AMERICAN UNIVERSITY OF BEIRUT
FACULTY OF ENGINEERING AND ARCHITECTURE
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT
EECE 340 – Signals & Systems
FINAL EXAM-Spring 2009
Open book
TIME: 2 HOURS
Wednesday, June 10, 2009
INSTRUCTORS: K. Kabalan, J. Saade, F. Karameh

NAME : ______________________________ **ID # :** ______________________

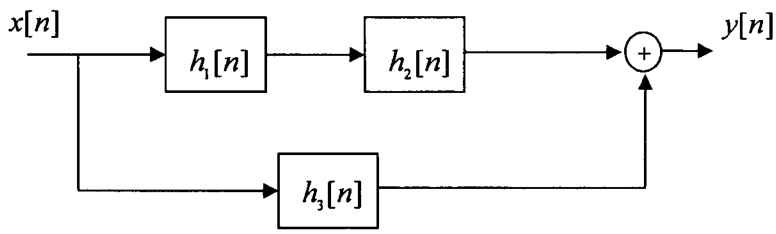
INSTRUCTIONS

- **Write your Name and ID # on this sheet, the computer card and the scratch booklet in the provided spaces.**
- **Provide your answer on the computer card and solution of each problem on the scratch booklet.**
- **Return the computer card, this question sheet and the scratch booklet when you finish the test.**
- **Only your answers on the computer card will be graded.**
- **All questions are equally weighted in grading.**

PROBLEM # 1

Determine the overall impulse response, h(n), of the DT system shown in the diagram below. The subsystems involved in the diagram are assumed linear and time-invariant and they are as given below:

$$h_1[n] = -3\delta[n+1] + 2\delta[n-2\}$$
$$h_2[n] = 2\delta[n+2] + \delta[n-1]$$
$$h_3[n] = 3\delta[n+1] - \delta[n] + 2\delta[n-1] + 7\delta[n-3] + 5\delta[n-5]$$



(a) $h(n)=-6\delta(n+3)+3\delta(n+1)+2\delta(n-1)+9\delta(n-3)+5\delta(n-5)$
(b) $h(n)=-6\delta(n+3)+2\delta(n+1)+2\delta(n-1)+8\delta(n-3)+5\delta(n-5)$
(c) $h(n)=-6\delta(n+3)+3\delta(n+1)+\delta(n-1)+9\delta(n-3)+\delta(n-5)$
(d) $h(n)=-5\delta(n+3)+3\delta(n+1)+2\delta(n-1)+6\delta(n-3)+5\delta(n-5)$
(e) None of the above.

PROBLEM # 2

The output y[n] of a discrete-time LTI system is related to its input x[n] by

$$y[n] = 2^{-n} \sum_{k=-\infty}^{k=\infty} 2^k x[k]$$

Examine the causality and stability of the system.

(a) The system is causal and stable.
(b) The system is causal and unstable.
(c) The system is non-causal and stable.
(d) The system is non-causal and unstable.
(e) None of the above.

PROBLEM # 3

The Z-transforms of the two signals x[n] and y[n] are given by:

$$X(z) = \frac{2z^2}{(z+1)^2};\ |z| > 1 \quad \text{and} \quad Y(z) = \frac{3z^2}{(z+1)^2(z-2)};\ 1 < |z| < 2$$

Determine the Z-transform, G(z), of g[n]=2x[n]+4y[n].

(a) $G(z) = \dfrac{4z^2}{(z+1)^2(z-2)},\ 1 < |z| < 2$

(b) $G(z) = \dfrac{4z^2}{(z-1)(z-2)},\ 1 < |z| < 2$

(c) $G(z) = \dfrac{4z^2}{(z+1)(z-2)},\ 1 < |z| < 2$

(d) $G(z) = \dfrac{4z^2}{(z+1)(z-2)^2},\ 1 < |z| < 2$

(e) None of the above.

