AMERICAN UNIVERSITY OF BEIRUT
Department of Electrical and Computer Engineering
EECE340 Signals and Systems -Summer 2011

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Problem Set 3

Out: Thursday July 14, 2011 Due: Thurs July 21, 2011

Work individually and write your complete solutions on clean paper. Start early, no extensions will be allowed.

Problem 1

Find the unit sample response $h[n]$ for each of the system functions given below:

$$(1) \qquad \tilde{H}(z) = \frac{1}{(1+\frac{1}{2}z^{-1})(1-4z^{-1})(1-3z^{-1})} \qquad \tilde{H}(z) \text{ is stable system}$$

$$(2) \qquad \tilde{H}(z) = \frac{z^3-2z}{z-2}, \qquad \tilde{H}(z) \text{ is an anti-causal system}$$

$$(3) \qquad \tilde{H}(z) = \frac{1}{(1-4z^{-1})(1-\frac{1}{3}z^{-1})}, \qquad \text{R.O.C.} \qquad \frac{1}{3} < |z| < 4$$

.

Problem 2

For each of the following pairs of input sequences $x[n]$ and unit sample responses $h[n]$, find the output $y[n] = x[n] \star h[n]$ using **convolution**. Then verify the result using the Z-transform.

a) $x[n] = \delta[n+1] + \frac{1}{2}\delta[n] + \delta[n-1] + \frac{3}{2}\delta[n-2]$, $\qquad h[n] = \delta[n+1] - \delta[n] + \delta[n-1]$.

b) $x[n] = (-3)^n u[-n-1]$, $\qquad h[n] = (\frac{1}{2})^n u[n+2]$.

Problem 3

Consider the cascade interconnection of three LTI causal DT systems shown in figure 1.
The unit sample response $g[n]$ of the equivalent overall system is shown in the same figure. The unit sample response $h_2[n]$ is given by

$$h_2[n] = u[n] - u[n-2]$$

Find $h_1[n]$ at $n = 2$. Clearly show your work.

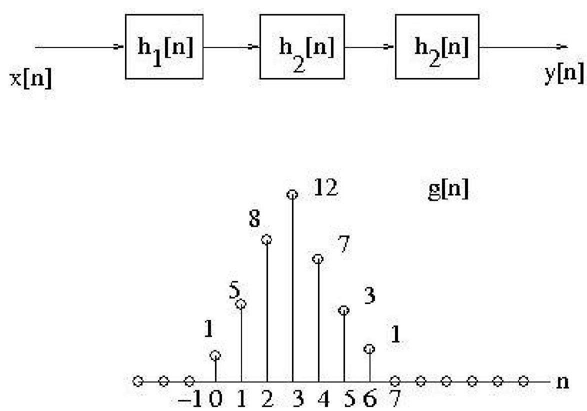

Figure 1:

Problem 4

Using *CT convolution*, compute the output of the system $y(t)$ for the given pairs of input $x(t)$ and impulse response $h(t)$.

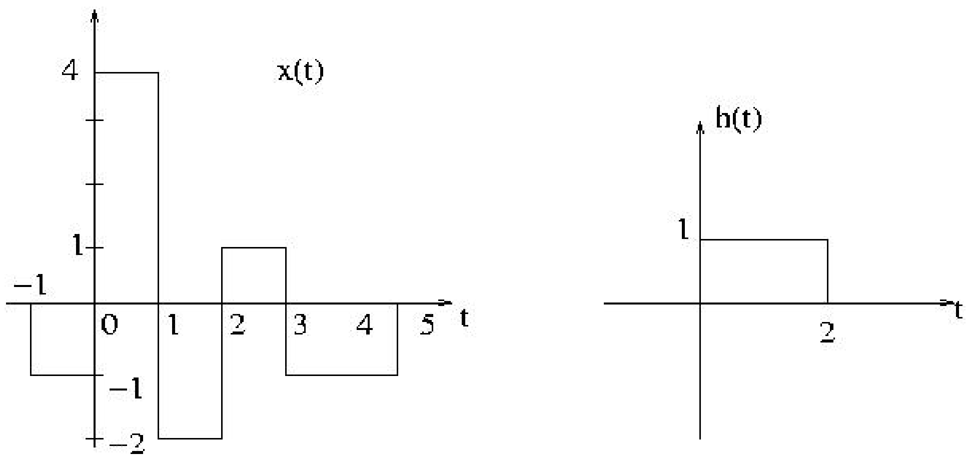

Figure 2: System Block Diagram Problem 4 a

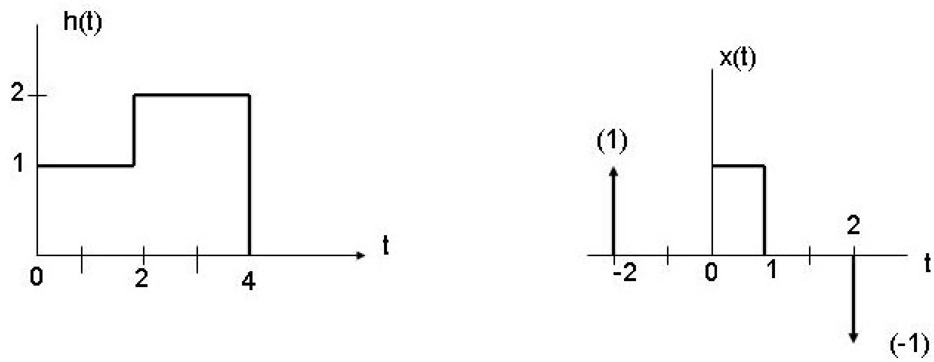

Figure 3: System Block Diagram Problem 4 c

a) $x(t)$, $h(t)$ as shown in figure 2.

b) $x(t) = e^{(-4+2t)}u(2-t)$ and $h(t) = 4e^{-t}u(t-1)$.

c) $x(t)$ and $h(t)$ as shown in figure 3.

