MECH 314

Quiz 1 Fall 2018

16 October 2018 Dr Oweis

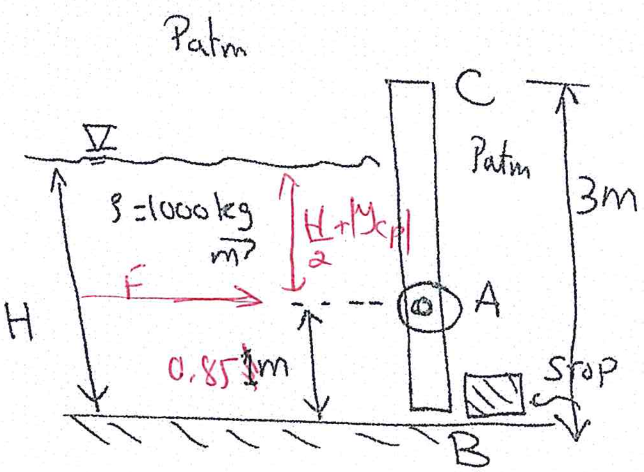
Problem 1: 30% Problem 2: 50% Problem 3: 20%

1- Vertical gate (BC) is 3 m tall and its mass is 200 kg. It has a width into the page of $b =$ 1.6 m. The gate is pinned at point (A) and is free to rotate about it. It is kept from opening by the stop at point B. Find the height of water H at which the gate will begin to open. The distance (AB) is 0.85 m. Water has a density of 1000 kg/m3. Assume no friction or vertical reaction force at (B).

Gate starts to open when

$\Sigma M_A = 0$

Weight of the gate does not contribute to the torque.



$\Rightarrow \Sigma M_A = 0$ when center of pressure $cp = A$ of water acts on A

$$y_{cp} = -\frac{\rho g \sin\theta I_{xx}}{F} \quad ; \quad F = \rho g h_{cG} A$$

$$\Rightarrow F = 9.8\times10^3 \times \frac{H}{2}(H \times \overset{1.6\,m}{b}) = 7.84\times10^3 H^2$$

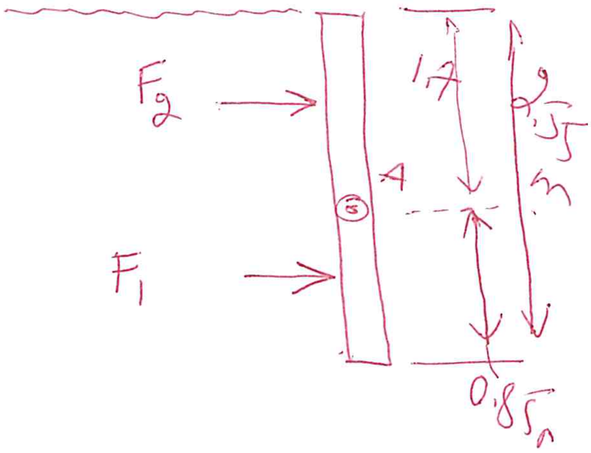
$$y_{cp} = -\frac{\cancel{9.8\times10^3} \sin 90 \times \cancel{b} H^3/12}{\cancel{9.8\times10^3} \times \frac{\cancel{b} H^2}{2}} = -\frac{H}{6}$$

$\Rightarrow$ From drawing: $H - \frac{H}{2} - |y_{cp}| = 0.85\,m$

$$\Rightarrow H - \frac{H}{2} - \frac{H}{6} = 0.85\,m$$

$$\boxed{H = 2.55\,m}$$

Checking Solution



$$F_1 = \rho g H_{CG} A$$
$$= 9.8\times 10^3 \left(1.7 + \frac{0.85}{2}\right)(0.85\times 1.6)$$
$$= 28.322 \text{ kN}$$

$$|y_{cp_1}| = \frac{\rho g \sin\theta \, I_{xx}}{F_1}$$

$$y_{cp_1} = \frac{\cancel{\rho g} \sin 90 \times \cancel{1.6} \times 0.85^3/12}{\cancel{\rho g}(0.85\times \cancel{1.6})\left(1.7+\frac{0.85}{2}\right)} = 0.0283 \text{ m}$$

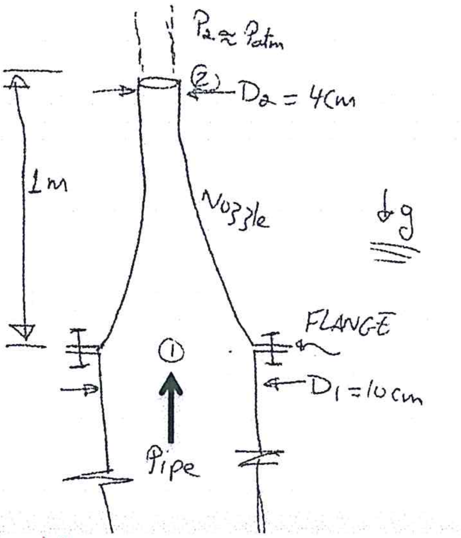
$$F_2 = 9.8\times 10^3 \times \frac{1.7}{2} \times 1.7 \times 1.6 = 22.658 \text{ kN}$$

$$y_{cp_2} = \frac{\cancel{\rho g} \sin 90 \times \cancel{1.6} \times 1.7^{\cancel{3}}/\cancel{12}\,6}{\cancel{\rho g}\left(\cancel{\frac{1.7}{2}}\right)(\cancel{1.7})(\cancel{1.6})} = 0.283 \text{ m}$$

$$\Sigma M_A = F_1 \times L_{1\perp} + F_2 \times L_{2\perp}$$
$$= 28.322 \times \left(\frac{0.85}{2} + 0.0283\right) - 22.658 \times \left(1.7 - \frac{1.7}{2} - 0.283\right)$$
$$= 12838 - 12847 \approx \text{zero}$$