PROBLEM # 4

Consider the discrete-time LTI system with transfer function given by

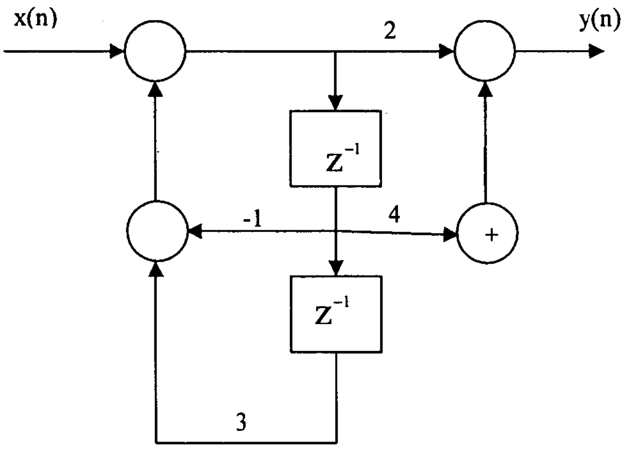
$$H(z) = \frac{1-2z^{-1}}{(z-1)(z+2)} \;;\quad |z| > 2$$

Determine the impulse response, $h_{eq}(n)$, of the new system formed by cascading H(z) with a second LTI system of impulse response $h_1[n] = \delta[n] - \delta[n-1]$.

(a) $heq(n)=\delta(n-2)-(-2)^{n-3}u(n-3)$
(b) $heq(n)=\delta(n+2)-(-2)^{n-1}u(n-3)$
(c) $heq(n)=\delta(n-2)-(-2)^{n-1}u(n-3)$
(d) $heq(n)=\delta(n+2)-(-2)^{n}u(n-3)$
(e) None of the above.

PROBLEM # 5

A causal discrete-time LTI system is represented by the Direct Form II block diagram given in the figure below. Determine the state space and output equations of the system.



$$(a)\begin{bmatrix} x_1(n+1) \\ x_2(n+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}x(n) \text{ and } y(n) = \begin{bmatrix} 6 & 2 \end{bmatrix}\begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix} + 2x(n)$$

$$(b)\begin{bmatrix} x_1(n+1) \\ x_2(n+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}x(n) \text{ and } y(n) = \begin{bmatrix} 6 & 2 \end{bmatrix}\begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix} + x(n)$$

$$(c)\begin{bmatrix} x_1(n+1) \\ x_2(n+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}x(n) \text{ and } y(n) = \begin{bmatrix} 6 & 5 \end{bmatrix}\begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix} + 2x(n)$$

$$(d)\begin{bmatrix} x_1(n+1) \\ x_2(n+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 5 & -1 \end{bmatrix}\begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}x(n) \text{ and } y(n) = \begin{bmatrix} 6 & 5 \end{bmatrix}\begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix} + x(n)$$

(e) None of the above.

PROBLEM # 6

A Discrete-time causal signal, h[n], has the following discrete-time Fourier transform (DTFT). Determine h[n]. Hint: you may use the inverse Z-transform.

$$H(\omega)=\frac{1+e^{-j\omega}}{1-0.5e^{-j\omega}}$$

(a) $h(n) = \delta(n)+3(1/2)^{n}u(n-1)$
(b) $h(n) = \delta(n)+3(1/2)^{n-1}u(n-1)$
(c) $h(n) = \delta(n)+3(1/2)^{n+1}u(n-1)$
(d) $h(n) = \delta(n)+3(1/2)^{n}u(n+1)$
(e) None of the above.

PROBLEM # 7

A LTI causal and stable discrete-time system is represented by the following difference equation:

$$0.25\, y[n-2] + y[n-1]+y[n]=x[n-2]$$

Determine the magnitude frequency response, $|H(\omega)|$, of the system.

(a) $|H(\omega)|=\dfrac{1}{2\cos\omega+2}$ (b) $|H(\omega)|=\dfrac{1}{\cos\omega+1/4}$
(c) $|H(\omega)|=\dfrac{1}{2\cos\omega+1}$ (d) $|H(\omega)|=\dfrac{1}{\cos\omega+5/4}$
(e) None of the above.

PROBLEM # 8

Consider the following finite-duration discrete-time signal:

$$x(n)=\begin{cases}\frac{1}{4}, & n=0,1,2,3\\ 0, & elsewhere\end{cases}$$

Determine the 4-point DFT of x(n).

(a) X(0)=1, X(1)=X(2)=X(3)=1/2.
(b) X(0)=1, X(1)=X(2)=X(3)=2.
(c) X(0)=1, X(1)=X(2)=X(3)=4.
(d) X(0)=1, X(1)=X(2)=X(3)=5.
(e) None of the above.