Problem 5

An CT LTI system has an input $x(t)$, a system function $H(s)$ and an output $y(t)$. We are given the following:

$$X(s) = \frac{s+5}{s-4}$$

$x(t) = 0$, $t > 0$ and the corresponding output

$$y(t) = -\frac{2}{3}e^{4t}u(-t) + \frac{1}{3}e^{-2t}u(t).$$

a) Determine $H(s)$ and its region of convergence.

b) Determine $h(t)$, the impulse response of the system.

Problem 6

(a) Consider a CT LTI system described by the following differential equation

$$\frac{d^2y}{dt^2} - 2\frac{dy(t)}{dt} - 3y(t) = x(t)$$

(a-1) Determine the impulse response $h(t)$ if this system stable.

(a-2) Determine the impulse response $h(t)$ if this system is neither causal nor stable.

(b) Consider a CT LTI system S with inputs $x(t)$ and output $y(t)$.

If an input $x_o(t) = 2e^{-3t}u(t-1)$ is applied, the corresponding (unknown) output is $y_o(t)$. If an input $x_1(t) = \frac{dx_o(t)}{dt}$, then the corresponding output is $y_1(t) = -3y_o(t) + e^{-2(t-1)}u(t-1)$.

Find the impulse response $h(t)$ of this system.

Problem 7

Determine the Continuous time signal corresponding to each of the following Fourier Transforms

(a) $X(w)$ is given by the magnitude and phase plots of figure 4

(b) $X(w) = \begin{cases} w-1, & 1 \le w < 2; \\ sign(w), & 2 < |w| \le 3; \\ w+1, & -2 \le w < -1; \\ 0, & \text{else.} \end{cases}$

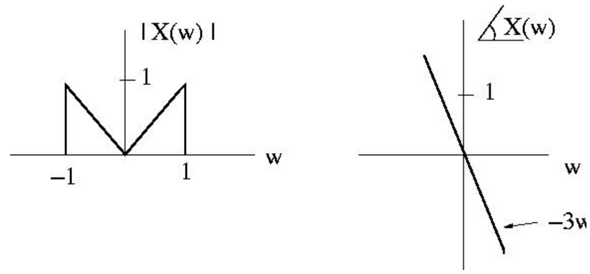

Figure 4: Problem 7

(c) $X(w) = e^{-2|w|}$

(d) $X(w) = \begin{cases} \cos(4w), & |w| < \frac{\pi}{4}; \\ 0, & \text{else.} \end{cases}$

(e) $X(w) = \dfrac{2sinw}{w(jw+2)}$.

(f) $X(w) = \dfrac{d}{dw}\left[4sin(4w)\dfrac{sin(2w)}{w}\right]$

Problem 8

Compute the Fourier Transform for the following CT signals

(a) $x(t) = \begin{cases} 1-t^2, & 0 < t < 1; \\ 0, & \text{else.} \end{cases}$

(b) $x(t) = e^{-3|t|}\sin 2t$

(c) $x(t) = \left(\dfrac{\sin(t)}{\pi t}\right) * \dfrac{d}{dt}\left[\left(\dfrac{sin(2t)}{\pi}\right)\right]$

(d) $x(t) = te^{-3|t-1|}$

Problem 9

Most of the modulation and demodulation schemes we discussed so far used multipliers as basic elements. Multipliers are often difficult to implement in practice. Therefore, many practical systems use a nonlinear element. In this problem we will be using a squaring device, as shown in figure 5.

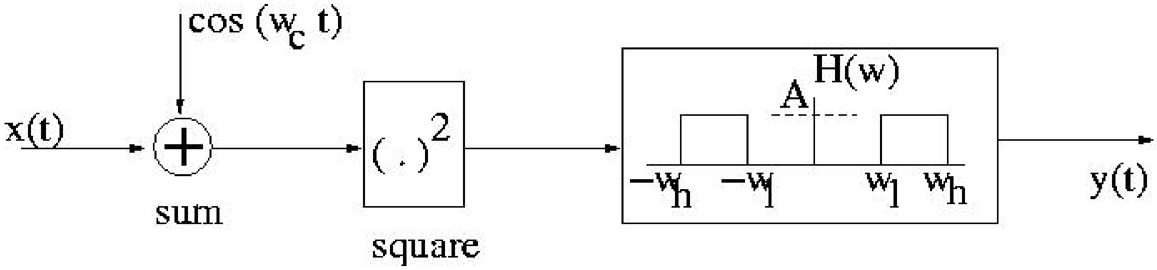

Figure 5: Problem 9

Assume $x(t)$ is a bandlimited signal such that $X(w) = 0$, $|w| > w_m$.

1. Determine the bandpass filter properties (w_l, w_h and A) such that $y(t)$ is a modulated version of $x(t)$. That is, $y(t) = x(t) cos w_c t$.

2. Specify the constraints, if any, on w_c, w_m.