AMERICAN UNIVERSITY OF BEIRUT
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT

EECE 340 SIGNALS AND SYSTEMS Spring 2007-2008
Prof. Kabalan Quiz II- Solution

Problem 1 (5 pts)
Find the Fourier transform of the signal

$$x(t) = \frac{d}{dt}\left[te^{-|t-1|}\right]$$

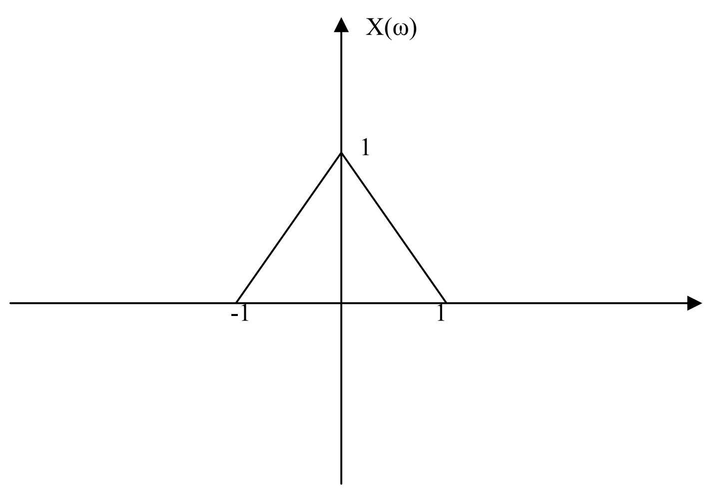
$e^{-|t|} \leftrightarrow \dfrac{2}{1+j\omega}$ **2 pts**

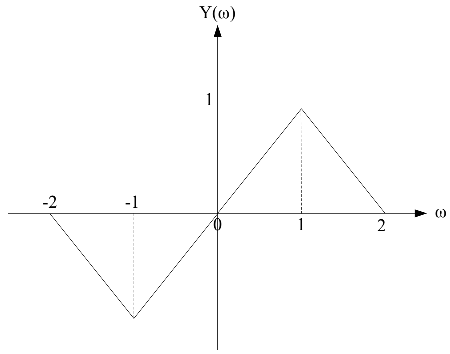
$e^{-|t-1|} \leftrightarrow \dfrac{2e^{-j\omega}}{1+j\omega}$ shifting property **1 pt**

$te^{-|t-1|} \leftrightarrow j\dfrac{d}{d\omega}\left(\dfrac{2e^{-j\omega}}{1+j\omega}\right)$ Multiplication by t **1 pt**

$\dfrac{d}{dt}te^{-|t-1|} \leftrightarrow j\omega \cdot j\dfrac{d}{d\omega}\left(\dfrac{2e^{-j\omega}}{1+j\omega}\right) = -\omega\dfrac{d}{d\omega}\left(\dfrac{2e^{-j\omega}}{1+j\omega}\right)$ derivative property **1 pt**

Problem 2 (5 pts)
The spectrum of the signals y(t) and x(t) are shown below. Write y(t) as a function of x(t).



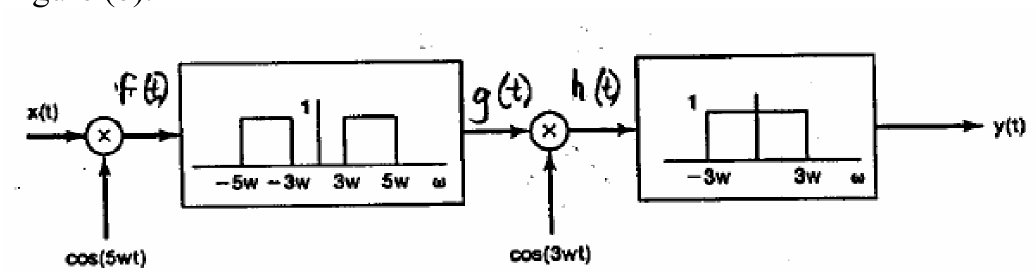


$Y(\omega) = X(\omega - 1) - X(\omega + 1)$ **2 pts**

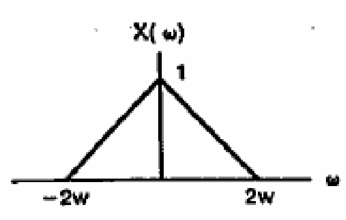
$y(t) = x(t)e^{jt} - x(t)e^{-jt}$ **3 pts**

Problem 3 (6 pts)

In Figure (a) below, a system is shown with input x(t) and output y(t). The input signal has the Fourier Transform (Spectrum) X(ω) shown in Figure (b).

