AMERICAN UNIVERSITY OF BEIRUT
Department of Electrical and Computer Engineering
EECE340 Signals and Systems -Summer 2011

Section 4 -Prof Karameh

Problem Set 5

Out: Monday August 1, 2011 NOT Due:

Problem 1

Compute the Fourier Transform of each of the following signals:

a $x[n] = \left(\frac{1}{3}\right)^{|n|} u[-n-2]$.

b $x[n] = \sin(\frac{\pi n}{2}) + \cos(n)$.

The following are the Fourier transforms of discrete-time signals. Find the corresponding signals for each transform.

(c) $\tilde{X}(e^{j\Omega}) = 4e^{j2\Omega} - e^{j\Omega} + 5 + 3e^{-j3\Omega} - 16e^{-j12\Omega}$.

(c) $\tilde{X}(e^{j\Omega}) = \begin{cases} 1, & 0 \leq |\Omega| < \frac{\pi}{4}, \frac{\pi}{2} < |\Omega| \leq \pi; \\ 0, & \frac{\pi}{4} < |\Omega| < \frac{\pi}{2}. \end{cases}$

(e) $\tilde{X}(e^{j\Omega}) = \dfrac{1 + 3e^{-j\Omega}}{1 + \frac{1}{4}e^{-j\Omega}}$.

Problem 2

Consider an LTI system with the following unit sample response:

$$h[n] = \frac{\sin(\pi n/3)}{\pi n}$$

What is the output of this system corresponding to the *periodic* square wave input shown in figure 1

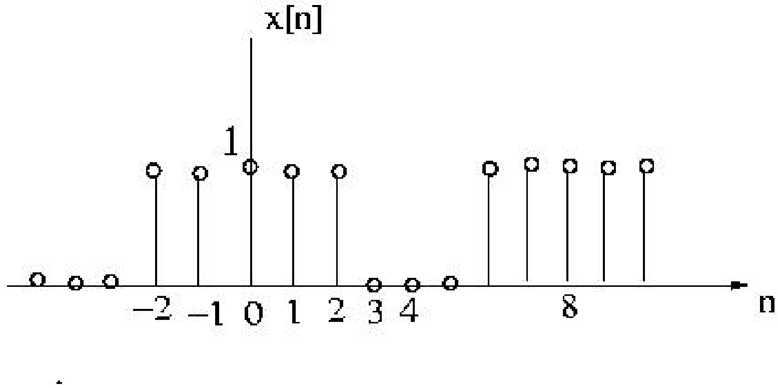

Figure 1: Problem 2

Problem 3

Consider the signal

$$x[n] = \sin\left(\frac{\pi n}{8}\right) - 2\cos\left(\frac{\pi n}{4}\right)$$

Suppose that this signal is the input to an LTI system whose impulse response is

$$h[n] = \frac{\sin(\pi n/8)\sin(\pi n/2)}{\pi^2 n^2}$$

Determine the corresponding output $y[n]$.

Problem 4

Consider a DT LTI system with the impulse response

$$h[n] = \left(\frac{1}{2}\right)^n u[n]$$

Determine, using DTFTs, the response to each of the following inputs:

(i) $x[n] = \left(\frac{3}{4}\right)^n u[n]$

(ii) $x[n] = (n+1)\left(\frac{1}{4}\right)^n u[n]$

(iii) $x[n] = (-1)^n u[n]$

Problem 5

Consider a system consisting of the cascade of two DT LTI systems with the frequency responses

$$H_1(e^{j\Omega}) = \frac{2 - e^{-j\Omega}}{1 + \frac{1}{2}e^{-j\Omega}}$$

and

$$H_2(e^{j\Omega}) = \frac{1}{1 - \frac{1}{2}e^{-j\Omega} + \frac{1}{4}e^{-j2\Omega}}$$

a) Find the difference equation describing the overall system.

b) Determine the impulse response of the overall system.

Problem 6

In this problem you will explore the computation of a signal frequency content using DFTs. In particular, you will be using the FFT implementation in Matlab to find the DFT.

(1) Recall that the DFT essentially computes the spectrum of a signal $x[n]$ at discrete steps in frequency. That is, given a signal $x[n]$ $0 \leq n \leq N-1$, then the N-pt DFT will compute

$$X[k] = \frac{1}{N} X(e^{j\Omega})|_{\Omega = \frac{2\pi}{N}}$$

(2) The matlab implementation (FFT) computes the DFT slightly differently than what you learned in class. If you type *help fft* at the command prompt in Matlab, you will get something like:

>> *help fft*

FFT Discrete Fourier transform.

