AMERICAN UNIVERSITY OF BEIRUT
Department of Electrical and Computer Engineering
EECE340- Spring 2011
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Problem 1

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

a- $$\frac{dy_1(t)}{dt} + 2y_1(t) = e^{2t}u(t)$$

Laplace Transform

$$\Rightarrow sY_1(s) - y_1(0) + 2Y_1(s) = \frac{1}{s-2}$$

$$\Rightarrow Y_1(s) = \frac{1}{(s-2)(s+2)} = \frac{1/4}{s-2} - \frac{1/4}{s+2}$$

$$\Rightarrow y_1(t) = \left(\frac{1}{4}e^{2t} - \frac{1}{4}e^{-2t}\right)u(t)$$

b- $$x_2(t) = [\gamma e^{2t} + e^{5t}]u(t)$$

$$\Rightarrow y_1'(t) + 2y(t) = (\gamma e^{2t} + e^{5t})u(t)$$

Laplace Transform

$$\Rightarrow sY_2(s) - Y_2(0) + 2Y_2(s) = \frac{\gamma}{s-2} + \frac{1}{s-5}$$

$$Y_2(s) = \frac{\gamma}{(s-2)(s+2)} + \frac{1}{(s-5)(s+2)} \cancel{\frac{\gamma/4}{s-2}}$$

$$= \frac{\gamma/4}{s-2} + \frac{-\gamma/4 - 1/7}{s+2} + \frac{1/7}{s-5}$$

$$\Rightarrow y_2(t) = \left[\frac{\gamma}{4}e^{2t} + \left(-\frac{\gamma}{4} - \frac{1}{7}\right)e^{-2t} + \frac{1}{7}e^{5t}\right]u(t)$$

c- $$x_3(t) = ke^{2(t-T)}u(t-T)$$

$$\frac{dy_3(t)}{dt} + 2y_3(t) = ke^{2(t-T)}u(t-T)$$

$$\Rightarrow sY_3(s) + y_3(0) + 2Y_3(s) = \frac{ke^{-sT}}{s-2}$$

$$Y_3(s) = \frac{ke^{-sT}}{(s+2)(s-2)} = -\frac{ke^{-sT}}{4(s+2)} + \frac{ke^{-sT}}{4(s-2)}$$

$$y_3(t) = \left[-\frac{k}{4}e^{-2(t-T)} + \frac{k}{4}e^{2(t-T)}\right]u(t-T)$$

from question (a)

$$y_1(t-T) = \left[\frac{1}{4}e^{2(t-T)} - \frac{1}{4}e^{-2(t-T)}\right]u(t-T)$$

for $k=1$, $y_3(t) = y_1(t-T)$

Problem 2

a.

$$v(t) = x_2(t) \quad (3)$$

$$\frac{dx_2(t)}{dt} = -\frac{7}{2}x_2(t) + \frac{1}{2}x_1(t) \quad (2)$$

$$\frac{dx_1(t)}{dt} = -6x_2(t) + u(t) \quad (1)$$

b. Laplace Transform to (1) and (2) and (3)

let $x_1(0) = x_1(0)$ and $x_2(0) = x_2(0)$

$$\begin{cases} s X_1(s) - x_1(0) = X(s) - 6X_2(s) \\ s X_2(s) - x_2(0) = \frac{1}{2}X_1(s) - \frac{7}{2}X_2(s) \end{cases}$$

$$V(s) = X_2(s)$$

$$\begin{cases} s X_1(s) + 6X_2(s) = X(s) + x_1(0) \\ -\frac{1}{2}X_1(s) + (s + \frac{7}{2})X_2(s) = x_2(0) \end{cases}$$

$$\Rightarrow X_2(s) = \frac{1}{2(s+2)(s+\frac{3}{2})}X(s) + \frac{\frac{1}{2}x_1(0) + s x_2(0)}{(s+2)(s+\frac{3}{2})}$$

since $V(s) = X_2(s)$

then $$V(s) = \frac{1}{2(s+2)(s+\frac{3}{2})}X(s) + \frac{\frac{1}{2}x_1(0) + s x_2(0)}{(s+2)(s+\frac{3}{2})} = Y(s)$$