2- The vertical nozzle shown in the figure delivers 36 m3/hr of water to the atmosphere. It has a mass of 35 kg including the water inside it, with D1=10 cm and D2 =4 cm. The nozzle is connected at its base at point 1 to a main pipe through flange and bolts. The friction head loss in the nozzle is estimated to be 1.5 m.

a- Find the gauge pressure of the water at the base of the nozzle p1
b- Find the total force of the flange bolts

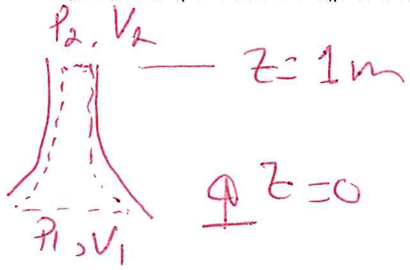


$Q = \frac{36}{3600} = 0.01\ \frac{m^3}{sec}$

$\dot m = \rho Q = 10\ kg/sec$

$V_1 = \frac{Q}{\frac{\pi}{4}D_1^2} = 1.27\ \frac{m}{sec}$

$V_2 = \frac{Q}{\frac{\pi}{4}D_2^2} = 7.96\ \frac{m}{sec}$



(a) CV as shown

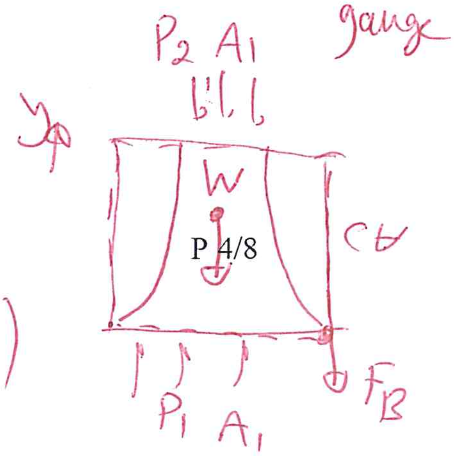
Energy:

$$-h_f = \left(\underbrace{\frac{P}{\rho g}}_{0\ gage} + \frac{V^2}{2g} + z\right)_2 - \left(\frac{P}{\rho g} + \frac{V^2}{2g} + z\right)_1$$

$$-1.5\,m = \left(\frac{7.96^2}{2\times 9.8} + 1\right) - \left(\frac{P_1}{9.8\times 10^3} + \frac{1.27^2}{2\times 9.8} + 0\right)$$

$$-1.5 = 3.23 + 1 - \left(\frac{P_1}{9.8\times 10^3} + 0.08\,m\right) \Rightarrow P_1 = 55.37\,kPa \text{ gauge}$$

(b) CV as shown $\sum F_y = \dot m (V_{y_o} - V_{y_{in}})$



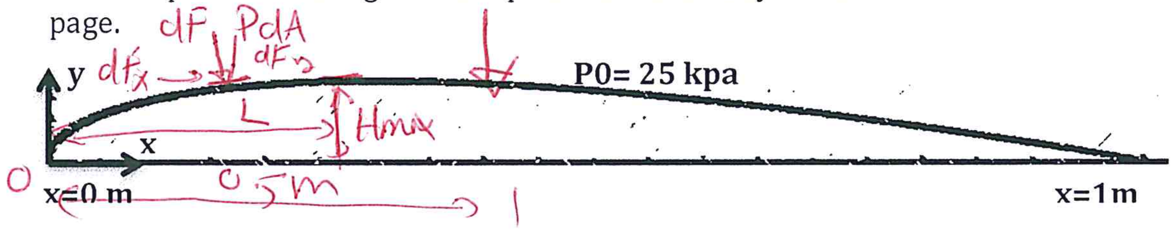
$$P_1 A_1 - P_2 A_1 - W - F_B = \dot m (V_2 - V_1)$$

$$55.37\times 10^3 \times \frac{\pi}{4}(0.1)^2 - 35\times 9.8 - F_B = 10(7.96 - 1.27\,\frac{m}{s})$$

3- The curved body shown in the graph can be described by the equation:

$$y = 5t\left[0.2969\sqrt{x} - 0.1260x - 0.3516x^2 + 0.2843x^3 - 0.1015x^4\right],$$

with t=0.15. Here x extends from 0 to 1 m and y is given in meters. The top side of this curved body is subjected to a uniform gauge pressure p0 of 25 kPa. You are asked to estimate the **location of the center of pressure** associated with this uniform pressure acting on the top surface. The body has a uniform width into the page.



$$L_1 = \frac{\sum M_o}{\sqrt{F_x^2 + F_y^2}}$$

$$F_x = \int_{x=0}^{x=1m} dF_x = \int_{y=0}^{y=0} p\,dy\,b = pb\int_0^0 dy = 0$$

$$F_y = \int_{x=0}^{x=1} pb\,dx = pb\,\Delta x$$

$$\int y\,dF_x = \int y\,pb\,dy = \int_{x=0}^{x=L} y\,pb\,dy + \int_{x=L}^{x=1} y\,pb\,dy = pb\left[\int_0^{H_{max}} y\,dy + \int_{H_{max}}^{0} y\,dy\right] = 0$$

$$\sum M_o = \overbrace{\int y\,dF_x}^{\text{ZERO}} + \int x\,dF_y$$

$$= \int_{x=0}^{x=1} x\,pb\,dx = \frac{x^2}{2}pb$$

$$L_1 = \frac{x^2/2\;pb}{pb\,x} = \frac{x}{2} = 0.5\,m$$