55.16 V/V.$$

**f) [5 points]** Using feedback techniques, find the input resistance  $R_{in}$ .

$$R_i = 1K//100K//(1K+50K//10K) = 895.14 \Omega$$

$$R_{if} = R_i/(1 + \beta A) = 895.14/66.53 = 13.45 \Omega$$

$$R_{in} = 1 / (1/R_{if} - 1/R_s) = 13.64 \Omega$$

**g) [5 points]** Using feedback techniques, find the output resistance  $R_{out}$ .

$$R_o = 5K//1K//(50K+1K//10K) = 819.91 \Omega$$

$$R_{of} = R_o/(1 + \beta A) = 819.91/66.53 = 12.324 \Omega$$

$$R_{out} = 1 / (1/R_{of} - 1/R_L) = 12.35 \Omega$$