c. $x(t) = 5\,u(t) \Rightarrow X(s) = \frac{5}{s}$

$x_1(0) = x_1(0) = 4 \qquad x_2(0) = -0.5 = x_2(0)$

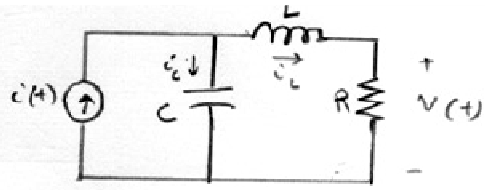
$Y(s)$ becomes

$$Y(s) = \frac{5/s}{2(s+2)(s+\frac{3}{2})} + \frac{\frac{1}{2}\times 4 - 0.5s}{(s+2)(s+\frac{3}{2})}$$

$$Y(s) = \frac{5/6}{s} - \frac{7/2}{s+2} + \frac{13/6}{s+\frac{3}{2}}$$

$$\Rightarrow y(t) = \frac{5}{6} - \frac{7}{2}e^{-2t} + \frac{13}{6}e^{-\frac{3}{2}t}$$

Problem 3



(a) WRITING

$$i_L(t) = i(t) - i_c(t)$$

$$= i(t) - C\frac{dv_c(t)}{dt}$$

AND $v_c(t) = L\frac{di_L(t)}{dt} + R i_L(t)$

THEN: $i_L(t) = i(t) - LC\frac{d^2}{dt^2} i_L(t) - RC\frac{d}{dt} i_L(t)$

OR: $LC\frac{d^2}{dt^2} i_L + RC\frac{d}{dt} i_L + i_L = i(t)$

TAKING LAPLACE XFORM:

$$LC\left(s^2 I_L(s) - s\, i_L(0) - i_L'(0)\right) + RC\left(s I_L(s) - i_L(0)\right) + I_L(s) = I(s)$$

$$\Rightarrow \quad I_L(s) = \frac{1}{LCs^2 + RCs + 1} \cdot I(s) + \frac{(LCs + RC)\, i_L(0) + LC\, i_L'(0)}{LCs^2 + RCs + 1}$$

$$V(s) = R\, I_L(s)$$

$\Rightarrow$ SYSTEM FUNCTION $H(s) = \frac{V(s)}{I(s)} = \frac{R}{LCs^2 + RCs + 1}$

(b) TO OBTAIN ZIR

WE HAVE : $i_L(0)=1$ (GIVEN) , $V_C(0)=4$, $R=4$, $L=1$

AND $V_C(0) = L\, i_L'(0) + R\, i_L(0)$

$\Rightarrow \quad 4 = i_L'(0) + 4 \Rightarrow i_L'(0) = 0$

FROM PART (a) , RECALL THE ZIR FOR $I_L(s)$

$\Rightarrow$ ZIR of $V(s) = R \times$ ZIR of $I(s)$

$$= \frac{4\left[\left(\frac{1}{4}s+1\right)\cdot i_L(0) + \frac{1}{4} i_L'(0)\right]}{\frac{1}{4}s^2+s+1}$$

$$\Rightarrow \quad V_{ZIR}(s) = \frac{4(s+4)}{s^2+4s+2} = 4\,\frac{(s+4)}{(s+2)^2}$$

$$= 4\left(\frac{1}{s+2} + \frac{2}{(s+2)^2}\right)$$

$$\Rightarrow \quad V_{ZIR}(t) = 4e^{-2t} + 8te^{-2t} \qquad t>0$$

(c) NOTE THAT THE INPUT CAN BE WRITTEN AS

$i(t) = t\,u(t) - (t-1)\,u(t-1) - (t-4)\,u(t-4) + (t-5)\,u(t-5$

THUS WE HAVE TO SOLVE FOR ONLY THE RESPONSE TO $i_1(t) = t\,u(t)$ AND WE CAN EASILY GET THE TOTAL SOLUTION. THAT is

$i(t) = i_1(t) + i_2(t) + i_3(t) + i_4(t)$

$= i_1(t) - i_1(t-1) - i_1(t-4) + i_1(t-5)$

LET $v_1(t)$ BE SOLUTION FOR SYSTEM WHEN $i_1(t)$ IS APPLIED. THEN:

