AMERICAN UNIVERSITY OF BEIRUT
Department of Electrical and Computer Engineering
EECE340 Signals and Systems -Summer 2011

Prof Karameh

Problem Set 2

Out: Thursday, June 30, 2011 Due: Thursday, July 7, 2011

Work individually and write your complete solutions on clean paper. .

Problem 1

(a) Using Routh Hurwitz criterion, find the range of the parameter K for which the characteristic polynomial

$$s^3 + s^2 + s + k = 0$$

represents a stable system.

(b) In general the characteristic equation of a third-order system is defined by

$$D(s) = a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0$$

Using R-H criterion, determine all the conditions that a_0, a_1, a_2, and a_3 must satisfy for the system to be stable

(c) Revisit part (a) in light of part (b).

Problem 2

Magnetic Levitation: Consider a system which is designed to suspend a weight in a magnetic field in free air. Such systems are often inherently unstable because the magnetic force on an armature grows rapidly as the gap is reduced. In this problem we will try to design a controller that can achieve such suspension task. As shown in figure 1, let Y_o be the position at which the magnetic force due to the current I_o just balances the gravitational force due to the weight.

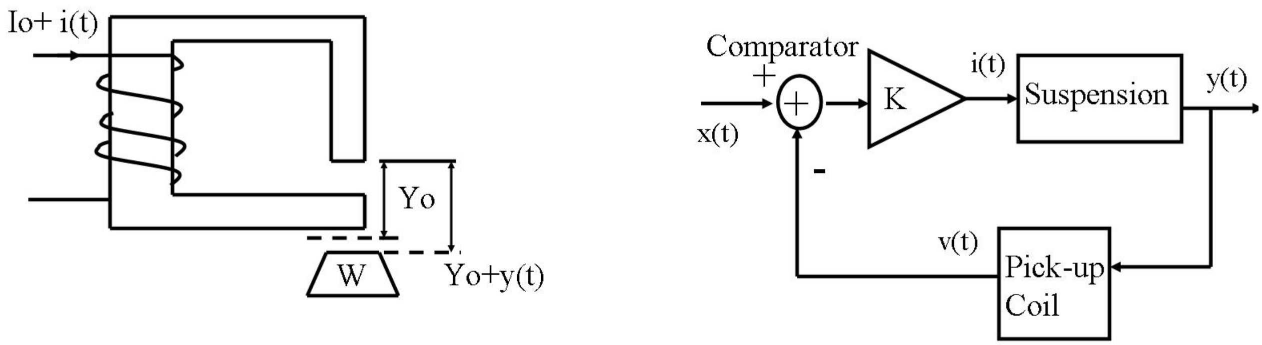

Figure 1: Left: Magnetic Levitation. Right: proposed control

For small changes from this *unstable* equilibrium, assume that the incremental current and position satisfy the following differential equation:

$$\frac{d^2y(t)}{dt} - 4y(t) = -10i(t)$$

To stabilize this system, suppose that we measure the movement from the equilibrium point with a pick up coil whose output is proportional to velocity,

$$v(t) = 2\frac{dy(t)}{dt}$$

We then propose to compare this signal with a desired velocity signal $x(t)$ and drive the coil with the amplified difference as show in figure 1 (right).

(a) show that the system is unstable for any value of K.

(b) The circuit shown in figure 2 is suggested as a controller for the system, to be inserted in the feedback path between the pick-up coil and the comparator. Determine the transfer function $\frac{V_c(s)}{V(s)}$ using assumptions of an ideal Op-amp (zero input current and V at + and − terminals are equal).

(c) Compute the system function for the suspension system with the compensator in the feedback loop.

(d) Find the range of values of K for which the controlled system is stable.

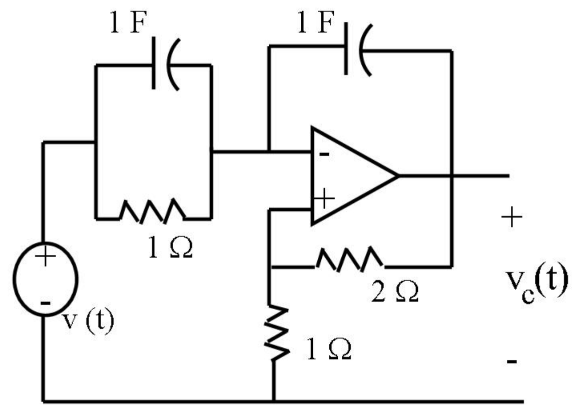

Figure 2: Circuit for control of magnetic levitation (ideal Op-amp)

Problem 3

Consider the feedback system shown in figure 3.

