AMERICAN UNIVERSITY OF BEIRUT
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT

EECE 340	SIGNALS AND SYSTEMS	Summer 2012
Prof. Kabalan	Midterm Exam	July 17, 2012

Problem 1 (6 pts)

Consider the signal shown below

$$x(t) = \begin{cases} \dfrac{1}{\sqrt{t}} & t > 1 \\ 0 & t \le 1 \end{cases}$$

a. Determine the total energy of this signal. Is x(t) an Energy Signal? (3 pts)

$E = \int_{1}^{\infty} \frac{1}{t} dt = \infty$. X(t) is not an energy signal

b. Determine the average power of this signal. Is x(t) a power signal. (3 pts)

$P = \lim_{T\to\infty} \frac{1}{T} \int_{-t/2}^{T/2} \frac{1}{t} dt = \text{does not exist}$. X(t) is not a power signal.

Problem 2 (10 pts)

The output y(t) of a continuous-time system is related to its input x(t) by

$$y(t) = \cos[2x(t+1)] + x(t)$$

a. Is the system linear? Justify your answer. (2 pts)
   No. due to the cosine function

b. Is the system time-invariant? Justify your answer. (2 pts)
   Yes, after we apply definition

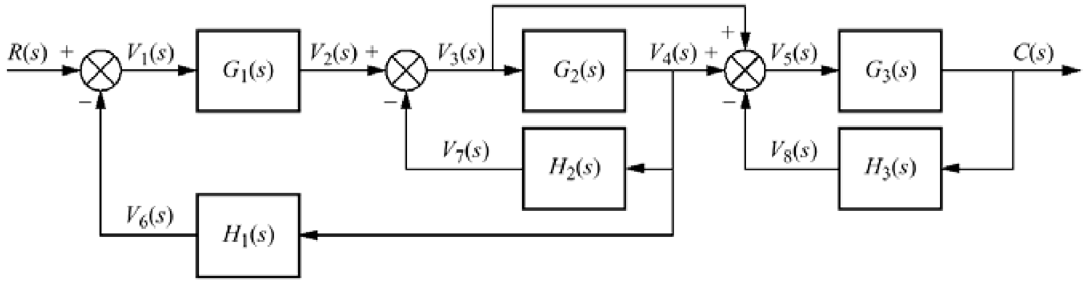
c. Is the system causal? Justify your answer. (2 pts)
   No, y(t) dependents on future values of x(t)

d. Is the system memoryless? Justify your answer. (2 pts)
   No, y(t) depends on future values of x(t)

e. Is the system stable? Justify your answer. (2 pts)
   Yes. X(t) is bounded and so is cos[2(x(t+1)]. This implies that y(t) is bounded.

Problem 3 (6 pts)

For the system shown below, determine the system transfer function



$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3 + G_1G_3}{1 + G_1G_2H_1 + G_2H_2 + G_3H_3 + G_1G_2G_3H_1H_3 + G_2G_3H_2H_3}$$

-2 for every incorrect term

Problem 4 (6 Pts)

Determine the forward transfer function of a unity feedback control system whose closed-loop transfer function is given by:

$$\frac{Y(s)}{R(s)} = \frac{s+1}{s^4 + 3s^3 + 4s^2 + 6s + 5}$$

$$G(s) = \frac{s+1}{s^4 + 3s^3 + 4s^2 + 5s + 4}$$

Problem 5 (6 pts)

The closed-loop transfer function of a unity feedback control system, whose input is Y(s) and output is R(s), is given by

$$\frac{Y(s)}{R(s)} = \frac{10}{s^5 + 7s^4 + 6s^3 + 42s^2 + 8s + 56}$$

Check the stability of the above system by determining the number of poles in each half of the complex plane.

Correct table 3 points

S^5	1	6	8
S^4	1	6	8
$\cancel{S^3}$	$\cancel{0}$	$\cancel{0}$	$A(s)=s^4+6s^2+8$
S^3	4	12	$A^{/}(s)=4s^3+12s$
S^2	3	8	
S^1	4		
S^0	8		

$$A(s) = 0 \Rightarrow s = \pm 2j, \text{ and } s = \pm\sqrt{2}j$$

Root Distributions: (1, 4, 0) **3 pts**

Problem 6 (6 pts)

A linear time-invariant system, whose output is y(t) and input r(t), is represented by the following transfer function

$$\frac{Y(s)}{R(s)} = \frac{10s+6}{2s^2+8s+56}$$

Write the state and output equations of this system

$$\begin{bmatrix} X_1'(t) \\ X_2'(t) \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ -28 & 0 \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix} + \begin{bmatrix} 5 \\ 3 \end{bmatrix} r(t) \quad \textbf{4 pts}$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix} + [0]r(t) \quad \textbf{2 pts}$$

