AMERICAN UNIVERSITY OF BEIRUT
Department of Electrical and Computer Engineering
EECE340- Signals and Systems –Summer 2011
Pset 3 Solutions

Problem 1

(1)

$$H(Z)=\frac{1}{(1+\frac{1}{2}Z^{-1})(1-4Z^{-1})(1-3Z^{-1})}$$

$$\Rightarrow H(Z)=\frac{A}{1+\frac{1}{2}Z^{-1}}+\frac{B}{1-4Z^{-1}}+\frac{C}{1-3Z^{-1}}$$

$$A=\left.\frac{1}{(1-4Z^{-1})(1-3Z^{-1})}\right]_{Z^{-1}=-2}=\frac{1}{9.7}=\frac{1}{63}$$

$$B=\left.\frac{1}{(1+\frac{1}{2}Z^{-1})(1-3Z^{-1})}\right]_{Z^{-1}=1/4}=\frac{1}{\frac{9}{8}.\frac{1}{4}}=\frac{32}{9}$$

$$C=\left.\frac{1}{(1+\frac{1}{2}Z^{-1})(1-4Z^{-1})}\right]_{Z^{-1}=\frac{1}{3}}=\frac{1}{\frac{7}{6}.\frac{-1}{3}}=\frac{-18}{7}$$

H(Z) is a stable system => ROC includes unit disc.
Let H(Z) = H1(Z) + H2(Z) + H3(Z).

***For H1(Z) :we have p = -1/2 < 1, so p belongs to a causal system and**

$$h_1[n]=\frac{1}{63}\left(-\frac{1}{2}\right)^n u(n)$$

***For H2(Z): we have p = 4 >1, and the system is stable so p belongs to a non causal system**

$$h_2[n]=\frac{-32}{9}(4)^n u(-n-1)$$

*** For H3(Z): we have p = 3 > 1, and the system is stable, so p belongs to a non causal system**

$$h_3[n]=\frac{18}{7}(3)^n u(-n-1)$$

$$\Rightarrow h[n] = \frac{1}{63}\left(-\frac{1}{2}\right)^n u(n) - \frac{32}{9}(4)^n u(-n-1) + \frac{18}{7}(3)^n u(-n-1)$$

(2)

$$H(Z) = \frac{Z^3 - 2Z}{z-2} \Rightarrow$$

$$H(Z) = \frac{Z^2 - 2}{1 - 2Z^{-1}} = \frac{Z^2}{1-2Z^{-1}} - \frac{2}{1-2Z^{-1}}$$

H(Z) is anti-causal

$$h[n] = -(2)^{n+2} u(-(n+2)-1) - 2.(2)^n u(-n-1)$$

$$=> h[n] = -(2)^{n+2} u(-n-3) - (2)^{n+1} u(-n-1)$$

(3)

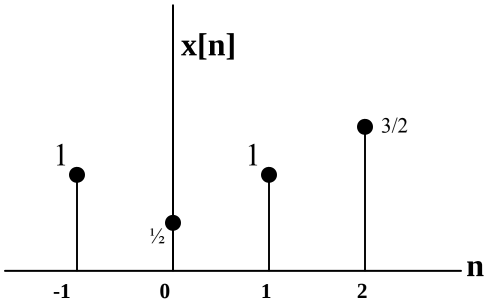
$$H(Z) = \frac{1}{(1-4Z^{-1})(1-\frac{1}{3}Z^{-1})}$$

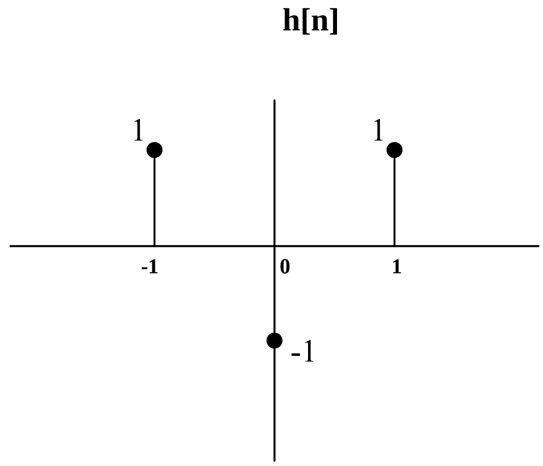
$$=> H(Z) = \frac{A}{1-4Z^{-1}} + \frac{B}{1-\frac{1}{3}Z^{-1}}$$

Problem 2:

a) $x[n] = \partial[n+1] + \frac{1}{2}\,\partial[n] + \partial[n-1] + {}^{3}/_{2}\; u[n-2]$

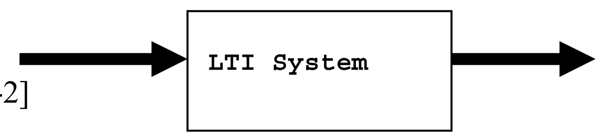
$h[n] = \partial[n+1] - \partial[n] + \partial[n-1]$