$$v(t) = \text{TOTAL SOLUTION} = v_1(t) - v_1(t-1) - v_1(t-4) + v_1(t-5)$$

RECALL THAT $H(s) = \dfrac{R}{LCs^2 + RCs + 1}$

FOR GIVEN $\Rightarrow$ $H(s) = \dfrac{16}{s^2 + 4s + 4}$

$$i_1(t) = t\,u(t) \overset{\mathcal{L}}{\Rightarrow} I_1(s) = \frac{1}{s^2}$$

THEN $V_1(s) = H(s)\,I_1(s) = \dfrac{16}{s^2(s^2+4s+4)} = \dfrac{16}{s^2(s+2)^2}$

$$V_1(s) = \frac{A}{(s+2)^2} + \frac{B}{(s+2)} + \frac{C}{s^2} + \frac{D}{s}$$ (PARTIAL FRACTION EXPANSION)

$$A = \frac{16}{4} = 4 \;;\quad B = \left[16\,\frac{(-2s)}{s^4}\right]_{s=-2} = 4$$

$$C = \frac{16}{4} = 4 \;;\quad D = \left[-16\,\frac{(s+2)(2)}{(s+2)^4}\right]_{s=0} = -4$$

$$\Rightarrow V_1(s) = 4\left[\frac{1}{(s+2)^2} + \frac{1}{(s+2)} + \frac{1}{s^2} - \frac{1}{s}\right]$$

AND $v_1(t) = 4\left(te^{-2t} + e^{-2t} + t - 1\right)u(t)$

$$\therefore\; v(t) = v_1(t) - v_1(t-1) - v_1(t-4) + v_1(t-5)$$

$$\begin{aligned} v(t) = {} & 4\left[te^{-2t} + e^{-2t} + t - 1\right]u(t) \\ & - 4\left[(t-1)e^{-2(t-1)} + e^{-2(t-1)} + t - 2\right]u(t-1) \\ & - 4\left[(t-4)e^{-2(t-4)} + e^{-2(t-4)} + t - 5\right]u(t-4) \\ & + 4\left[(t-5)e^{-2(t-5)} + e^{-2(t-5)} + t - 6\right]u(t-5) \end{aligned}$$

Problem 4

FROM EXPERIMENT 1:

$$x(t) = e^{-4t} \quad t>0$$

$$\Rightarrow y(t) = \underbrace{5e^{-4t}}_{H(-4)e^{-4t}} + \underbrace{k_0 e^{-t} + k_1 t e^{-t}}_{\text{natural frequencies}} \quad t>0$$

$$\Rightarrow H(4) = 5 \text{ and } H(s) \approx \frac{C(s)}{(s+1)^2}$$

For some $C(s)$

FROM EXPERIMENT 2:

$$x(t) = e^{-2t} \Rightarrow y(t) = \underbrace{k_2 e^{-t} + k_3 t e^{-t}}_{\text{natural frequencies}} + \underbrace{0 . e^{-2t}}_{H(-2)=0}$$

since e^{-2t} does not appear in output

$\Rightarrow s = -2$ is a zero of $H(s)$

$$\therefore H(s) = \frac{k(s+2)}{(s+1)^2}$$

$$H(-4) = 5 \Rightarrow k\frac{(-4+2)}{(-4+1)^2} = 5 \Rightarrow k\frac{(-2)}{(-3)^2} = 5$$

$$\Rightarrow k = -22.5$$

Thus, $H(s) = \dfrac{-22.5(s+3)}{(s+1)^2}$

Problem 5

$X(t) = A\cos(w_0 t + \theta) \Rightarrow y(x) = ?$

$X(t) = (A/2)e^{j\theta}e^{jw0t} + (A/2)e^{-j\theta}e^{-jw0t}$

$H(jw) = |H(jw)|e^{j\Phi H}$

$Y(S) = H(S)X(S)$

According to the eigen property e^{jwot} into $H(s)$ gives $H(jwo)e^{(jwot)}$

$\Rightarrow y(t) = (A/2)|H(jw_0)|\, e^{(jwot)}e^{j\theta}\, e^{j\Phi H(w0)} + (A/2)|H(-jw_0)|\, e^{(-jwot)}e^{-j\theta}\, e^{-j\Phi H(w_0)}$