Problem 7 (6 pts)

A sinusoidal signal (Carrier) has a period of 10 ms. At *t*=0, the signal has a value of 20 V. It reaches its first positive peak 2 ms later. Determine the equation of the signal.

$$c(t) = A\cos(\omega_c t + \phi) \text{ with } T = 10^{-2}\text{s}, \ \omega_c = 2\pi f_c = 200\pi \text{ rad/s} = 628.3 \text{ rad/s} \quad 1 \text{ pt}$$

$$c(0) = A\cos(\phi) = 20,$$

$$c(2\cdot 10^{-3}) = A = A\cos(0.4\pi + \phi) \Rightarrow \phi = -0.4\pi = -1.2566 \quad \textbf{2 pts}$$

$$A = \frac{20}{\cos(-0.4\pi)} = 64.72$$

$$c(t) = 64.72\cos(628.3t - 1.257) \text{ Volts} \quad \textbf{2 pts}$$

Problem 8 (6 pts)

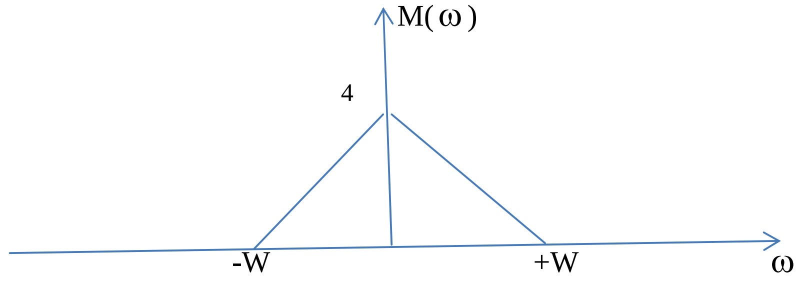
Using the FT properties, determine the Fourier transform of the following function

$$y(t) = 5e^{3t}u(-t)$$

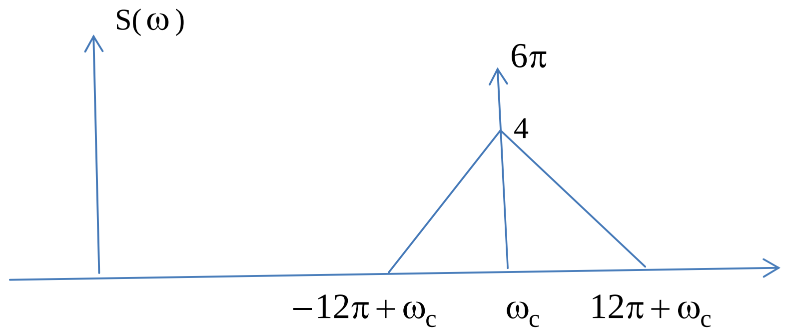
Where u(t) is the unit step function. Please simplify your expression for full points.

$$Y(\omega) = \int_{-\infty}^{0} 5e^{3t}e^{-j\omega t}dt = \frac{5}{3 - j\omega}$$

Problem 9 (6 pts)
The spectrum of the message signal m(t) is shown below



a. Write the time domain representation s(t) of the signal shown below. (4 pts)



This is the positive frequency part of a DSB-LC signal. $s(t) = 12\cos(\omega_c t) + 2m(t)\cos(\omega_c t)$.

It is given by:

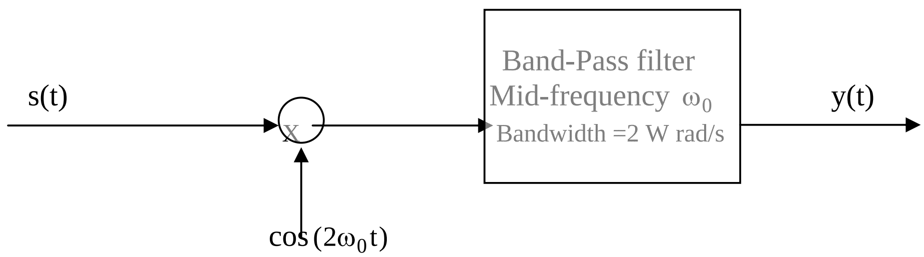
$$\frac{1}{2}\left[s(t) + j\hat{s}(t)\right] = 6\cos(\omega_c t) + m(t)\cos(\omega_c t) + j6\sin(\omega_c t) + jm(t)\sin(\omega_c t)$$

$$= 6e^{j\omega_c t} + 4m(t)e^{j\omega_c t}$$

b. Is s(t) is a real-valued signal? Explain. (2 pts)
No as no symmetry about the zero frequency.