Method one:

$\partial[n+1] + \frac{1}{2}\,\partial[n]$
$+ \partial[n-1] + {}^{3}/_{2}\; u[n-2]$

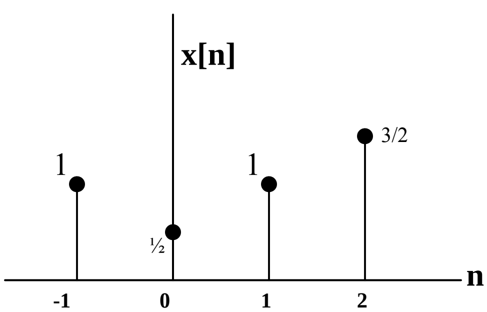


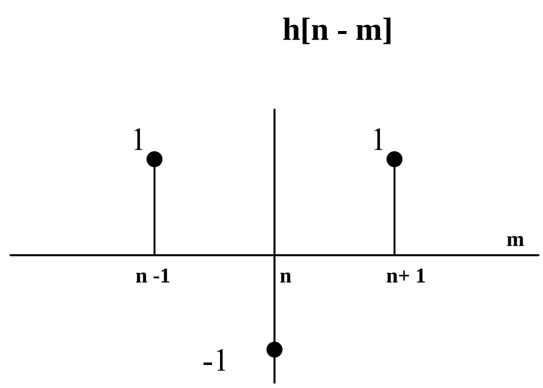
$\partial[n+1] - \partial[n] + \partial[n-1]$

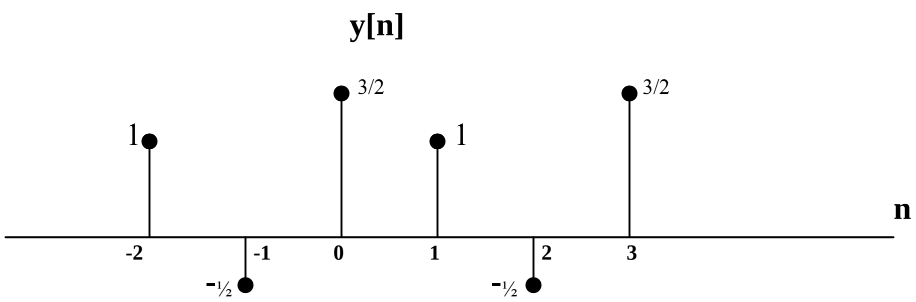
Method two:

Flip and Slide:

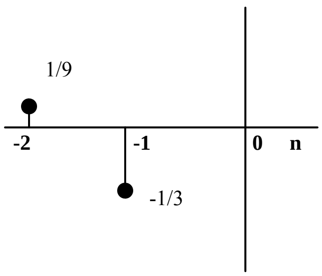
$$Y[n] = \sum_{m=0}^{n} x[m]\, h[n-m]$$

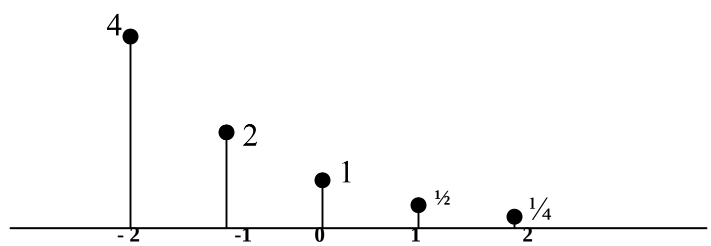






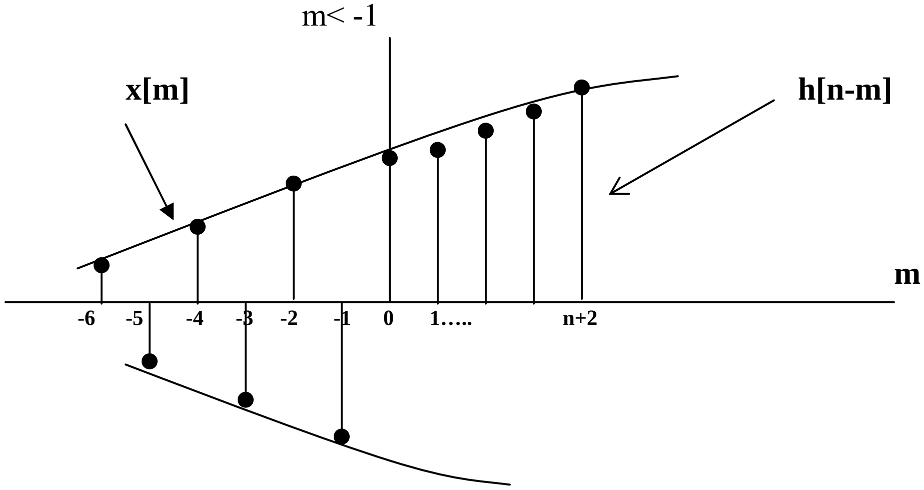
b) $x[n] = (-3)^n u[-n-1]$ $\qquad$ $h[n] = (½)^n u[n+2]$





$$Y[n] = \sum_{m=-\infty}^{\infty} x[m]\, h[n-m] = \sum_{m=-\infty}^{\infty} (-3)^n u[-m-1] (½)^{n-m} u[n-m+2]$$