FFT(X) is the discrete Fourier transform (DFT) of vector X. For matrices, the FFT operation is applied to each column. For N-D arrays, the FFT operation operates on the first non-singleton dimension.

FFT(X,N) is the N-point FFT, padded with zeros if X has less than N points and truncated if it has more.

For length N input vector x, the DFT is a length N vector X, with elements:

$$X(k) = \sum_{n=1}^{N} x(n) * exp^{-j\frac{2*\pi*(k-1)*(n-1)}{N}} \qquad 1 \leq k \leq N \qquad (1)$$

The inverse DFT (computed by IFFT) is given by

$$x[n] = \frac{1}{N} \sum_{k=1}^{N} X[k] * exp^{j\frac{2*\pi*(k-1)*(n-1)}{N}} \qquad 1 \leq n \leq N \qquad (2)$$

This is different in (a) the fraction $\frac{1}{N}$ is included in the IDFT formula and not in the DFT as we learned in class. This variation is common and acceptable as long as $\frac{1}{N}$ is included in either analysis or synthesis formulas. (The fraction is also sometimes split into two $\frac{1}{\sqrt{N}}$ in both formulas). In addition, since matlab cannot handle vectors with zero index such as $X[0]$, the summation is over $[1, N]$ and not $[0, N-1]$, which is used such that $X[1]$ corresponds to zero frequency.

We are now ready to start applying FFT.

Assume that we sampled a CT signal $x(t) = \cos(2\pi f_o t)$ (f_o Hz tone) at a rate of 200 Hz for 0.5 seconds. Assume that $f_o = 55Hz$ is unknown and we would like to estimate it using Matlab.

a- Create the corresponding DT signal $x[n]$. Plot this signal with respect to time ($1/Fs \leq t \leq 0.5$) sec. Verify that you have 100 samples. An efficient way to create the time vector here is:

$$\mathbf{t = 1/Fs : 1/Fs : 0.5};$$

b- Now use the function *fft* to compute the DFT of $x[n]$. That is, type $fx1 = fft(x)$. This will compute the 100 pt DFT of $x[n]$. Since the DFT $X[k]$ is in general a complex quantity, plot its absolute magnitude. *hint*: since you are plotting discrete points, use the function *stem* instead of *plot*. An efficient way of doing this is:

$$\mathbf{fx1 = abs(fft(x)); stem(fx1)};$$

c- Verify that $X[k]$ is plotted for $k = 1 \ldots N$. For what values of Ω (discrete frequency) does each sample k correspond to?

d- For what values of w (continuous frequencies) does each sample correspond to? Plot $X[k]$ versus w. Recall that the $w_s = 2\pi 200$ rad/sec sampling frequency corresponds to $\Omega = 2\pi$. That is, we are moving in the DFT as k=1:1:N; In the DTFT as $\mathrm{fs_d} = (1/\mathrm{N} : 1/\mathrm{N} : 1) * 2\pi$; and in the Continuous domain as $ws_c = (1/N : 1/N : 1) * 2\pi Fs$ (in rad/sec) or $fs_c = (1/N : 1/N : 1) * Fs$ in Hz.

e- We will now estimate the frequency f_o based on the DFT plot. Recall that the obtained peaks in $X[k]$ are proportional to samples of the spectrum $X(e^{j\Omega}$. Verify that the peak(s) you obtained is (are) close to the expected frequency of $2\pi f_o = 2\pi 55$. Is the result accurate? Why/why not?

f- We will now increase the size of the DFT to 200 points. That is, we are effectively zero-padding the signal $x[n]$ and then computing the DFT. For this, compute $fx2 = fft(x, 200)$. Plot fx2 versus w. How did your results change from before? Are they more accurate? Why/why not? Explain carefully.

g- What happens if the frequency we are trying to estimate is $f_o = 110$ Hz instead of 55 Hz ? Plot the DFT and explain what you obtain.

Problem 7

$y_1[n]$ and $y_2[n]$ are two real 8-pt sequences. You are given the first five points of their DFTs as

$$\tilde{Y}_1[k] = [2 \quad 1 \quad 0 \quad 1 \quad 2]$$

$$\tilde{Y}_2[k] = [0 \quad 1 \quad 2 \quad 0 \quad 0]$$

- Find the complete sequence $\tilde{Y}_1[k]$ and $\tilde{Y}_2[k]$. *Hint:* Exploit knowledge of DFT properties for real sequences.

- Find the sequence formed by the circular convolution of y_1 and y_2. That is, find

$$z[n] = y_1[n] \odot y_2[n].$$