$= A|H(jw_0)|\cos(w_0 t + \theta + \Phi_H(w_0))$

Problem 6:

```
% first define the numerator and denominator polynomials
   num=[100]; den=[1 15 10];

% then define the system
   sys=tf(num,den);

% open a new figure
   figure(1);
% perform and plot step response. Add title. Set the axis of the figure

   step(sys); title('Figure 1: Step response of the system');axis([0 10 0 11]);

%now create the input vector for g(t) and the time vector

   g=[ ones(1,4001) -ones(1,6000)];
   t=0:0.01:100;

   figure(2)

% now simulate the system with input g(t)
   lsim(sys,g,t);title('Figure 2: Response of system to g(t)');
axis([0 100 -10.5 10.5]);


%now lets create another system with an underdamped or oscillatory response
   num1=[100]; den1=[1 2 50];
   sys1=tf(num1,den1);
   figure(3);
   lsim(sys1,g,t);title('Figure 3: Response of an oscillatory system to
g(t)');axis([0 100 -6.6 6.5]);

% finally we can create a system with a sluggish of overdamped response

   num1=[100]; den1=[1 105 20];
   sys1=tf(num1,den1);
%here we are gonna store the output vector and time of the simulation, and
%subsequently plot with some more options

   [yout,tout]=lsim(sys1,g,t);
   figure(4);
      subplot(211); % split the figure into two (211= 2 by 1, 1st one)
      plot(tout,yout, 'k', t,g,'-.r');
      title('Figure 4a: Response of an overdamped system to g(t)');
      axis([0 100 -5.5 5.5]);
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```
[ystep,tstep]=step(sys1);
 subplot(212);
  plot(tstep,ystep);
  title('Figure 4b: Step Response of an overdamped system ');
  axis([0 30 0 5.5]);
% add some more info to the last plot
    xlabel('time (sec)'); ylabel('y(t)');
```

OUTPUT:

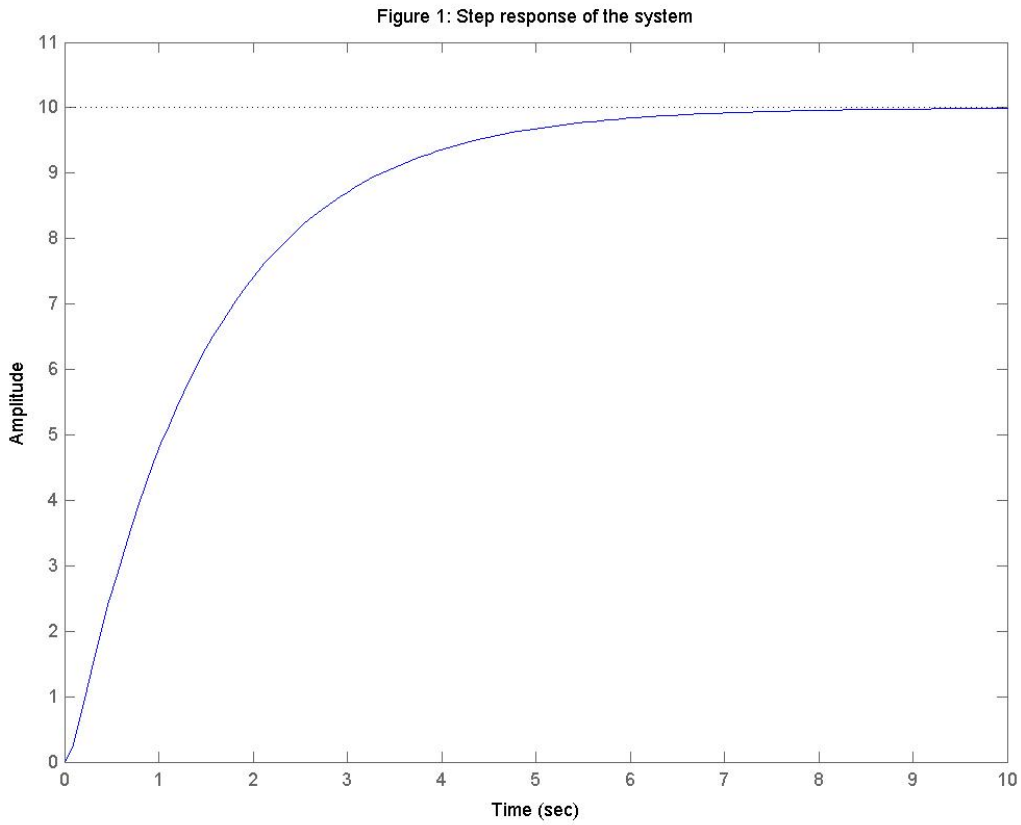

Figure 2: Response of system to g(t)

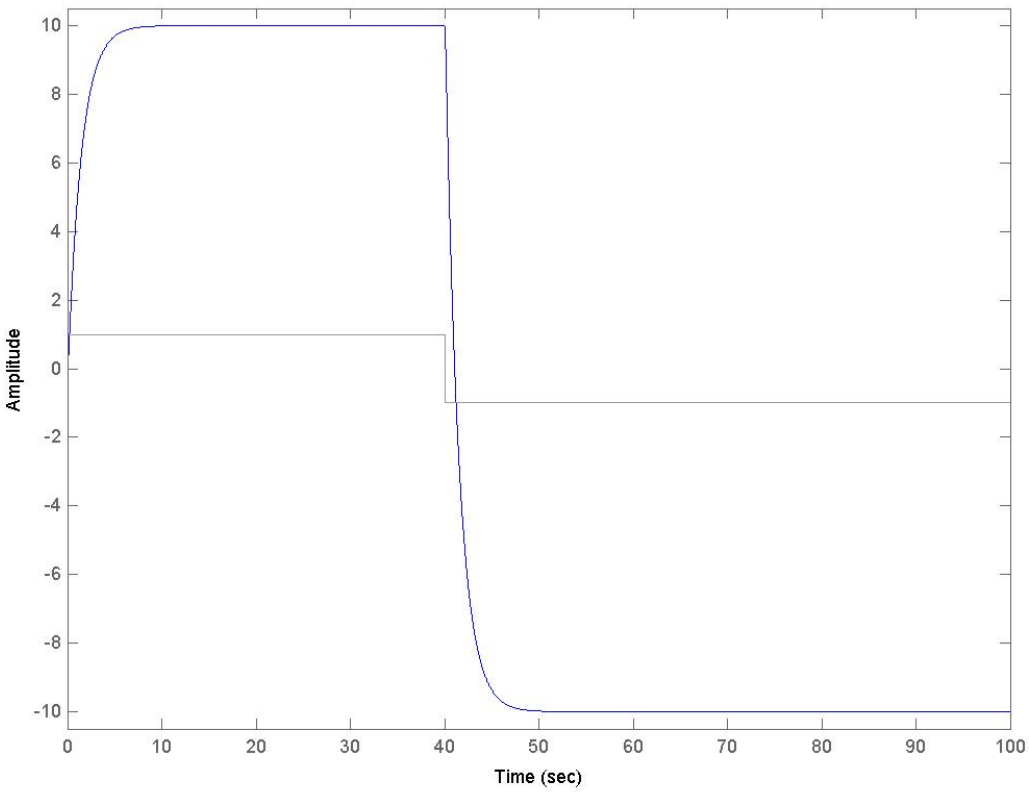

Figure 3: Response of an oscillatory system to g(t)