$-m-1 > 0$, $m < -1$ $\qquad$ $n-m+2 > 0$, $m < n+2$



We have 2 regions we must consider:

For n+2 ≥ -1, the convolution summation is:

$$Y[n] = \sum_{m=-\infty}^{-1} (-3)^m (½)^{n-m} = (½)^n \sum_{m=1}^{\infty} (-{}^1/_6)^m = (½)^n \left[\frac{1}{1+{}^1/_6} - 1 \right] = - {}^1/_7 (½)^n, \quad n \geq -3$$

For n+2 < -1 , the convolution summation is:

$$Y[n] = \sum_{m=-\infty}^{n+2} (-3)^m (½)^{n-m} = (½)^n \sum_{m=-n-2}^{\infty} (-{}^1/_6)^m = (½)^n \left[\sum_{m=0}^{\infty} (-{}^1/_6)^m - \sum_{m=0}^{-n-3} (-{}^1/_6)^m \right]$$

$$= (½)^n \left[\frac{1}{1+{}^1/_6} - \frac{1 -({}^1/_6)^{-n-2}}{1 + {}^1/_6} \right]$$

$y[n] = (½)^n [{}^6/_7 - {}^6/_7 + {}^6/_7 (- {}^1/_6)^{-n-2}] = (½)^n [{}^6/_7 (- {}^1/_6)^{-n-2}] = ({}^6/_7)(36)(-3)^n, \; n< -3$

total $y[n] = -{}^1/_7 (½)^n u [n+3] + {}^{216}/_7 (-3)^n u [-n -4]$

Now a & b verified with Z-transforms:

a) $X(z) = z + ½ + z^{-1} + {}^3/_2 z^{-2}$ $\qquad$ $H(z) = z -1+ z^{-1}$

$$Y(z) = (z + ½ + z^{-1} + {}^3/_2 z^{-2}) (z -1+ z^{-1})$$
$$= z^2 - z +1 + ½ z - ½ + ½ z^{-1} +1 - z^{-1} + z^{-2} + {}^3/_2 z^{-1} - {}^3/_2 z^{-2} + {}^3/_2 z^{-3}$$
$$= z^2 - ½ z + {}^3/_2 + z^{-1} - ½ z^{-2} + {}^3/_2 z^{-3}$$

$y[n] = \partial[n+2] - ½ \partial[n+1] + {}^3/_2 \partial[n] + \partial[n-1] - ½ \partial[n-2] + {}^3/_2 \partial[n-3]$

b) $X(z) = -1 / (1+3z^{-1}) \quad |z| < 3$ $\qquad$ $H(z) = Z^{-1}\{ h[n]\} = Z^{-1}\{ (½)^{-2}(½)^{n+2} u[n+2]$

$$H(z) = \frac{4 z^2}{1 - ½ z^{-1}}, \; |z| > ½$$

$$Y(z) = \frac{- 4 z^2}{(1+ 3z^{-1})(1- ½ z^{-1})}, \quad ½ < |z| < 3$$

$$Y(z) = \frac{A}{(1+ 3z^{-1})} + \frac{B}{(1- ½ z^{-1})}, \quad ½ < |z| < 3$$

$$\text{with } A = \left. \frac{-4}{1- ½ z^{-1}} \right|_{z^{-1} = -1/3} = - 24 / 7$$

$$B = \left. \frac{-4}{1+ 3 z^{-1}} \right|_{z^{-1} = 2} = -4 / 7$$

$$Y(z) = \frac{-24/7}{(1+3z^{-1})} + \frac{-4/7}{(1-\frac{1}{2}z^{-1})}, \qquad \tfrac{1}{2} < |z| < 3$$

$$y'[n] = {}^{24}/_{7}\,(-3)^{n}\,u[-n-1] + {}^{-4}/_{7}\,(\tfrac{1}{2})^{n}\,u[n]$$
$$y[n] = y'[n+2] = {}^{24}/_{7}\,(-3)^{n+2}\,u[-n-3] + {}^{-4}/_{7}\,(\tfrac{1}{2})^{n+2}\,u[n+2]$$