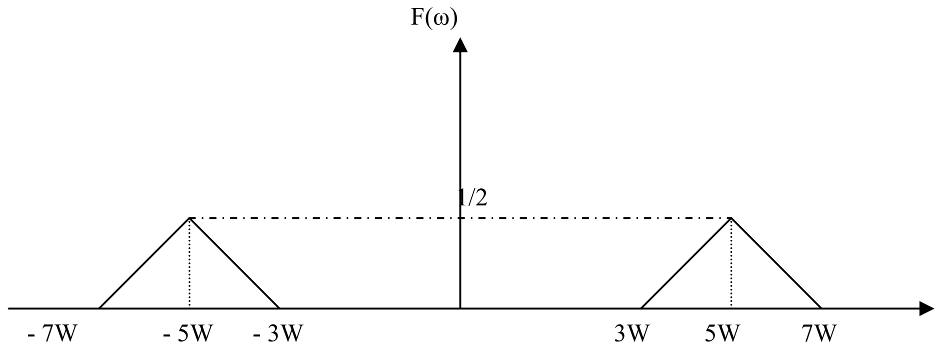


(a)

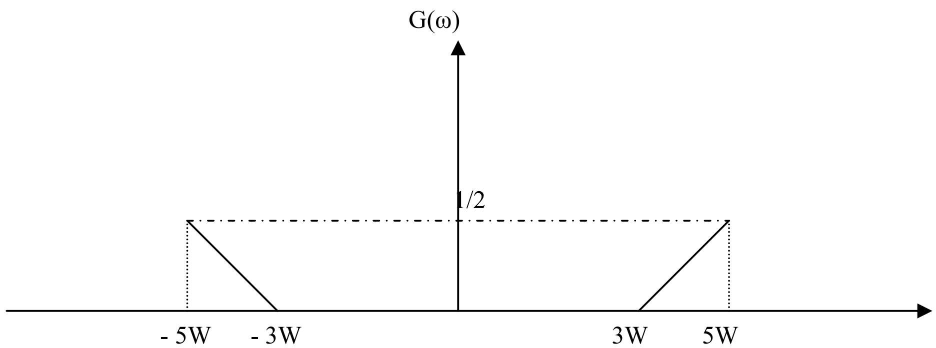


(b)

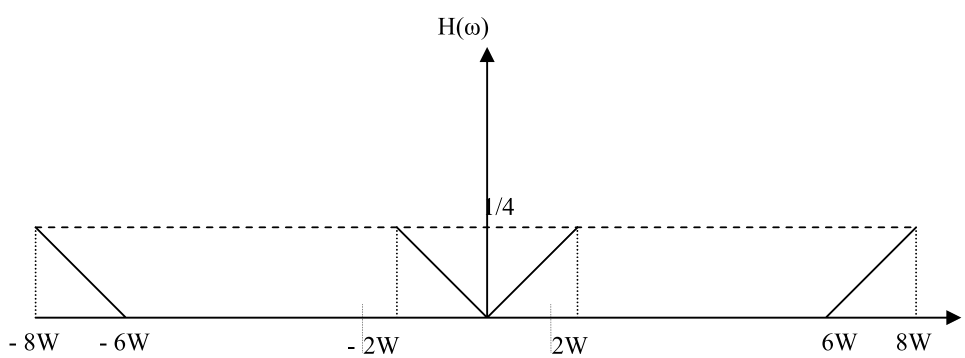
a. Determine and plot F(ω) (2 pts)



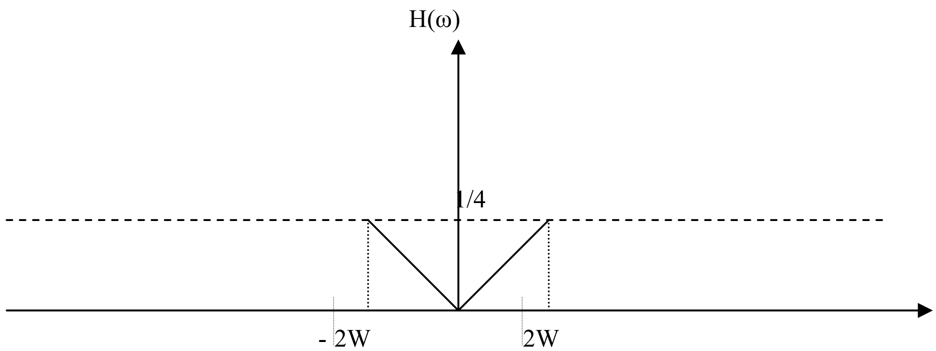
b. Plot G(ω) 1 pt



c. Plot H(ω) **2 pts**



c. Determine and plot Y(ω) 1 pt



<u>Problem 4 (6 pts)</u>

Consider the filter with impulse response $h(t) = 2e^{-t}u(t)$.