Problem 10 (8 pts)

Let m(t) be a band-limited signal of bandwidth W rad/s. Using m(t), we construct the signal $s(t) = A[1 + m(t)]\cos(\omega_0 t)$ where $\omega_0 >> W$ rad/s. The signal s(t) is imputed to the following system. Let y(t) be the output of the system as shown below.



a. Determine y(t). (4 pts)

The output of the multiplier:

$$s(t)\cos(\omega_c t) = A_c[1 + m(t)]\cos(\omega_c t)\cos(2\omega_c t)$$

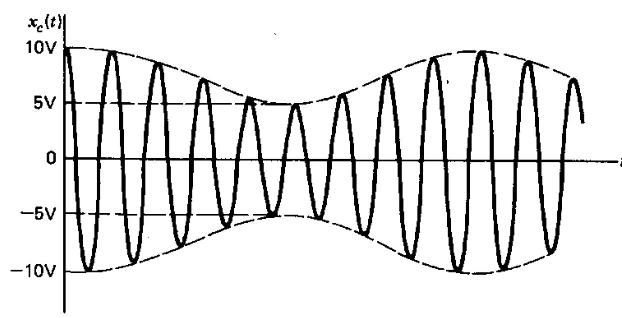
$$= \frac{A_c}{2}[1 + m(t)]\cos(\omega_c t) + \frac{A_c}{2}[1 + m(t)]\cos(3\omega_c t)$$

$$y(t) = \frac{A_c}{2}[1 + m(t)]\cos(\omega_c t)$$

b. Determine the bandwidth of y(t). (4 pts)

BW=2W rad/s.

<u>Problem 11 (8 pts)</u>



An AM (DSB-LC) waveform, denoted by $x_c(t)$, is shown above. Assume that the message signal is sinusoidal.

1. Find the modulation index. (4 Pts)

$A_{max} = 10V$, $A_{min} = 5V$

$$\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} = \frac{5}{15} = \frac{1}{3}$$

2. Calculate the carrier power. (4 Pts)

$$A_{max} = A_c[1+\mu] \Rightarrow 10 = A_c\left[1+\frac{1}{3}\right] \Rightarrow A_c = \frac{15}{2}\text{Volts}$$

$$P_{av} = \frac{1}{2}\left(\frac{15}{2}\right)^2 = \frac{225}{8} = 28.125\text{ Watts}$$

Problem 12 (6 pts)

A Frequency modulated (FM) signal of carrier frequency 2 MHz, frequency sensitivity k_f =100 Hz/V, and information signal m(t) given by

$$m(t) = 100\cos(150\pi t) + 200\cos(300\pi t)$$

Write the time domain expression of corresponding FM signal.

$$s(t) = A_c \cos\left[4\pi \cdot 10^6 t + \frac{400}{3}(\sin 150\pi t + \sin 300\pi t)\right]$$

-2 pts if the above term is wrong

Problem 13 (10 pts)

An FM signal is given by:

$$s(t) = 100\cos\left[2\pi \cdot 10^5 t + 3\sin(100\pi t)\right]$$

a. Determine the bandwidth of this FM signal using the 1% rule. (4 pts)
   $\beta = 3$, using Bessel table, $BW = 10\omega_m = 1000\pi \text{ rad/s}$
b. Determine bandwidth of this FM signal using Carson's rule. (4 pts
   $\beta = 3,\ BW = 2\omega_m(1+\beta) = 8\omega_m = 800\pi \text{ rad/s}$
c. Compare your results and explain. (2 pts)
   Carson's rule is an approximate rule.

Problem 14 (10 pts)

An FM signal is described by

$$s(t) = 50\cos\left[2\pi \cdot 10^6 t + 10^6 t + 0.001\cos(1000\pi t)\right]$$

a. Determine the instantaneous frequency of the signal. (4 pts)
   $$s(t) = 50\cos\left[(1+2\pi)\cdot 10^6 t + 0.001\cos(1000\pi t)\right]$$
   $$\omega_i(t) = (1+2\pi)\cdot 10^6 - \pi\cos(1000\pi t)$$
   -1 if the above is incorrect
b. Is the signal a narrowband or wideband frequency modulation? (4 pts)
   Narrow band as $\beta = 0.001 <<< 1$.
c. Determine the information signal. (2 pts)
   $$\omega_i(t) = \omega_c + k_f m(t) = (1+2\pi)\cdot 10^6 - \pi\cos(1000\pi t)$$
   $$m(t) = -\frac{\pi}{k_f}\cos(100\pi t)\ \text{ Volts}$$