Problem 3

(a)

$$h_2(n) = u(n) - u(n-2) = \delta(n) + \delta(n-1)$$

$$h_3 = h_2 * h_2 = (\delta(n) + \delta(n-1)) * (\delta(n) + \delta(n-1))$$

$$= \delta[n] + 2\delta(n-1) + \delta(n-2)$$

$$g(n) = h_1(n) * h_3(n) = h_1(n) * (\delta(n) + 2\delta(n-1) + \delta[n-2])$$

$$g(n) = h_1(n) + 2h_1(n-1) + h_1(n-2)$$

system is causal $\rightarrow h_1(n) = g(n) = 0 \quad n < 0$

$$g[0] = h_1[0] = 1$$

$$g(1) = h_1(1) + 2h_1[0] \Rightarrow h_1(1) = 5 - 2 = 3$$

$$g(2) = h_1(2) + 2h_1(1) + h_1(0) \rightarrow h_1(2) = 3 - 6 - 1 = 1$$

Problem 4:

y(t)=x(t)*h(t).

Using Flip and Slide, we observe different regions of time that give different areas as one graph (x(t) for example) moves along the t-axis and getting into intersection with the graph of the other signal (h(t)).

We get the following output signal:

$$y(t)=\begin{cases} 0 & t<-1 \\ 1-t & -1<t<0 \\ -1+4t & 0<t<1 \\ -t+4 & 1<t<2 \\ -3t+8 & 2<t<3 \\ t-4 & 3<t<4 \\ 8-2t & 4<t<5 \\ t-7 & 5<t<7 \\ 0 & t>7 \end{cases}$$

b) $x(t)=e^{-2(2-t)}u(2-t)$ and $h(t)=4e^{-t}u(t-1)$

$$y(t)=x(t)*h(t)=\int_{.-\infty}^{\infty} e^{-2(2-s)}u(2-s).4e^{-(t-s)}u(t-s-1)=4e^{-4-t}\int_{-\infty}^{\infty} e^{3s}u(2-s)u(t-s-1)$$

$$u(2-s).u(t-s-1)=\begin{cases} u(2-s) & t-1>2 \\ u(t-s-1) & t-1<2 \end{cases}$$

then

$$y(t)=4e^{-4-t}\left[\left(\int_{-\infty}^{2} e^{3s}ds\right)u(t-3)+\left(\int_{-\infty}^{t-1} e^{3s}ds\right)u(3-t)\right]=\frac{4}{3}e^{-4-t}\left(e^{6}u(t-3)+e^{3t-3}u(3-t)\right)$$

$$\Rightarrow y(t)=\frac{4}{3}e^{2-t}u(t-3)+\frac{4}{3}e^{2t-7}u(3-t)$$

c)

$$
\begin{aligned}
&\text{for } t+2<0 && t<-2\\
& && m(t)=0\\
&\text{for } t+2<2 && -2\le t<0\\
& && m(t)=1\\
&\text{for } t-1<0 && 0\le t<1\\
& && m(t)=\int_0^t d\tau+2=t+2\\
&\text{for } t<2 && 1\le t<2\\
& && m(t)=\int_{t-1}^t d\tau+2=3
\end{aligned}
$$

Problem 5

a) $X(s)=\dfrac{s+5}{s-4}$

x(t)=0, t>0 => ROC: Re(s)<4

$$y(t)=-\frac{2}{3}e^{4t}u(-t)+\frac{1}{3}e^{-2t}u(t)$$

$$=> Y(S)=\frac{2/3}{s-4}+\frac{1/3}{s+2}$$

ROC: -2 < Re(s)<4

$$H(s)=\frac{Y(s)}{X(s)}=\frac{2/3}{s+5}+\frac{1/3\,(s-4)}{(s+2)(s+5)}$$

=> $=> H(s)=\dfrac{s}{(s+5)(s+2)}$

$$ROC:\mathrm{Re}(s)>-2$$

$$H(s)=\frac{s}{(s+5)(s+2)}$$

b)

$$=> H(s)=\frac{5/3}{s+5}+\frac{-2/3}{s+2}$$

and ROC is Re(s)>-2

Problem 6

(a) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} - 3y(t) = x(t)$

$(s^2 - 2s - 3)Y(s) = X(s) \Rightarrow H(s) = \frac{1}{s^2-2s-3} = \frac{1}{(s-3)(s+1)}$

$H(s) = \frac{\frac{1}{4}}{s-3} + \frac{-\frac{1}{4}}{s+1}$

a-1) system stable

$\Rightarrow h(t) = -\frac{1}{4}e^{3t}u(-t) - \frac{1}{4}e^{-t}u(t)$

(a-2) system neither causal nor stable:

$h(t) = +\frac{1}{4}e^{3t}u(-t) + \frac{1}{4}e^{-t}u(-t)$

is neither

(b)

$$x_0(t) = 2e^{-3t}u(t-1) \longrightarrow y_0(t)$$

$$\frac{dx_0}{dt} = x_1(t) \longrightarrow y_1(t) = -3y_0(t) + e^{-2(t-1)}u(t-1)$$

$$x_1(t) = \frac{dx_0}{dt} = -6e^{-3t}u(t-1) + 2e^{-3t}\delta(t-1)$$

$$x_1(t) = -3x_0(t) + 2e^{-3t}\delta(t-1)$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$y_1(t) \qquad -3y_0(t) \qquad 2e^{-3}h(t-1) \qquad \text{by linearity}$$

$$\therefore\ 2e^{-3}h(t-1) = e^{-2(t-1)}u(t-1)$$

$$\Rightarrow\ h(t) = \frac{1}{2e^{-3}}e^{-2t}u(t) = \frac{1}{2}e^{-2t+3}u(t)$$

Problem 7

a) $X(\omega) = |X(\omega)| \, e^{j\angle X(\omega)} = \begin{cases} -\omega \, e^{-3j\omega} & , -1 \le \omega \le 0 \\ \omega \, e^{-3j\omega} & , 0 \le \omega \le 1 \end{cases}$

$$\Rightarrow x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) \, e^{j\omega t} \, d\omega = \frac{1}{2\pi} \left[\int_{-1}^{0} -\omega \, e^{j\omega(t-3)} \, d\omega + \int_{0}^{1} \omega \, e^{j\omega(t-3)} \, d\omega \right.$$

$$= \frac{1}{2\pi} \left[\frac{-1}{-(t-3)^2} \, e^{j\omega(t-3)} \left(j\omega(t-3) - 1 \right) \Big|_{-1}^{0} + \frac{1}{-(t-3)^2} \, e^{j\omega(t-3)} \left(j\omega(t-3) - 1 \right) \Big|_{0}^{1} \right.$$

$$= \frac{1}{2\pi} \left(\frac{-1}{(t-3)^2} - \frac{1}{(t-3)^2} \, e^{-j(t-3)} \left(-j(t-3) - 1 \right) + \frac{1}{(t-3)^2} - \frac{1}{(t-3)^2} \, e^{j(t-3)} \left(j(t-3) \right. \right.$$

$$= \frac{1}{2\pi} \left(\frac{+1}{(t-3)^2} \right) \left[-(t-3) j \left(e^{j(t-3)} - e^{-j(t-3)} \right) + \left(e^{+j(t-3)} + e^{-j(t-3)} \right) \right]$$

$$= \frac{+1}{\pi (t-3)^2} \left[(t-3) \sin(t-3) + \cos(t-3) \right]$$

$$= \frac{\sin(t-3)}{\pi (t-3)} + \frac{\cos(t-3)}{(t-3)^2} \qquad \frac{-1}{\pi (t-3)^2}$$

b)

$$X(\omega) = \begin{cases} \omega - 1, & 1 \leq \omega < 2 \\ \text{sign}(\omega), & 2 < |\omega| \leq 3 \\ \omega + 1, & -2 \leq \omega < -1 \\ 0, & \text{else} \end{cases}$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega = \frac{1}{2\pi} \left[-\int_{-3}^{-2} e^{j\omega t} d\omega + \int_{-2}^{-1} (\omega+1) e^{j\omega t} d\omega + \int_{1}^{2} (\omega-1) e^{j\omega t} d\omega + \int_{2}^{3} e^{j\omega t} d\omega \right]$$

$$= \frac{1}{2\pi} \left[-\frac{1}{jt} e^{j\omega t} \Big|_{-3}^{-2} + \frac{\omega+1}{jt} e^{j\omega t} \Big|_{-2}^{-1} - \int_{-2}^{-1} \frac{e^{j\omega t}}{(jt)^2} d\omega + \frac{\omega-1}{jt} e^{j\omega t} \Big|_{1}^{2} - \int_{1}^{2} \frac{e^{j\omega t}}{(jt)^2} d\omega + \frac{1}{jt} e^{j\omega t} \Big|_{2}^{3} \right]$$

$$= \frac{1}{2\pi} \left[\frac{-1}{jt} \left(e^{-2jt} - e^{-3jt} \right) + \frac{1}{jt} e^{-2jt} - \frac{1}{(jt)^2} \left(e^{-jt} - e^{-2jt} \right) + \frac{1}{jt} e^{2jt} - \frac{1}{(jt)^2} \left[e^{2jt} - e^{jt} \right] + \frac{1}{jt} \left[e^{j2t} - e^{3jt} \right] \right]$$

$$= \frac{1}{2\pi} \left[\frac{1}{jt} e^{-3jt} - \frac{1}{(jt)^2} \left[e^{-jt} - e^{jt} \right] - \frac{1}{(jt)^2} \left(e^{2jt} - e^{-2jt} \right) + \frac{1}{jt} \left(e^{2jt} - e^{j2t} \right) + \frac{1}{jt} e^{j3t} \right]$$