PROBLEM # 9

Consider the following 2 finite-duration discrete-time signals:

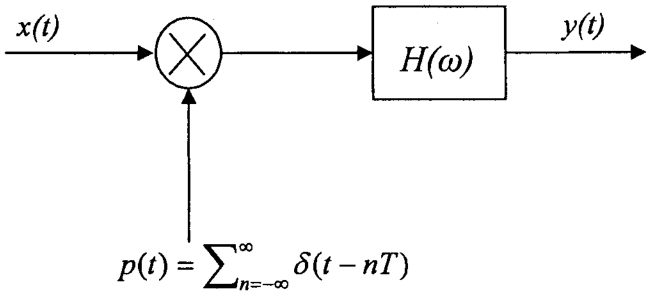
$$f(n) = 3\delta(n) + \delta(n-1) + 2\delta(n-2) - \delta(n-3)$$
$$g(n) = 3\delta(n) + 2\delta(n-1) + \delta(n-2)$$

Let F(k) and G(k) be the 4-point DFT's of f(n) and g(n) respectively. Determine the discrete-time signal y(n) such that its 4-point DFT is given by the product of F(k) and G(k); i.e., Y(k)=F(k)G(k). Note here that y(n) in this case is the circular convolution of f(n) and g(n). Also, note that linear convolution with an impulse provides time shift.

(a) $y(n)=9\delta(n)+9\delta(n-1)+11\delta(n-2)+\delta(n-3)$
(b) $y(n)=9\delta(n)+8\delta(n-1)+11\delta(n-2)+2\delta(n-3)$
(c) $y(n)=7\delta(n)+8\delta(n-1)+9\delta(n-2)+2\delta(n-3)$
(d) $y(n)=9\delta(n)+8\delta(n-1)+5\delta(n-2)+\delta(n-3)$
(e) None of the above.

PROBLEM # 10

Consider the system depicted below where T=1. Let $x(t)=\cos(3\pi t/2)$ and $H(\omega)$ be an ideal LPF with bandwidth equal to π rad/s. Determine the output of the system denoted by y(t).



(a) $y(t)=\cos(\pi t/5)$
(b) $y(t)=\cos(\pi t/3)$
(c) $y(t)=\cos(\pi t/2)$
(d) $y(t)=\cos(\pi t)$
(e) None of the above.

PROBLEM # 11

Consider two LTI discrete-time systems with the following unit sample responses:

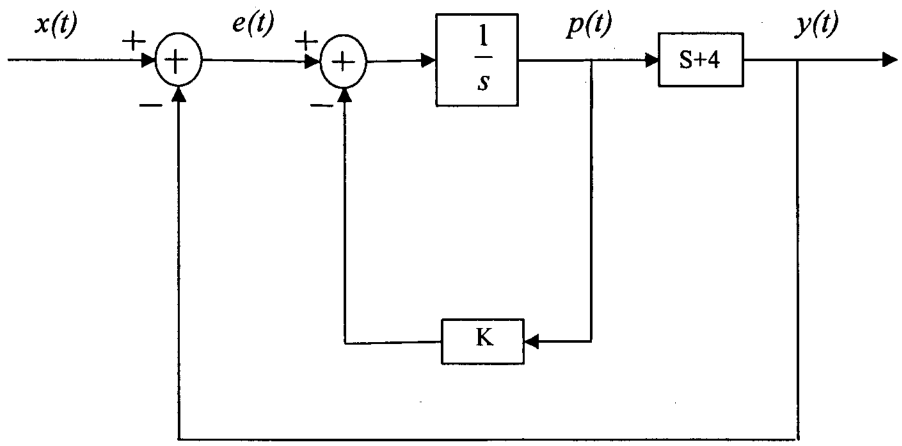
$$h_1[n]=\frac{4}{7}2^{n-1}u[-n] \text{ and } h_2[n]=\delta[n]-3^{n-1}u[-n]$$

Let $h[n]$ be the impulse response of the system which is equivalent to the cascaded connection of $h_1[n]$ and $h_2[n]$. Determine whether $h[n]$ is causal and stable.

(a) The equivalent system is non-causal and unstable.
(b) The equivalent system is causal and stable.
(c) The equivalent system is causal and unstable.
(d) The equivalent system is non-causal and stable.
(e) None of the above.

