EECE 340 Signals and Systems | Jun. 11, 2015
American University of Beirut, Summer 2014–2015 | Handout #4
Due: Wednesday, Jun. 17, 2015

Problem Set 2

Reminder on Collaboration Policy

The following is an acceptable form of collaboration: discuss with your classmates possible approaches to solving the problems, and then have *each one* fill in the details and *hand-write her/his own* solutions *independently*.

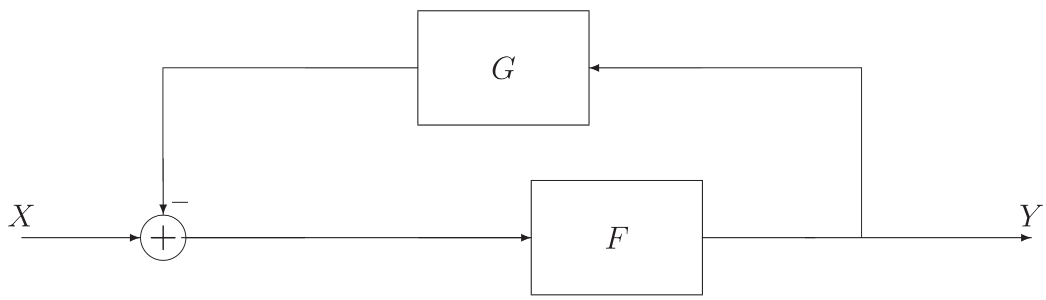
An unacceptable form of dealing with homework is to copy a solution that someone else has written.

At the top of each homework you turn in, list all sources of information you used, apart of course from the text, books on reserve for this course or discussions with the Prof. A brief note such as "did problem 7 with May Berite in study group" would be sufficient.

In general, we expect students to adhere to basic, common sense concepts of academic honesty. Presenting another's work as if it were your own, or cheating in exams will not be tolerated.

Problem 2.1

Consider the following block diagram of a DT system,



where instead of using explicitly a multiplier for "-1", we simply wrote a "minus" sign in the sum diagram.

Assume that X is an impulse (and the system is naturally at rest).

(a) Determine the system function Y/X function of $F(D)$ and $G(D)$.

(b) Assume that $F(D)$ can be written as a ratio of polynomials in D. Let N_F and D_F represent the numerator and denominator polynomial, respectively. Similarly, assume that $G(D)$ can be written as a ratio of polynomials N_G and D_G. Express the system functional in terms of N_F , D_F , N_G, and D_G.

Problem 2.2

Having spent most of your time in college studying, and with prom night (a.k.a. Gala) approaching, you realize that you have not mastered the art of dancing; something many people claim to be very important especially during the prom.

Being well versed in the art of Signals and Systems instead, you decide to develop dancing routines guided by well thought-of dynamics. More precisely, the position of your left leg (say $y[n]$) is after all nothing but a signal! The same holds for the right one of course, but the subject of this exercise is the left leg only.

Part I:

Let $x[n]$ be the position of the right leg of your partner, and you figure that dancing (the ballroom-type at least) is nothing but moving your left leg according to the following equation:

$$y[n] = x[n] + 10,$$

where positions are measured in centimeters.

(a) Find the system function of your system.

(b) Find the impulse response of your system.

Hint: Feel free to redefine the "input" if you'd like to.

Part II:

The drawback of your dancing moves in Part I, is that if your partner stops moving for some reason, you will abruptly stop as well. To make your dancing smooth(er), you decide now to move (your left leg) as follows:

$$y[n] = x[n] + 10 + ay[n-1],$$

for some constant non-zero scalar a to be determined.

(c) Draw a block diagram of your system, where $x[.]$ and $y[.]$ are the input and output respectively.

(d) Would you choose a complex value for a? Explain.

(e) You decide to use a real value for a, and say you really like the guy. Would you rather choose a positive or negative? Explain.

(f) What is the range of values of a that you may want to choose from? Explain.

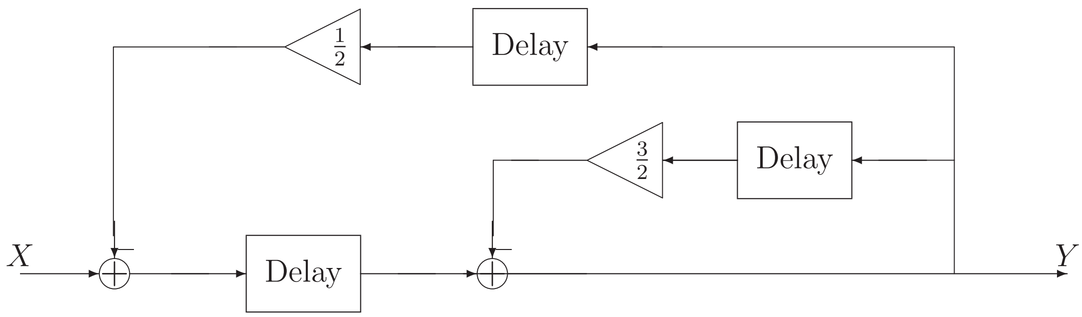
Part III:

Odds are, your partner is not a good dancer either. You decide to take the lead. Your movement will therefore depend on the music and not his steps. For every "beat" (hits on derbakke or drums), you want to generate a "wavy" movement of frequency ω_o. More precisely a movement resembling $\cos(\omega_o n)$. Also naturally, you want your wavy movements to gracefully die out after the party ends: The amplitude of your "waves" should be cut in half in roughly 5 steps.

(g) Write an appropriate difference equation for your new dance. Make sure to specify all the parameters in order to meet the conditions above.

Problem 2.3

Consider the system defined by the following block diagram



(a) Derive a difference equation describing the input/output relationship.

(b) Determine the system function.

(c) Determine the poles of the system.

Problem 2.4

Consider a first order system function

$$\frac{1}{1-\alpha D}.$$

(a) Assume α is a real number (equal to 0.5 for example), and that the input is a complex-valued signal.

Draw the corresponding block diagram using only *real* signals and components.

(b) Now assume α is a complex number (equal to $0.5(1+j)$ for example), and that the input is a real-valued signal.

Draw the corresponding block diagram using only *real* signals and components.

Problem 2.5

Consider a DT system with the following system function:

$$\frac{Y}{X} = \frac{2 - 10D + 12D^2}{1 - D - (1/4)D^2 + (1/4)D^3}.$$

(a) Draw a block diagram of your system.

(b) Write a difference equation of your system.

(c) Decompose your system into a (serial) cascade of 0-th and 1-st order systems.

(d) Decompose your system into parallel 0-th and 1-st order systems.

Problem 2.6

We have seen in class that when we have a repeated root α twice, the impulse response is $(n+1)\alpha^n$ for $n \geq 0$.

The objective of this problem is to figure out what the impulse response is when the repeated root has a higher order.

(a) Consider a DT system with system function

$$\frac{Y}{X} = \frac{1}{(1 - \alpha D)^3}.$$

Using the perturbation method, determine the impulse response of the system.

(b) If the system function were

$$\frac{Y}{X} = \frac{1}{(1 - \alpha D)^k},$$

use your favorite technique to determine the impulse response of the system. It is okay if your answer is not expressed in closed form.