$$= \frac{1}{2\pi} \left[\frac{1}{jt} \left(e^{3jt} + e^{-3jt} \right) + \frac{2}{jt^2} \sin t - \frac{2}{jt^2} \sin 2t \right]$$

$$\Rightarrow x(t) = \frac{1}{j\pi t} \cos 3t + \frac{1}{j\pi t^2} (\sin t - \sin 2t)$$

$$X(\omega) = e^{-2|\omega|} = \begin{cases} e^{-2\omega}, & \omega \geq 0 \\ e^{2\omega}, & \omega \leq 0 \end{cases}$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega = \frac{1}{2\pi}\left[\int_{-\infty}^{0} e^{\omega(2+jt)}\, d\omega + \int_{0}^{\infty} e^{\omega(jt-2)}\, d\omega\right.$$

$$= \frac{1}{2\pi}\left[\frac{1}{2+jt}\, e^{\omega(2+jt)}\Big|_{-\infty}^{0} + \frac{1}{jt-2}\, e^{\omega(-2+jt)}\Big|_{0}^{\infty}\right]$$

$$= \frac{1}{2\pi}\left(\frac{1}{2+jt} - \frac{1}{jt-2}\right) = \frac{1}{2\pi}\,\frac{2-jt-(-2-jt)}{4+t^2} = \frac{2}{\pi(4+t^2)}$$

$$X(\omega) = \begin{cases} \cos(4\omega), & |\omega| < \frac{\pi}{4} \\ 0, & \text{else} \end{cases}$$

$$x(t) = \frac{1}{2\pi}\int_{-\frac{\pi}{4}}^{\pi/4} \cos(4\omega)\, e^{j\omega t}\, d\omega = \frac{1}{4\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(e^{j4\omega} + e^{-j4\omega}\right) e^{j\omega t}\, d\omega$$

$$= \frac{1}{4\pi}\left[\frac{1}{j(t+4)}\, e^{(t+4)j\omega} + \frac{1}{(t-4)j}\, e^{j(t-4)\omega}\right]_{-\frac{\pi}{4}}^{\pi/4}$$

$$= \frac{1}{2\pi}\left[\frac{1}{2j(t+4)}\left(e^{j(t+4)\frac{\pi}{4}} - e^{-j(t+4)\frac{\pi}{4}}\right) + \frac{1}{2j(t-4)}\left(e^{j(t-4)\frac{\pi}{4}} - e^{-j(t-4)\frac{\pi}{4}}\right)\right]$$

$$= \frac{1}{2\pi}\left(\frac{1}{t+4}\sin\left(\frac{\pi}{4}t+\pi\right) + \frac{1}{t-4}\sin\left(\frac{\pi}{4}t-\pi\right)\right]$$

$$= \frac{1}{2\pi}\left(\frac{-1}{t+4}\sin\left(\frac{\pi}{4}t\right) + \frac{-1}{t-4}\sin\left(\frac{\pi}{4}t\right)\right]$$

$$= \frac{t}{\pi(t^2-16)}\sin\left(\frac{\pi}{4}t\right).$$

e) $X(\omega) = \dfrac{2 \sin \omega}{\omega (j\omega + 2)}$

Let $x_1(t) = \text{rect}\left[\frac{t}{2}\right] \Rightarrow X_1(\omega) = \dfrac{2 \sin \omega}{\omega}$

$x_2(t) = e^{-2t} u(t) \Rightarrow X_2(\omega) = \dfrac{1}{j\omega + 2}$

$$\Rightarrow x(t) = x_1(t) * x_2(t) = \int_{-\infty}^{\infty} \text{rect}\left(\frac{\tau}{2}\right) \cdot e^{-2(t-\tau)} u(t-\tau)\, d\tau$$

- if $t < -1$, $\tau < t \Rightarrow \tau < -1 \Rightarrow \text{rect}\left(\frac{\tau}{2}\right) = 0 \Rightarrow x(t) = 0$
- if $t > 1 \Rightarrow x(t) = \int_{-1}^{1} e^{-2(t-\tau)}\, d\tau = \frac{1}{2} e^{2\tau - 2t}\Big|_{-1}^{1} = \frac{1}{2} e^{2-2t} - \frac{1}{2} e^{-2-2t}$
- if $-1 < t < 1 \Rightarrow x(t) = \int_{-1}^{t} e^{-2(t-\tau)}\, d\tau = \frac{1}{2} e^{2\tau - 2t}\Big|_{-1}^{t} = \frac{1}{2} - \frac{1}{2} e^{-2-2t}$

$$\Rightarrow x(t) = \left(\frac{1}{2} - \frac{1}{2} e^{-2-2t}\right) \text{Rect}\left[\frac{t}{2}\right] + \frac{1}{2} e^{-2t} \left(e^{2} - e^{-2}\right) u(t-1)$$

f) $X(\omega) = \dfrac{d}{d\omega}\left[4 \sin(4\omega) \dfrac{\sin 2\omega}{\omega}\right] = \dfrac{d}{d\omega}\left[\sin(4\omega) \cdot \dfrac{8 \sin 2\omega}{2\omega}\right]$