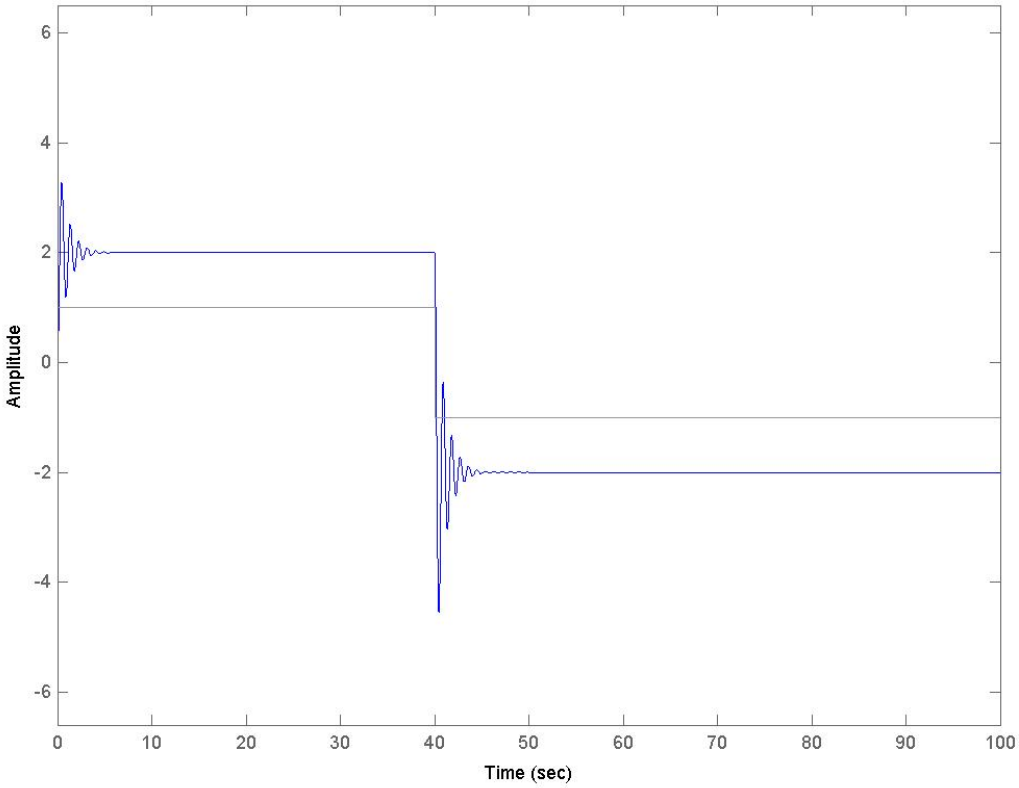

Figure 4a: Response of an overdamped system to g(t)

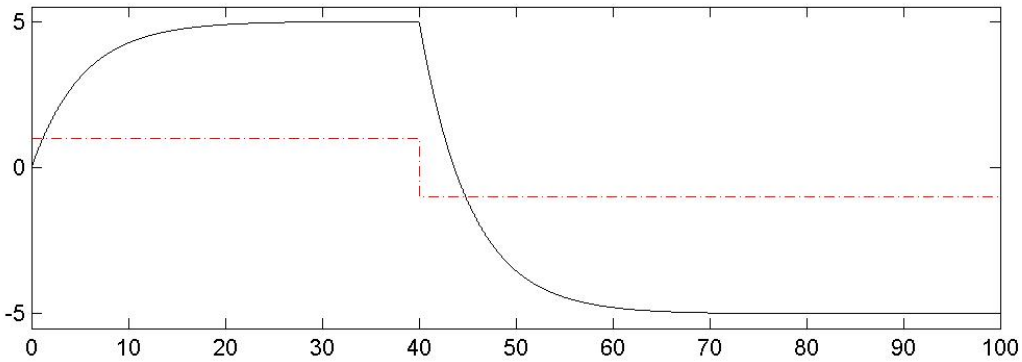

Figure 4b: Step Response of an overdamped system