PROBLEM # 12

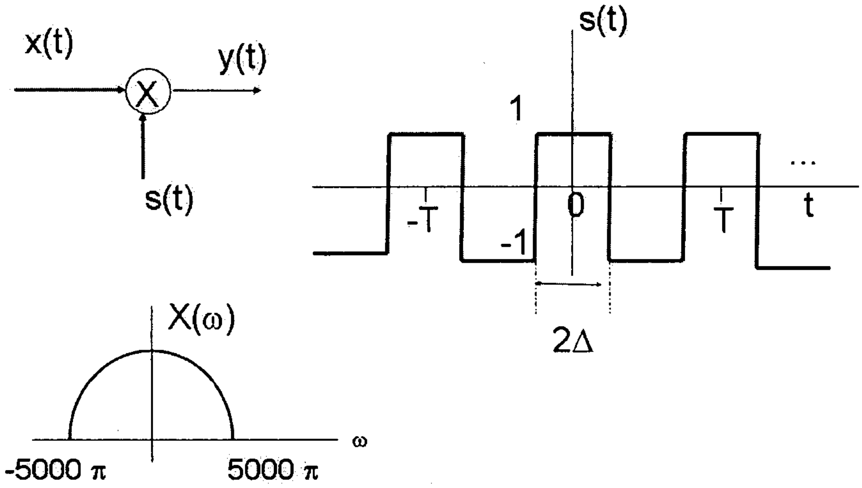
Consider the continuous-time LTI causal system shown in the figure below. Obtain the overall transfer function of the system; i.e., H(s)=Y(s)/X(s), and then determine the range of the real constant K for which the system is stable. Hint: To obtain H(s), obtain first the transfer function of the inner loop; i.e., P(s)/E(s).



(a) K<-2 (b) K>-4 (c) K >-1/4 (d) K<-1/2
(e) None of the above.

PROBLEM # 13

A signal x(t) is sampled through a non-ideal sampler using the periodic signal s(t) of period T as shown in the figure below. The signal x(t) is assumed band-limited (also as shown in the figure). Let $s(t)=s_1(t)-1$ and determine $s_1(t)$ first. Then obtain the Fourier transform spectrum of s(t) using that of $s_1(t)$ and as a result obtain the spectrum of the sampled signal, y(t). Determine the smallest possible value of the sampling rate (1/T) for which there is NO ALIASING in the sampled signal. USE $\Delta=T/4$ in all computations.



(a) The smallest sampling rate is 7500 samples/sec.
(b) The smallest sampling rate is 5000 samples/sec.
(c) The smallest sampling rate is 2500 samples/sec.
(d) The smallest sampling rate is 10000 samples/sec.
(e) None of the above.

PROBLEM # 14

Consider a discrete-time LTI system with unit sample response, h(n), given by:

$$h(n) = a^{-n}u(-n-1) \text{ with } |a| < 1.$$

Determine the unit-step response, $y_u(n)$, of the system using discrete-time convolution.

$(a)\ y_u(n) = \frac{1}{1-a}\left[a\,u(n) + a^{-n}u(n-1)\right]$ $(b)\ y_u(n) = \frac{1}{1-a}\left[a\,u(n) + a^{n}u(-n-1)\right]$

$(c)\ y_u(n) = \frac{1}{1-a}\left[a^{n}\,u(n) + a^{-n}u(-n-1)\right]$ $(d)\ y_u(n) = \frac{1}{1-a}\left[a\,u(n) + a^{-n}u(-n-1)\right]$

(e) None of the above.

PROBLEM # 15

Consider the following finite-length discrete-time signal:

$$x(n)=u(n)-u(n-6)$$

Let the Z-transform of $x(n)$ be sampled at $z = e^{j\frac{2\pi}{8}k}$, $k = 0, 1, 2, ..., 7$. The sampled Z-transform represents the discrete Fourier series coefficients of a periodic discrete-time signal, $\tilde{x}(n)$. Determine $\tilde{x}(n)R_8(n)$, where $R_8(n)=1$, for n=0, 1, 2, ..., 7 and 0, elsewhere.

$(a)\ \tilde{x}(n)R_8(n) = u(n) - u(n-6)$

$(b)\ \tilde{x}(n)R_8(n) = u(n) - u(n-8)$

$(c)\ \tilde{x}(n)R_8(n) = u(n-1) - u(n-6)$

$(d)\ \tilde{x}(n)R_8(n) = u(n) - u(n-5)$

(e) None of the above.

PROBLEM # 16

Consider a causal, LTI and unstable discrete-time system with transfer function given by

$$H(z) = \frac{z}{(z-1)(z-2)}, \quad |z| > 2$$

We want to transform this system to a stable one by multiplying its impulse response, $h(n)$, by a^n. Determine the condition on $|a|$ that provides a system with impulse response $h_1(n)=a^n h(n)$ that is stable. Use the properties of the Z-transform.

$(a)\ |a| < 1/3$ $(b)\ |a| < 1/4$ $(c)\ |a| < 2$ $(d)\ |a| < 1/2$

(e) None of the above.