$= \dfrac{d}{d\omega}\left[\dfrac{e^{j4\omega} - e^{-j4\omega}}{2j} \cdot \dfrac{8 \sin 2\omega}{2\omega}\right] = j \dfrac{d}{d\omega}\left[\left(e^{-j4\omega} - e^{j4\omega}\right) \cdot \dfrac{4 \sin 2\omega}{2\omega}\right]$

let $X_1(\omega)$ s.t. $X(\omega) = j \dfrac{d}{d\omega}\left(X_1(\omega)\right) \Rightarrow x(t) = t\, x_1(t)$

$X_1(\omega) = \left(e^{-j4\omega} - e^{j4\omega}\right) \dfrac{4 \sin 2\omega}{2\omega}$

Using time-shift property & $F^{-1}\left(\dfrac{4 \sin 2\omega}{2\omega}\right) = \text{Rect}\left(\dfrac{t}{4}\right)$

we get $x_1(t) = \text{Rect}\left[\dfrac{t-4}{4}\right] - \text{Rect}\left[\dfrac{t+4}{4}\right] = \begin{cases} -1, & -2 \le t \le -6 \\ 1, & 2 \le t \le 6 \\ 0, & \text{else} \end{cases}$

$$\Rightarrow x(t) = t\, x_1(t) = \begin{cases} -t, & -2 \le t \le -6 \\ t, & 2 \le t \le 6 \\ 0, & \text{else.} \end{cases}$$

Problem 8

(a) $x(t) = \begin{cases} 1-t^2, & 0 < t < 1; \\ 0, & \text{else.} \end{cases}$

$$X(w) = \int_{-\infty}^{\infty} x(t) e^{-jwt}\, dt$$
$$= \int_0^1 (1-t^2) e^{-jwt}\, dt$$
$$= \int_0^1 e^{-jwt}\, dt - \int_0^1 t^2 e^{-jwt}\, dt$$
$$= \frac{-1}{jw}(e^{-jw}-1) - \left(\frac{t^2}{-jw} e^{-jwt}\right] + \int_0^1 \frac{2t}{jw} e^{-jwt}\, dt)$$
$$= \frac{-1}{jw}(e^{-jw}-1) + \frac{1}{jw} e^{-jw} + [(\frac{2t}{w^2} e^{-jwt}\bigg] - \int_0^1 \frac{2}{(jw)^2} e^{-jwt}\, dt$$
$$= \frac{1}{jw} + \frac{2}{w^2} e^{-jw} + \frac{2}{(jw)^3}(e^{-jw}-1)$$
$$= \frac{1}{jw} + \frac{2}{w^2} e^{-jw} + \frac{2}{(jw)^3} e^{-jw} - \frac{2}{(jw)^3}$$

(b) $x(t) = e^{-3|t|}\sin 2t$

$$X(w) = \int_{-\infty}^{+\infty} x(t)e^{-jwt}dt$$

$$= \int_{-\infty}^{0} e^{-jwt}e^{3t}\sin 2t + \int_{0}^{\infty} e^{-jwt}e^{-3t}\sin 2t$$

$$= \int_{-\infty}^{0} e^{-jwt}e^{3t}\frac{e^{2jt}-e^{-2jt}}{2j}dt + \int_{0}^{\infty} e^{-jwt}e^{-3t}\frac{e^{2jt}-e^{-2jt}}{2j}dt$$

$$= \frac{1}{2j}\left(\int_{-\infty}^{0} e^{-jwt}e^{(2j+3)t} - e^{-jwt}e^{(-2j+3)t}dt + \int_{0}^{\infty} e^{-jwt}e^{(2j-3)t} - e^{-jwt}e^{(-2j-3)t}dt\right)$$

$$= \frac{1}{2j}\left(\frac{1}{3+j(2-w)} - \frac{1}{3+j(-2-w)}\right) + \frac{1}{2j}\left(\frac{1}{3-j(2-w)} - \frac{1}{3-j(-2-w)}\right)$$

$$= \frac{1}{2j}\left(\frac{1}{3+j(2-w)} + \frac{1}{3-j(2-w)}\right) - \frac{1}{2j}\left(\frac{1}{3+j(-2-w)} + \frac{1}{3-j(-2-w)}\right)$$

$$= \frac{1}{2j}\left(\frac{6}{9+(2-w)^2} - \frac{6}{9+(-2-w)^2}\right)$$

(c) $x(t) = \left(\frac{\sin(t)}{\pi t}\right) * \frac{d}{dt}\left[\left(\frac{sin(2t)}{\pi}\right)\right]$