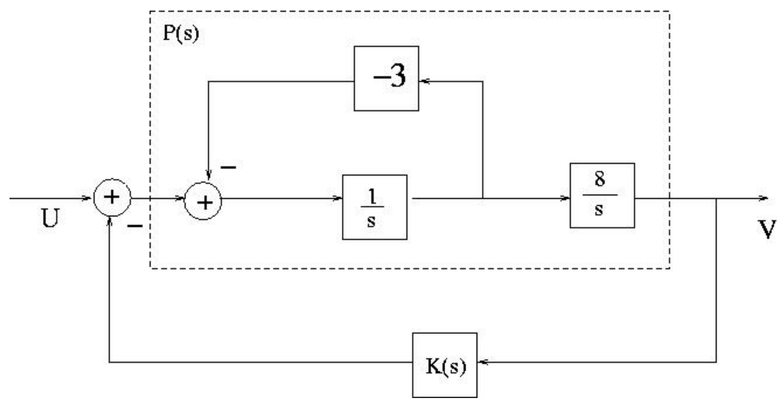

Figure 3: simple control scheme

(a) Is the plant $P(s)$ stable? explain.

(b) If $K(s) = K_o$, find the range of K_o for which this overall system is stable. This is called proportional control.

(c) If $K(s) = K_o + K_1 s$, find the range of K_o, K_1 for which the overall system is stable. This is called proportional derivative control (PD).

Problem 4

Homer Simpson is having trouble reducing his weight with all the donuts he eats. His friend Mo is trying to help him loose weight by going on a strict exercise program. Mo and Homer figured out the following.

A- Homer's weight tends to *increase* in proportion to his current weight x_1 and to *decrease* with the increase in his ability to exercise x_2 (Assume all proportionality factors here are unity).

B- Homer's exercise ability x_2 *decreases* in proportion to *twice* his current weight.

In the following questions, we will observe Homer's weight progress.

a) a-1: Write down the differential equations which describe the dynamics of his weight increase x_1 and his exercise ability x_2.

a-2: Draw a block diagram representation of the ordinary differential equation governing x_1 with a minimum number of integrators.

b) Find the weight of Homer for all time as his exercise program progresses. Assume that his weight initially was $x_1(0) = 120$ Kg and the rate $x^{'}(0) = 0$. Is there a problem with this exercise plan?

c) Lisa Simpson observed how her father's weight is progressing and decided to help him. In addition to the earlier exercise dynamics (B above), she noticed that the rate of his exercise ability $x_2^{'}$ will increase in direct proportion to its current state x_2. Therefore, she decided to give him some reward to boost this ability (such as watching TV at the end of each exercise). In this case, the rate $x_2^{'}$ will be increasing at a steady constant rate R. That is,

$$x_2^{'} = -2x_1 + k_2 x_2 + k_1 x_1 + Ru(t)$$

Could this scheme work? If so, find the constants R, k_1 and k_2 such that his weight stabilizes at half its original, that is, $x_1 = \frac{1}{2}(x_1(0)) = 60$ Kg.

Problem 5

Consider the linear difference equation describing a DT LTI system:

$$y[n] + \frac{2}{9}y[n-2] - y[n-1] = x[n] - 2x[n-1]$$

a) Find a block diagram realization of this system using *a minimum number* of delays.

b) Find the zero state response of this system if $x[n] = u[n] - u[n-2]$.

c) Find the output of this system if the input is $x[n] = (-2)^n$, $\forall n$.

Problem 6

Determine the homogeneous solution for each of the systems described by the following linear difference equations:

(a) $y[n] - \alpha y[n-1] = 2x[n] + 3x[n-5]$

(b) $y[n] + y[n-1] + \frac{1}{4}y[n-2] = x[n] + 2x[n-1]$

Problem 7

Determine the particular solution for the system described by the following linear difference equation for the given inputs:

$$y[n] - \frac{2}{5}y[n-1] = 2x[n]$$

i) $x[n] = 2u[n]$

ii) $x[n] = -(\frac{1}{2})^n u[n]$