a. Find the system transfer function. (2 pts)

$$H(\omega) = \frac{2}{1 + j\omega}$$

b. Find the output spectrum Y(ω) when the input signal x(t) is given by: x(t)=3sint. (2 pts)

$$Y(\omega) = \frac{2}{1 + j\omega} \cdot \frac{3\pi}{i}\left[\delta(\omega - 1) - \delta(\omega + 1)\right]$$

c. Find y(t) (2 pts)

y(t)=3 sint – 3 cost

Problem 5 (5 pts)

Consider a DSB-LC wave where the carrier has a 500-KHz frequency with a peak amplitude of 20 V_p. The message is a single tone with frequency of 10-KHz modulating that is of sufficient amplitude to cause a peak change in the output wave of ±7.5 V.

a. Determine the USF and the LSF frequencies **1 pt**
   USF=510 KHz and LSF=490 KHZ
b. Determine the modulation factor **1 p**t
   $A_{\max} = 27.5 \text{ V}, \quad A_{\min} = 12.5 \text{ Volts}$
   $$\mu = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}} = 0.375 \text{ V},$$
c. Determine the peak amplitude of the USB component of the AM signal **1 pt**
   $$= \frac{A_c \mu}{2} = 3.75 \text{Volts}$$
d. Determine the maximum and the minimum amplitudes of the AM envelope. **1 pt**

   $$A_{\max} = 27.5 \text{ V}, \quad A_{\min} = 12.5 \text{ Volts}$$
e. Determine the expression of the modulated AM wave **1 pt**
   $s(t) = A_c[1 + 0.375\cos(20000\pi t]\cos(1000000\pi t) \text{ Volts}$

Problem 6 (4 pts)

Consider the DSB-LC signal

$$s(t) = A_c[1 + 2(\cos^2 20\pi t + \cos 40\pi t)]\cos(\omega_c t)$$

with ω_c>>40π. Determine the percentage modulation of s(t).

Problem 6

$$s(t) = A_c\left[1 + 2\left(\frac{1}{2} + \frac{1}{2}\cos(40\pi t) + \cos(40\pi t)\right)\right]\cos(\omega_c t)$$

$$s(t) = A_c[1 + 1 + (3\cos(40\pi t))]\cos(\omega_c t)$$

$$s(t) = 2A_c\left[1 + \frac{3}{2}\cos(40\pi t)\right]\cos(\omega_c t)$$ **2 pts up to here**

From above:

$\mu = \frac{3}{2} = 1.5$, implies 150% modulation **2 pts**

Problem 7 (4 pts)

Consider a single-tome modulated DSB-LC signal. The percentage modulation of this signal is 120%. Determine the lowest amplitude value of the envelope of this DSB-LC signal. The highest value of the envelope is assumed to be equal to 2 Volts.

As it is an over-modulated signal, then the envelope is equal to 0 volts. Please note that envelope can not be negative. **If so, -3 pts**

Problem 8 (5 pts)

A sinusoidal carrier is frequency modulated by a 4 KHz sinusoidal wave resulting in an FM signal having a maximum frequency of 107.41 MHz and a minimum frequency of 107.196 MHz.

a. Determine the carrier frequency **2 pts**

$$f_c = \frac{f_{max} + f_{min}}{2} = 107.303 \text{ MHz}$$

b. Determine the number of impulses in the spectrum of s(t) within its bandwidth, above and not including the carrier frequency **3 pts**

$B_T = f_{max} - f_{min} = 214 \text{ KHz}$

Number of impulses on each side of the carrier = (107/4) = 26 Impulses. Total number of impulses excluding the carrier is: 52

Problem 9 (5 pts)

A base-band modulating signal m(t) has a bandwidth of 10Khz. m(t) is modulated using FM modulation techniques. Let the transmission bandwidth of the obtained FM signal be 180Khz and the frequency sensitivity k_f=5000$(\text{Vsec})^{-1}$. Determine the maximum value of m(t). Use Carson's rule for the computation of the transmission bandwidth.

Problem 9

$B_T = 2(\Delta f + f_m) = 2(\Delta f + 10) = 180 \text{ KHz}$

$\Delta f = 80KHz = 80000Hz$

But

$\Delta f = k_f a_m \Rightarrow a_m = 16 \text{ Volts}$ **-1 for no unit**

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<u>Problem 10 (5 pts)</u>

Consider a single tone FM signal with amplitude 10 Volts, frequency 5 KHz, and modulation index $\beta = 2$. Determine the ratio of the average power of the frequency components of this FM signal contained within its bandwidth to the total signal average power. Use Carson's rule for bandwidth computation.

$J_0(2) = 0.2239, J_1(2) = 0.5767, J_2(2) = 0.3528, J_3(2) = 0.1289,$

$J_4(2) = 0.0340, J_5(2) = 0.007, J_6(2) = 0.0012$

All got it true