$$X(w) = \int_{-\infty}^{\infty} x(t)e^{-jwt}dt$$

$$x(t) = x1(t) * x2(t)$$

$$=> X(w) = X1(w).X2(w)$$

$$x1(t) = \frac{1}{\pi}\sin c(t) => X1(w) = \frac{1}{\pi}\pi \operatorname{Re}ct(\frac{w}{2}) = \operatorname{Re}ct(\frac{w}{2})$$

and

$$x2(t) = \frac{1}{\pi}\frac{d}{dt}\sin(2t) => X2(w) = \frac{1}{\pi}jw(F(\sin(2t)))$$

$$and\ \sin 2t = \frac{e^{j2t}-e^{-j2t}}{2j}$$

X(w) = Y(w).G(w)
Y(w)= rect (w/2)

g (t) = df/dt => G(w) = jw F(w) where f)t) = sin(2t)/π

⇨ G(w) = w[δ(w-2) + δ(w+2)]

Therefore,
⇨ X(w) = rect(w/2). w[δ(w-2) + δ(w+2)]
⇨ X(w) =0

(d) $x(t) = te^{-3|t-1|}$

let

$x1(t) = te^{-3|t|}$

$=> if t < 0, x1(t) = e^{3t}$

$if t > 0, x1(t) = e^{-3t}$

$$=> X1(w) = \int_{-\infty}^{0} e^{(3-jw)t} dt + \int_{0}^{\infty} e^{(-3-jw)t} dt$$

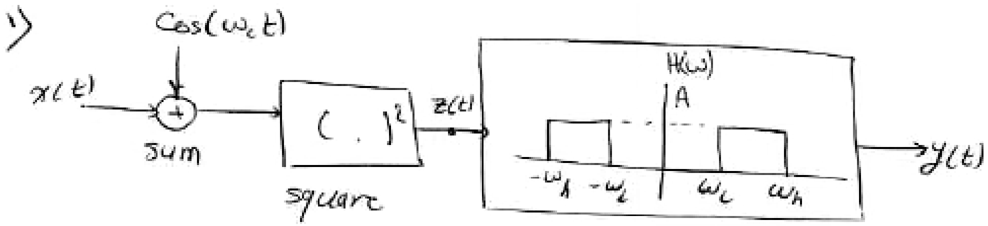
$$= \frac{1}{3-jw} + \frac{1}{3+jw}$$

$$=> X(w) = j\frac{d}{dw}(e^{-jw}\frac{1}{3-jw} + \frac{1}{3+jw})$$

$$= \frac{6e^{-jw}(9+w^2-2jw)}{(9+w^2)^2}$$

Problem 9

1)



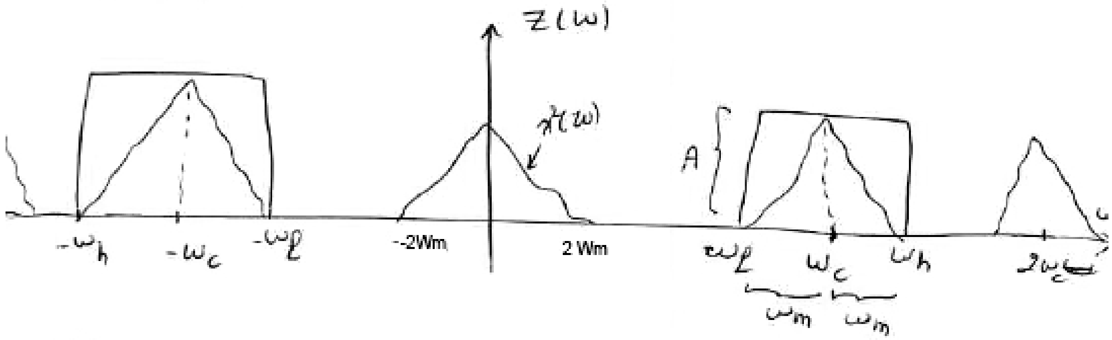
$$z(t) = [x(t) + \cos(\omega_c t)]^2$$

$$= x^2(t) + \cos^2(\omega_c t) + 2x(t)\cos(\omega_c t).$$

$$= x^2(t) + \frac{1}{2}\cos(2\omega_c t) + \frac{1}{2} + 2x(t)\cos(\omega_c t).$$

but $y(t) = x(t)\cos(\omega_c t)$.

∴ The filter has to keep the signal with frequency near ω_c.



From the above graph ⇒ $\omega_h = \omega_c + \omega_m$

$\omega_l = \omega_c - \omega_m$

$A = \frac{1}{2}$. (the signal has to be reduced to half in its amplitude)

2) Notice that W_c should be greater than 3Wm in order for two signals $x^2(t)$ & $x(t)\cos(\omega_c t)$ not to interfere.

⇒ Wc > 3 Wm